May 27th, 1910. Vol. V., No. 1.

東京帝國大學
工科大學紀要

第五冊第壹號

明治四十三年五月

JOURNAL

OF THE

COLLEGE OF ENGINEERING,

IMPERIAL UNIVERSITY OF TOKYO.

TOKYO.

Published by the University.

43RD YEAR OF MEIJI.

(1910.)

EDITING COMMITTEE.

All communications relating to this Journal should be addressed to the Chairman of the Committee.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
VOL. V. No. 1.

Published May, 1910.

On Vibration of Steamers.*

By

Prof. Seinen Yokota, *Kōgakushi, Kōgakuhakushi.*

With Plates 1 to IX.

Article I.

General Consideration.

This article and the next were written mainly in 1903.

The problem in them is, for simplicity sake, limited to the vertical vibrations of hull, that is, the lateral vibrations about a horizontal plane, (but of course similar reasoning applies to the horizontal vibrations for a first approximation, although a ship is generally devoid of symmetry in this latter case).

On this understanding, I consider a ship as a free-free bar symmetrical about the centre line plane and treat her as subjected to forced vibrations under the influence of unbalanced forces of engines (both main and auxiliary) and propellers, but not taking account of wave effects of the surrounding water.

For the general theory of vibration, I follow the course in Lord Rayleigh's theory of sound and use his notations.

Denoting the potential energy V of the strained state and the kinetic energy T of a vibrating system in terms of normal coordinates, we have

$$\left.\begin{aligned} V &= \tfrac{1}{2}c_1\varphi_1^2 + \tfrac{1}{2}c_2\varphi_2^2 + \cdots\cdots, \\ T &= \tfrac{1}{2}a_1\dot{\varphi}_1^2 + \tfrac{1}{2}a_2\dot{\varphi}_2^2 + \cdots\cdots, \end{aligned}\right\} \qquad (1)$$

where $\varphi_1, \varphi_2, \ldots$ are the normal coordinates and $\dot{\varphi}_1, \dot{\varphi}_2, \ldots$ denote the differential coefficients of $\varphi_1, \varphi_2, \ldots$ with respect to time, and $c_1, c_2, \ldots$ $a_1, a_2, \ldots$ are all constants, provided the displacements are small.

Next, I assume that the dissipative function F is also expressible in the form

$$F = \tfrac{1}{2}b_1\dot{\varphi}_1^2 + \tfrac{1}{2}b_2\dot{\varphi}_2^2 + \ldots\ldots, \} \qquad (2)$$

where $b_1, b_2, \ldots$ are constants. This function does not include the effect of the surrounding water, but simply the effects as commonly allowed for the vibrations in the air.

Then the Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_r}\right) - \frac{\partial T}{\partial \varphi_r} + \frac{\partial F}{\partial \dot{\varphi}_r} + \frac{\partial V}{\partial \varphi_r} = \Phi_r, \qquad r = 1, 2, \ldots$$

where t is the time and Φ_r is the component of external forces in the direction of φ_r, give for the equations of motion

$$a_r\ddot{\varphi}_r + b_r\dot{\varphi}_r + c_r\varphi_r = \Phi_r, \qquad (r = 1, 2, \ldots\ldots) \ \ldots\ldots \ (3)$$

The general solutions of the above equations are

$$\varphi_r = A_r e^{-\frac{1}{2}\varkappa_r t}\cos(n'_r t - B_r)$$
$$+ \frac{1}{a_r n'_r}\int^t e^{-\frac{\varkappa_r}{2}(t-t')}\sin n'_r(t-t')\Phi'_r dt', \quad r = 1, 2, \ldots\ldots \ (4)$$

where

$$\varkappa_r = \frac{b_r}{a_r}, \qquad n_r^2 = \frac{c_r}{a_r}, \qquad n_r' = (n_r^2 - \tfrac{1}{4}\varkappa_r^2)^{\frac{1}{2}},$$

and Φ_r' is the value of Φ_r at time t'.

The lower limit of the integral of the last term must be taken so that the integrated expression vanishes at that limit. A_r and B_r are arbitrary constants.

In the case of a steamer, Φ_r are the forces due to the unbalanced forces of engines and propellers, and these may be expressed in terms of harmonic functions by Fourier's theorem such as $E_1 \cos(p_1 t - \varepsilon_1)$, $E_2 \cos(p_2 t - \varepsilon_2)$, &c., acting at points x_1, x_2,... of the ship; of these some may coincide when the forces Φ_r are not all strictly harmonic.

Thus, for a reciprocating engine moving at a uniform angular velocity, the force may be decomposed into a harmonic force of the same period as the time of one revolution and its even upper harmonics, and for a propeller into harmonic forces of periods equal to the period of one revolution divided by the integral multiples of the number of blades.

The above consideration includes the couples caused by any set of unbalanced forces.

Hence the general expression for Φ_r is

$$\Phi_r = \sum_1^s \int u_r E_s \cos(p_s t - \varepsilon_s) dx$$

$$= \sum_1^s u_r(x_s) E_s \cos(p_s t - \varepsilon_s), \qquad s = 1, 2, \ldots\ldots\ldots\ldots \quad (5)$$

where u_r is the normal function corresponding to φ_r, and $u_r(x_s)$ denotes the value of u_r at the point x_s.

Substituting this value in (4) and working out the necessary calculations, we have the well known expression

$$\left.\varphi_r = \sum_1^s \frac{u_r(x_s) E_s}{a_r \sqrt{(n_r^2 - p_s^2)^2 + \varkappa_r^2 p_s^2}} \cos(p_s t - \varepsilon_s - \theta_s),\right\} \ldots\ldots\ldots \quad (6)$$

where $\quad \tan \theta_s = \dfrac{\varkappa_r p_s}{n_r^2 - p_s^2}$.

In the above expression for φ_r, the term containing $e^{-\frac{1}{2}\varkappa_r t}$ is

omitted, as we have to deal with vibrations in the stationary state.

Hence we see from (6), that the period of a forced vibration must be the same as that of the impressed force; *i.e.*, a harmonic force can only excite vibrations having their period equal to that of the force, and can never excite vibrations with periods different from that of the force either shorter or longer.

The amplitude of vibration of each period is directly proportional to the magnitude of the impressed force, and also to the value of the normal function at that point, namely, at the point at which the impressed force acts. It is inversely proportional to the mass corresponding to φ_r and depends upon the relative periods of the force and the mode. If the point of application of the force be at a node, $u_r=o$, and consequently φ_r is independent of that force, showing that a force acting at a node in any mode of vibration cannot generate that mode, however great be the magnitude of that force. If $n_r=p_s$, the amplitude becomes a maximum; synchronism is established.

To find the normal functions, I have proceeded as follows:

The potential energy of bending a ship is

$$V=\tfrac{1}{2}\int qk^2\omega\frac{ds}{R^2}$$

integrated over the whole length of the ship; where q is the Young's modulus; k, the radius of gyration of the variable section, ω, the sectional area; R, the radius of curvature; and ds, a length element of the neutral axis.

Or approximately

$$V=\tfrac{1}{2}\int qk^2\omega\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

if we take for the x-axis the neutral axis of the ship supposed straight at its position of equilibrium. The expression (7) assumes that the neutral axis of ship remains always nearly horizontal, and the square of the tangent of inclination of that axis at any instant to the x-axis is small compared to 1.

The kinetic energy is

$$T=\tfrac{1}{2}\int\rho\omega\left(\frac{\partial y}{\partial t}\right)^2dx+\tfrac{1}{2}\int\rho k^2\omega\left(\frac{\partial}{\partial t}\frac{\partial y}{\partial x}\right)^2dx, \quad\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where ρ is the mass of unit volume. The second term of the right-hand member denotes the effect due to rotatory inertia.

Then

With the Compliments of the Director of the Engineering College.

$$\frac{\partial^2}{\partial x^2}\left(qk^2\omega\frac{\partial^2 y}{\partial x^2}\right)+\rho\omega\frac{\partial^2 y}{\partial t^2}-\frac{\partial}{\partial x}\left(\rho k^2\omega\frac{\partial^3 y}{\partial t^2\partial x}\right)=0, \quad\ldots\ldots\ldots (9)$$

and at both ends,

$$\left.\begin{aligned}&\rho k_2\omega\frac{\partial^3 y}{\partial t^2\partial x}-\frac{\partial}{\partial x}\left(qk^2\omega\frac{\partial^2 y}{\partial x^2}\right)=0,\\&\frac{\partial^2 y}{\partial x^2}=0.\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Suppose $y=\text{Const.}\times u\cos nt$, where u is a function of x only, then we have from (9),

$$\frac{d^2}{dx^2}\left(qk^2\omega\frac{d^2u}{dx^2}\right)-\rho\omega n^2u+n^2\frac{d}{dx}\left(\rho k^2\omega\frac{du}{dx}\right)=0. \quad\ldots\ldots\ldots\ldots (9')$$

If we neglect the part of the kinetic energy due to the effect

omitted, as we have to deal with vibrations in the stationary state.

Hence we see from (6), that the period of a forced vibration must be the same as that of the impressed force; *i.e.*, a harmonic force can only excite vibrations having their period equal to that of the force, and can never excite vibrations with periods different from that of the force either shorter or longer.

The amplitude of vibration of each period is directly proportional to the magnitude of the impressed force, and also to the value of the normal function at that point, namely, at the point at which the impressed force acts. It is inversely proportional to

To find the normal functions, I have proceeded as follows: The potential energy of bending a ship is

$$V=\tfrac{1}{2}\int qk^2\omega\frac{ds}{R^2}$$

integrated over the whole length of the ship; where q is the Young's modulus; k, the radius of gyration of the variable section, ω, the sectional area; R, the radius of curvature; and ds, a length element of the neutral axis.

Or approximately

$$V=\tfrac{1}{2}\int qk^2\omega\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

if we take for the x-axis the neutral axis of the ship supposed straight at its position of equilibrium. The expression (7) assumes that the neutral axis of ship remains always nearly horizontal, and the square of the tangent of inclination of that axis at any instant to the x-axis is small compared to 1.

The kinetic energy is

$$T=\tfrac{1}{2}\int\rho\omega\left(\frac{\partial y}{\partial t}\right)^2dx+\tfrac{1}{2}\int\rho k^2\omega\left(\frac{\partial}{\partial t}\frac{\partial y}{\partial x}\right)^2dx,\dots\dots\dots\dots\dots\dots \quad (8)$$

where ρ is the mass of unit volume. The second term of the right-hand member denotes the effect due to rotatory inertia.

Then,

$$\delta(V+T)=\int\left\{\frac{\partial^2}{\partial x^2}\left(qk^2\omega\frac{\partial^2 y}{\partial x^2}\right)+\rho\omega\frac{\partial^2 y}{\partial t^2}-\frac{\partial}{\partial x}\left(\rho k^2\omega\frac{\partial^3 y}{\partial t^2\partial x}\right)\right\}\delta y\,dx$$
$$+\left\{\rho k^2\omega\frac{\partial^3 y}{\partial t^2\partial x}-\frac{\partial}{\partial x}\left(qk^2\omega\frac{\partial^2 y}{\partial x^2}\right)\right\}\delta y+qk^2\omega\frac{\partial^2 y}{\partial x^2}\delta\left(\frac{\partial y}{\partial x}\right)=0.$$

Therefore, since δy and $\delta\left(\frac{\partial y}{\partial x}\right)$ are both arbitrary, we must have, throughout the range of integration

$$\frac{\partial^2}{\partial x^2}\left(qk^2\omega\frac{\partial^2 y}{\partial x^2}\right)+\rho\omega\frac{\partial^2 y}{\partial t^2}-\frac{\partial}{\partial x}\left(\rho k^2\omega\frac{\partial^3 y}{\partial t^2\partial x}\right)=0,\dots\dots\dots \quad (9)$$

and at both ends,

$$\left.\begin{aligned}&\rho k_2\omega\frac{\partial^3 y}{\partial t^2\partial x}-\frac{\partial}{\partial x}\left(qk^2\omega\frac{\partial^2 y}{\partial x^2}\right)=0,\\ &\frac{\partial^2 y}{\partial x^2}=0.\end{aligned}\right\}\dots\dots\dots\dots\dots\dots\dots\dots \quad (10)$$

Suppose $y=\text{Const.}\times u\cos nt$, where u is a function of x only, then we have from (9),

$$\frac{d^2}{dx^2}\left(qk^2\omega\frac{d^2u}{dx^2}\right)-\rho\omega n^2u+n^2\frac{d}{dx}\left(\rho k^2\omega\frac{du}{dx}\right)=0.\ \dots\dots\dots\dots \quad (9')$$

If we neglect the part of the kinetic energy due to the effect

of rotatory inertia, the equation (9′) becomes simpler and takes the form

$$\frac{d^2}{dx^2}\left(qk^2\omega\frac{d^2u}{dx^2}\right)=\rho wn^2u \qquad (9'')$$

Also then the conditions (10) become

$$\frac{d^3u}{dx^3}=0, \qquad \frac{d^2u}{dx^2}=0. \qquad (10')$$

Neglecting the effect of the rotatory inertia may sometimes be considered serious in such cases as ships with military tops, &c., but my present object being to find out the normal functions to the first order of approximations, I started with the equations (9″) and the conditions (10′) at both ends.

Although the equation (9″) cannot generally be solved in analytical form for a complex figure as a ship, we may solve it graphically with sufficient approximation by means of mechanical integration found out by Lord Kelvin. (See Thomson & Tait-Natural Philosophy Part I, p. 500.)

He used his brother's disc-, globe-, and cylinder- integrators, but I substituted for them Abdank Abakanowicz's integraphs, as the latter are more familiar to us, the naval architects. (See Abdank Abakanowicz-Les integraphes, la courbe integrale et ses applications. See also Pollard et Dudebout—Theorie du navire. Tome 1, chapitre III.)

Take four integraphs and connect the recording point of the first so as to give a vertical motion, namely, a motion perpendicular to the base line, equal to its own to the tracing point of the second.

Similarly connect the recording point of the second with the tracing point of the third and so on up to the fourth. Let y, y_1,

y_2, y_3 be the distances of the tracing points of the first, second,… ……integraphs from respective base lines at any time. Let infinitesimal horizontal motions $(\rho\omega n^2)dx$, dx, $\frac{dx}{qk^2\omega}$, dx be given simultaneously to the first, second,………tracing points respectively. The vertical motions of the recording points thus produced are

$$dy_1=y(\rho\omega n^2)dx, \qquad dy_2=y_1dx, \qquad dy_3=\frac{y_2dx}{qk^2\omega}, \qquad dy_4=y_3dx.$$

Therefore,

$$(\rho\omega n^2)y=\frac{d^2}{dx^2}(qk^2\omega\frac{d^2y_4}{dx^2}).$$

Now couple the last recording point with the first tracing point, so that their vertical motions shall be equal—that is to say, $y=y_4$. Then putting u to denote the common value of these variables, we have

$$\frac{d^2}{dx^2}\left(qk^2\omega\frac{d^2u}{dx^2}\right)=\rho\omega n^2u,$$

namely, the curve recorded by the recording point of the last integraph is the graphic solution of the equation (9″).

Moreover, as we may choose quite arbitrarily the initial values of the four quantities y, y_1, y_2, y_3, the above solution is the most general solution.

A machine fulfilling the above conditions may be constructed, but as I had only one integraph available, I used a graphical integration according to the above principle.

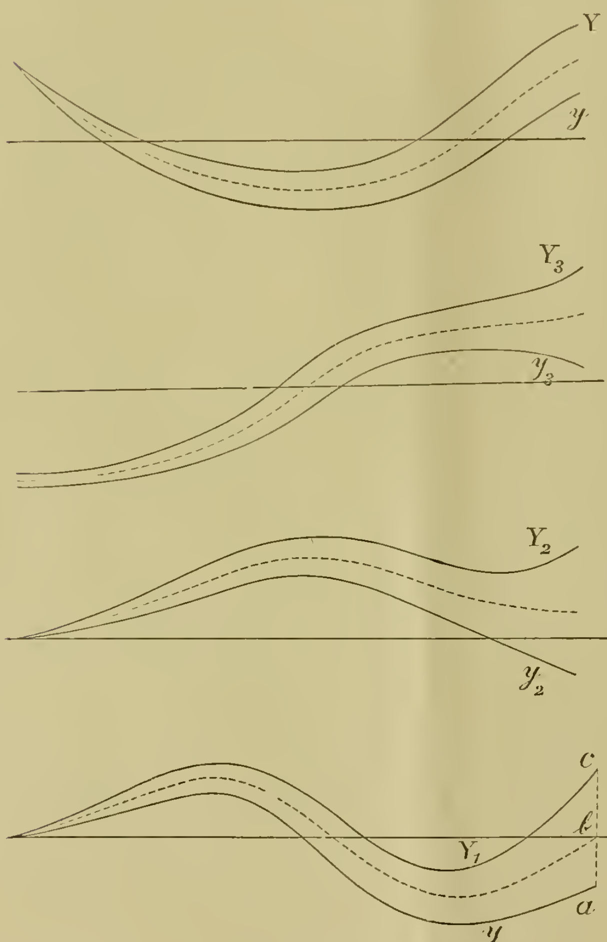
We have to impose upon the solution the conditions (10′). This is effected by taking the initial positions of the recording points of the first and second integraphs coincident with the respective base lines, so that at one end of the ship $\frac{d^2u}{dx^2}=\frac{d^3u}{dx^3}=0$.

We have still to satisfy the condition (10′) at the other end.

The initial value of u must still remain arbitrary, since we are quite at liberty to select an arbitrary constant to be multiplied to u to satisfy the equation (9″).

Therefore we have to choose the initial value of $\frac{du}{dx}$ and the value of n^2 so as to satisfy the conditions at the other end.

At first I thought the above choice would be very difficult, but while working out the example at the end of this article, I discovered that a very rough approximate values for $\frac{du}{dx}$ and n^2 are sufficient to find exactly the normal functions and also that the true value of n^2 are determined at the same time. This was rather unexpected. The reason for the simplification is as follows :

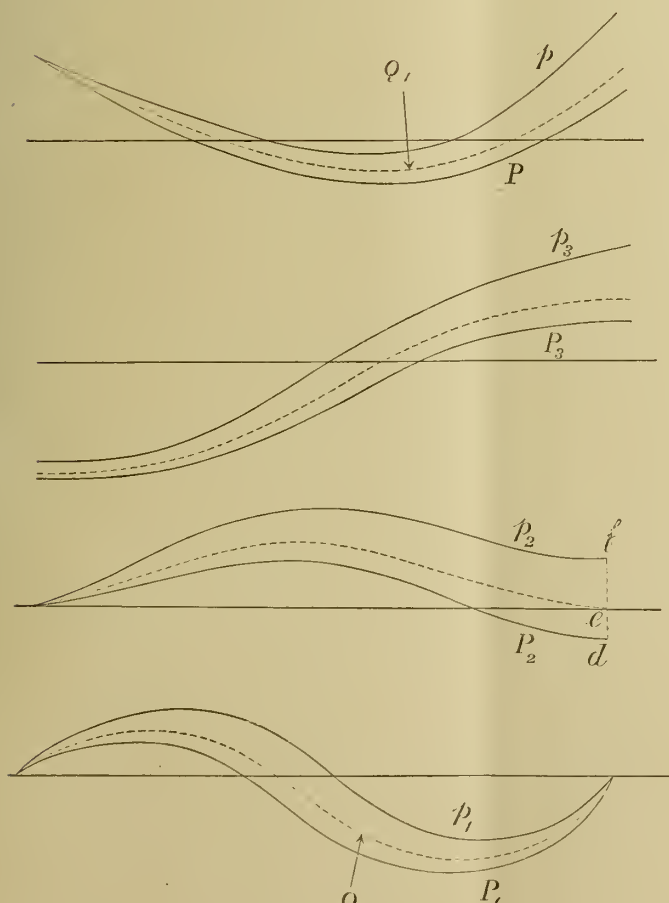


Suppose y_1, y_2, y_3, y and Y_1, Y_2, Y_3, Y are corresponding curves traced out by integraphs with the same value of n^2 but with different initial values of $\frac{du}{dx}$.

The respective

intervals between these two sets of curves must correspond to each other by the properties of the integral and differential curves. Hence divide the intervals throughout in the ratio *ab : bc*, and draw curves as shown by dotted lines. These newly drawn curves must also correspond to each other as the original sets, but with the additional condition $\frac{d^3u}{dx^3}=0$ also satisfied at the other end.

Similarly construct another set of curves with a different value for n^2 and satisfying th condition $\frac{d^3u}{dx^3}=0$ at the other end.



Let p_1, p_2, p_3, p and P_1, P_2, P_3, P be these two sets of new curves, both satisfying the condition $\frac{d^3u}{dx^3}=0$ at the other end.

Again divide the respective intervals throughout in the ratio *de:ef*, and draw curves through these points as before. These new curves correspond to each other. The con-

ditions at both ends of the ship are now all satisfied.

It now only remains to determine n^2.

Define n^2 so that the first and the last curves, namely Q and Q_1 in the figure, correspond to each other. This determines the true admissible value of n^2.

The above reasoning corresponds to the fact that any particular solution of a linear differential equation may be obtained by means of linearly independent particular solutions equal in number to the order of that equation.

Therefore we need only draw four sets of curves, each two corresponding to a definite value of n^2, to find a normal function and the true corresponding value of n^2.

The very rough approximate values of n^2 necessary during the above calculation may be guessed at from the value for a uniform bar, or from the results of experiments already known to us.

Thus we have found out all the necessary quantities to determine the motion of a ship as a whole at any instant.

Hence if ζ denotes the vertical displacement of a point in the ship at time t, we have

$$\zeta=\varphi_1 u_1+\varphi_2 u_2+\varphi_3 u_3+\ldots\ldots, \qquad \ldots\ldots\ldots\ldots (11)$$

where

$$\left.\begin{aligned}\varphi_r&=\sum_1 {}_s\frac{u_r(x_s)E_s}{a_r\sqrt{(n_r^2-p_s^2)^2+\varkappa_r^2 p_s^2}}\cos(p_s t-\varepsilon_s-\theta_s),\\ \tan\theta_s&=\frac{\varkappa_r p_s}{n_r^2-p_s^2}.\end{aligned}\right\}\ldots\ldots\ldots (6)$$

The value of ζ does not depend upon the scales of the normal functions u, as the expression for a_r is $\int\rho\omega u_r^2dx$, these factors in the numerator and the denominator cancel each other.

Example.

The ship of which the following calculations are worked is a torpedo boat destroyer. The principal dimensions are:

Length between perpendiculars	$\overset{m}{69.2}$
Breadth moulded	$\overset{m}{6.55}$
Depth moulded	$\overset{m}{4.04}$
Draught (forward and aft)	$\overset{m}{1.83}$

The length over all is about 71 metres. Plate I shows the curve of weight. This I was able to get from our Admiralty through the kindness of Captain M. Kondo (now Admiral constructor).

The weight of this boat calculated from this curve is 366 tonnes.

Throughout the following calculations, the units of mass, time and length are taken to be tonne, second and metre respectively.

Plate II shows the curve of $gk^2\omega$. The moment of inertia of the variable section was calculated for the following fifteen sections:

At frame No. 1	428×10^{-4}(metre)^4
,, ,, ,, 6	$1214 \times$,,
,, ,, ,, 14	$1755 \times$,,
,, ,, ,, 20	$2185 \times$,,
,, ,, ,, 26	$2342 \times$,,
,, ,, ,, 36	$2244 \times$,,
,, ,, ,, 38	$3127 \times$,,
,, ,, ,, 58	$3685 \times$,,
,, ,, ,, 65	$3565 \times$,,
,, ,, ,, 85	$3207 \times$,,
,, ,, ,, 93	$2513 \times$,,

At frame	No.	100	1861×10^{-4} (metre)[4]	
,,	,,	,, 112	$1010 \times$,,	
,,	,,	,, 124	$693 \times$,,	
,,	,,	,, 131	$313 \times$,, .	

The value of q was taken at 216×10^6 (absolute units). In the same plate, the curve for $\frac{1}{qk^2\omega}$ is drawn.

Plates III & IV show the actual process of solving the differential equation for the first and the third mode respectively.

The correct values of n^2 were found to be

$$n_1^2 = 560, \qquad n_2^2 = 2450, \qquad n_3^2 = 7890.$$

Consequently the number of vibrations of each mode is

$$\frac{23 \cdot 6}{\pi} \times 30 = 225 \text{ per minute, with } 2 \text{ nodes,}$$

$$\frac{49 \cdot 5}{\pi} \times 30 = 470 \text{ per minute, with } 3 \text{ nodes,}$$

$$\frac{88 \cdot 8}{\pi} \times 30 = 850 \text{ per minute, with } 4 \text{ nodes.}$$

Plate V shows the normal functions.

It appears from the above result that the numbers of vibrations with 3 and 4 nodes are nearer in harmonics with the number of vibration of 2 nodes than in the case of a uniform bar

Verification of the above calculation by means of a model.

Our university has a model of the above destroyer made of brass plates and angles, with detail of fittings complete. The weight of this model without engines and boilers is 23·1 kilogrammes and its size 1/25 of the actual ship. Hence the corresponding

displacement of this model would be $23{\cdot}1 \times \overline{25}^3 = 360$ tonnes. Thus without the engines and boilers, this model has approximately the corresponding displacement with the above destroyer.

I suspended this model with two belts of elastic cords. When a tuning fork of variable pitch is set in vibration and its stem is applied at a few inches distance on either side of a node and the pitch is in harmony with that of the model, a decided sign of synchronism was both audible and visible. The positions of the cords had almost no effect upon the vibration, provided they are near the nodes.

By this means, I observed the pitch of this model to be as follows:

No. of vibrations per second	
60	with two nodes,
125	with three nodes.

Taking the ratio of the square roots of the Young's modulii for steel and brass to be $1{\cdot}5:1$ and calculating the corresponding numbers of vibrations for the case when this model is enlarged to the full size and the material changed from brass to steel, we have

No. of vibrations per minute	
$\dfrac{60 \times 1 \cdot 5 \times 60}{25} = 216$	with two nodes,
$\dfrac{125 \times 1 \cdot 5 \times 60}{25} = 450$	with three nodes.

while the calculated values for the destroyer are 225 and 470 resp.

Considering the dissimilarity of the model and the actual ship, one without engines and boilers and the other with them, the result may be considered as being fairly in agreement.

The ratio of the numbers of vibrations of the first and the second modes for the model is $60:125=1:2{\cdot}09$ while for the destroyer as above calculated is $225:470=1:2{\cdot}09$, the same as for the model.

As these ratios ought to be a mere number for a uniform bar, the agreement shows that the calculation is probably correct.

The vibrations of the model with four or more nodes were mixed up with local vibrations and not distinctly observable.

Article 2.

Change of structural stress due to vibration.

As one application of the preceding article, we may trace the change of structural stress due to vibration. This change of stress is periodic and repeats itself after each complete vibration, its magnitude varying constantly from a certain maximum to a certain minimum. To calculate this change, it is necessary to find the second differential coefficients of the normal functions with respect to x. The values of these coefficients may be found almost at once from the drawings used for finding the normal functions. Thin full lines in Plate V show the approximate values of these coefficients.

If y denotes the distance from the neutral axis of the part under consideration, the change of stress per square metre is given by $qy\frac{d^2u}{dx^2}$.

i) Consider the case when the vibration of the first mode only occurs. To fix our ideas, suppose the amplitude of vibration at the aft end of ship is $\frac{1}{2}$ centimetre, so that the total range of vibration is 1 centimetre.

In this case, the maximum curvature, as measured from

Plate V is approximately at the midship section and the value is $\frac{0 \cdot 022}{2 \times 825} = 0 \cdot 0000133$. The largest value of Y is 2·3 metres, supposing the neutral axis to be at half the depth of the section.

Hence the maximum change of stress per square metre $= 0 \cdot 66 \times 10^4$ each way. Or the stress changes between $\frac{\pm 0 \cdot 66}{9 \cdot 8} = \pm 0 \cdot 067$ tonne per square centimetre or between $\pm 0 \cdot 43$ ton per square inch.

If the amplitude of vibration be one half of that supposed just now, the corresponding change of stress will be one half the value above obtained and so on proportionately.

ii) Next, considering the case when the second mode only occurs, the amplitude at the aft end being supposed to be $\frac{1}{2}$ centimetre as before, we get the maximum curvatures 0·000034 and 0·000032 at about 53 and 18 metres from the fore end of the ship respectively. The maximum values of y at these points are 2·1 and 2·2 metres, so that the change of stress lies between $\pm 0 \cdot 157$ tonne per square centimetre or between $\pm 1 \cdot 01$ ton per square inch for the former, and between $\pm 0 \cdot 155$ tonne per square centimetre or between $\pm 1 \cdot 0$ ton per square inch for the latter.

iii) Thirdly consider the case when the third mode only occurs. The amplitude of vibration is, as before, supposed to be $\frac{1}{2}$ centimetre at the aft end.

Proceeding in the same manner, we find the maximum stresses at 60, 39 and 16 metres from the fore end, and the values are

$\pm 0 \cdot 38$ tonne per square centimetre $= \pm 2 \cdot 45$ tons per square inch

$\pm 0 \cdot 304$ tonne per square centimetre $= \pm 1 \cdot 94$ tons per square inch

$\pm 0 \cdot 46$ tonne per square centimetre $= \pm 2 \cdot 86$ tons per square inch

respectively.

In the above three cases, the vibration is supposed to be of

one specified mode only, but actually several modes of vibrations occur simultaneously. In this case, the curvature at a point may be found by simply measuring the curvatures of several modes and taking the algebraic sum of them, and hence the change of stress at that point may be calculated as above.

Strictly speaking, a maximum stress does not necessarily occur at a point of maximum curvature; since the maximun stress is given by the joint product of y, the curvature, and the Young's modulus, it is the y multiplied by the curvature that determines the point of maximum stress. Hence if we draw a curve of $y \times \left(\frac{d^2u}{dx^2}\right)$, the maximum point or points of that curve correspond to the point or points of maximum stress. But my present object being to show how the change of stress due to vibration may be calculated, I did not proceed further.

Article 3.

Frequency Analyser.

As a ship's vibration usually consists of a number of harmonic vibrations all having the periods of the acting forces, it becomes a matter of importance to analyse the complicated vibration into simple elements in order that the cause of vibration may be traced. A usually constructed vibrograph is not delicate enough to record all these vibrations, and even if it records them, the process of analysing and finding out the component periods involves more or less difficulty.

To obviate this, a form of frequency analyser was contrived. The principle of this instrument is based upon the resonance phenomena. It consists of several thin steel bars with lead attached to the free end of each and the other end rigidly fixed to

a foundation. Plate VI is a general view of this instrument (with the cover removed), as manufactured by Messrs Kelvin and James white, Glasgow. The figures on the free end denote the numbers of vibrations per minute of the respective bars. In the present instrument the range of vibration is from 200 to 2700 per minute.

When this instrument is placed on a vibrating body, the bars having the same or nearly the same numbers of vibration as the body vibrate, and so the instrument automatically analyses the complicated vibrations, namely, the instrument performs the function of a harmonic analyser. The general equation (6) of the article 1, which applies to vibrating bodies in general, shows that the periods of the vibrating body must always be the same as the periods of the acting harmonic forces ; hence we see that the vibrating bars of the instrument all correspond to the component forces and that the bars corresponding to either lower or higher harmonics of the acting forces are not vibrating.

I made several experiments with the instrument on board ships and always found it faithfully analysing the vibration into their component elements. It is quite interesting to watch several vibrating bars transfering their vibrations to their neighbours as the revolutions of engines increase or decrease.

Construction of the instrument.—The thickness of the steel bars were made constant as far as possible and the periods adjusted by means of lead attached to the free ends.

The rough calculation for each bar was made by using the equation

$$p^2=\frac{3qk^2\omega}{Ml^3}, \quad \text{(Rayleigh-Theory of sound. Vol. I. § 182.)}$$

where $p=2\pi\times$ number of vibrations per unit time,

$l=$ length of bar,

and M=Mass at the free end.

This corresponds to the case when the mass of the thin steel bar is neglected. My first intention was to push the lower limit of vibration to about 50 per minute, but finding the deflection, when placed as in the photograph, to be too great, I sought a relation between the number of vibrations and the deflection, in order to see what effect the material, the dimensions of the bar and the quantity of lead would have on the deflection, for the same number of vibrations per unit time, and, if possible, to select them so as to give it a minimum deflection.

The number of vibrations N per unit time is given by

$$N=\frac{1}{2\pi}\sqrt{\frac{3qk^2\omega}{Ml^3}},$$

while the deflection D at the free end would be

$$D=\frac{Mgl^3}{3qk^2\omega},$$

so that

$$N=\frac{1}{2\pi}\sqrt{\frac{g}{D}}.$$

Hence, under the above supposition, the number of vibrations is independent of the length, the dimensions, and the material of the bar and only depends upon the deflection D. So that, being unable to find a remedy without introducing complication, I fixed the lower limit at 200 per minute, in order to avoid the excessive deflection. In this connection, Dr. Bottomley of Glasgow pointed out to me that the number of vibrations in this case may be remembered as the same as that of a simple pendulum with its length equal to the deflection D.

Article 4.

A Vibrograph.

Until recently, I used a combined instrument consisting of Prof. Tanakadate's vertical vibration recorder and of Prof. Ohmori's horizontal pendulum for recording a vibration on board a ship. Although this combined instrument worked nicely when the sea was calm, it was rather bulky, and when the ship rolled, the horizontal pendulum would oscillate to and fro until it rested on one side or the other, making the recording impossible.

To avoid these drawbacks, and to make the whole thing as portable and compact as possible, a new form of instrument, with a modified Prof. Tanakadate's vertical recorder and with a convenient type of pendulum for the horizontal recorder, was contrived. Plate VII is two photographic views of the instrument. This instrument is quite small, the whole being kept in a box about 8 inches square and one foot long. The effect of rolling on the instrument is made small, so that recording is possible even while the ship is rolling.

Article 5.

The effect of water on vibration of steamers.

The effect of the surrounding water upon the period of vibration of a steamer has, as far as I am aware, received but little attention. My view until recently was the commonly accepted one that the effect would be small. In fact, a damping force has, in general, but a small effect upon the period of vibration of a system, and for forced vibrations the maximum am-

plitude is attained when the period of the acting force (supposed harmonic) synchronizes with the natural free (undamped) period of the system.

A few years ago, however, Dr. Terada, of the Science College, Tokio Imperial University, showed experimentally that a body floating on water, undergoes a considerable change in its natural periods of vibration compared to the corresponding periods in air, owing to the wave effect of surrounding water vibrating with the body. At the International Congress in Naval Architecture held at Bordeaux, 1907, I mentioned this effect in reference to M. Wehmeyer's paper. (See pp. 156–157, Bulletin de l'assoc. tech. mar. 1907.)

The question now arises whether the change in its natural periods is also observable under forced vibrations or only under free vibrations.

To decide this point, I took a nine feet model (above mentioned) of a Torpedo-boat destroyer, made of brass plates and angles, and determined its natural periods both in air and on water by applying the stem of a tuning fork of variable pitch. The natural numbers of vibration of this model were as follows:—

No. of vibrations per second.

Vertical.		Horizontal.		
In air	On water	In air	On water	
60	43	90	80	(2 nodes)
125	100	208	185	(3 nodes)

The numbers of vibrations being thus determined, I fitted a small crank at the stern of the model and made it revolve with a fan motor.

The model gave a decided sign both audible and visible of sychronism with vertical vibration of two nodes when the number

of revolutions of the motor was from 2600 to 2550 per minute, or 43 per second; thus showing that the water has the same effect upon the change of periods under forced, as under free vibrations. The positions of nodes were distinctly visible from the surrounding ripples made by the vibrating model. Plate VIII is from a photograph of the vibrating model as above described.

Owing to want of suitable apparatus, I was unable to push the revolutions higher, and so test the similar effects for vibrations of higher order; but there seems every probability that the change would be present also for vertical vibrations with three or more nodes and also for horizontal vibrations. In fact, the vibration caused by applying the stem of a vibrating tuning fork, is a kind of forced vibration with a constrained displacement at that point.

Thus it appears, in calculating or experimenting upon the vibrations of a steamer, that it is important to take this effect into consideration and to note that the water plays an important part upon the number of vibrations of the steamer.

Article 6.

Determination of the proper periods of a despatch boat.

With the permission and the kind cooperation of our Admiralty officials, I carried out an experiment on board one of our warships to determine her proper periods both horizontal and vertical.

For this purpose, I fitted a crank on the stern of that boat and made it revolve with an electric motor. At the end of the crank, variable weights from 2 to 50 pounds, according to the number of revolutions per minute, were fastened with bolts.

The axis of revolution of the crank was arranged parallel to the longitudinal axis of the boat, so that by its revolution both horizontal and vertical periodic forces might act upon her to cause these vibrations. Thus, the revolutions of the crank were gradually increased from 50 to 2700 per minute. Plate IX shows a general plan for the above arrangement.

When the number of revolutions synchronized with a proper number of vibrations of the ship, the amplitude of vibration was remarkable, and by this, I was able to determine the proper periods of vibration of this boat to be as follows :—

Horizontal	Vertical
166 per minute	105 per minute (2 nodes).
360 ,, ,,	225 ,, ,, (3 nodes).
	390 ,, ,, (4 nodes).

These vibration were recorded by both of the two instruments, one set at the stern and the other at the stem of the boat; so that there is no doubt as to these being the vibrations of the ship as a whole. Besides the above vibrations, the two instruments recorded a proper number of 780 per minute, both for the horizontal and for the vertical, but this number was not so distinct as those in the above list. Probably this corresponds to the 4 nodal horizontal, and the five nodal vertical vibrations. Also the number 780 involves the maximum probable error of something like ± 30, so that it is by no means certain that the horizontal and the vertical vibrations just cited have actually the same number of vibrations per minute.

At revolutions from 1900 to 2300 per minute, though the instrument at the stem recorded nothing, there was very violent shaking at the stern. Even the sound of the vibration was dis-

tinctly andible. When the revolutions per minute became higher, however, the vibration and the sound died out and these were not again distinct up to the highest number of revolutions then obtained (2700 or 2800 per minute). This shaking, or vibration, occured both horizontally and vertically. The nature of this vibration remains to be studied, though it is certain that this is a proper vibration of the ship whether structural or local, and that a three bladed propeller revolving 650 to 750 times per minute under the stern is very effective in causing it.

The frequency analyser, described in Art. 3, was used throughout this experiment for reading the number of vibrations, and by that we were guided to regulate the electric current to make the revolutions vary continuously, so that the analyser served us as a revolution counter in this case.

A remarkable fact observed during this experiment was that when the ship was vibrating with one of its proper periods, the electric power to maintain this vibration was rather high, sometimes absolving as much as 30 kilowatts, while the maximum amplitude never exceeded one thirtieth of an inch. Lieutenant constructor Tawara, who was assisting me during the experiment, and myself saw at once that this loss of energy was obviously due to the dissipation caused by the resistance in the ship and of the water.

If we suppose the energy of vibration to be proportional to the square of amplitude for the same number of vibrations per unit time, it appears that the dissipation of energy due to the vibration of a ship may sometimes be bigger than what we might expect.

The above experiment was carried out while the ship was at

anchor. When the displacement varied, the number of vibrations was found to be roughly inversely proportional to the square root of the displacement.

Dec., 1909.

PLATE I.

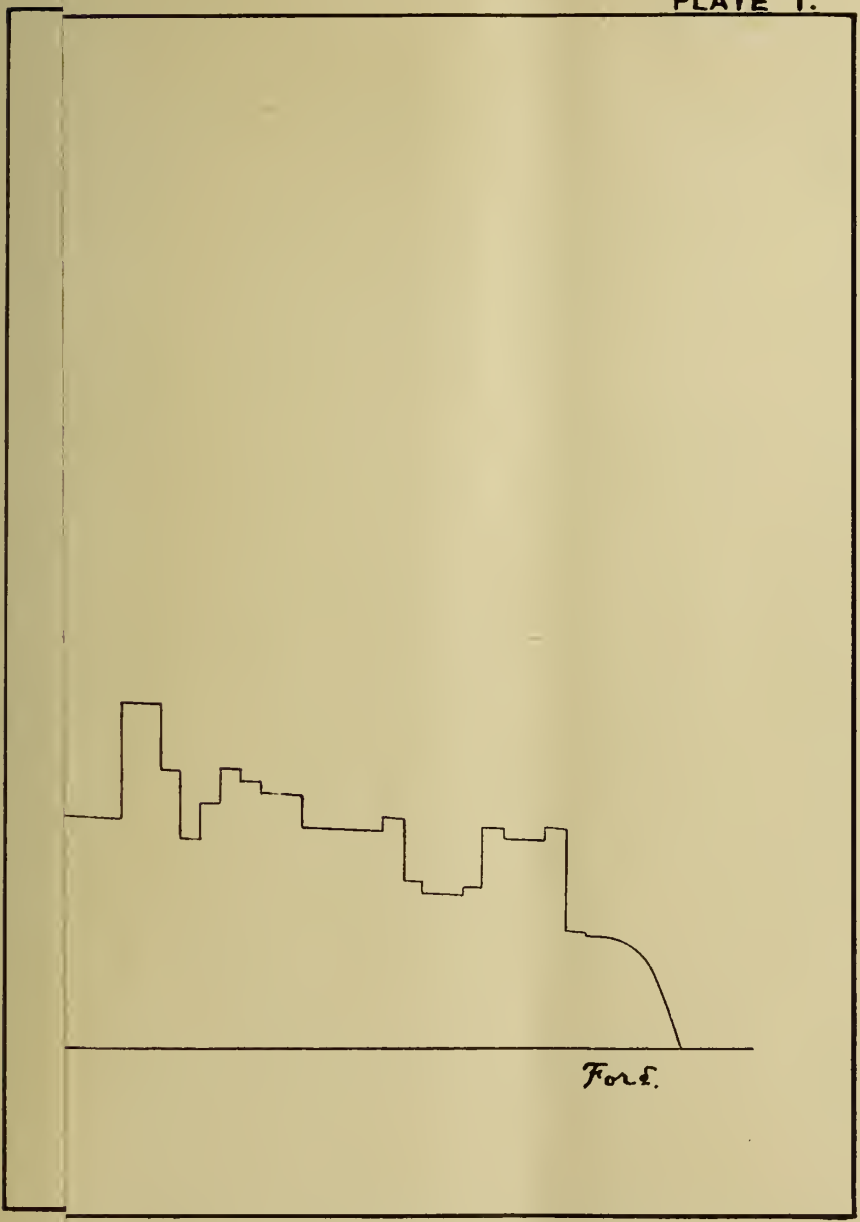
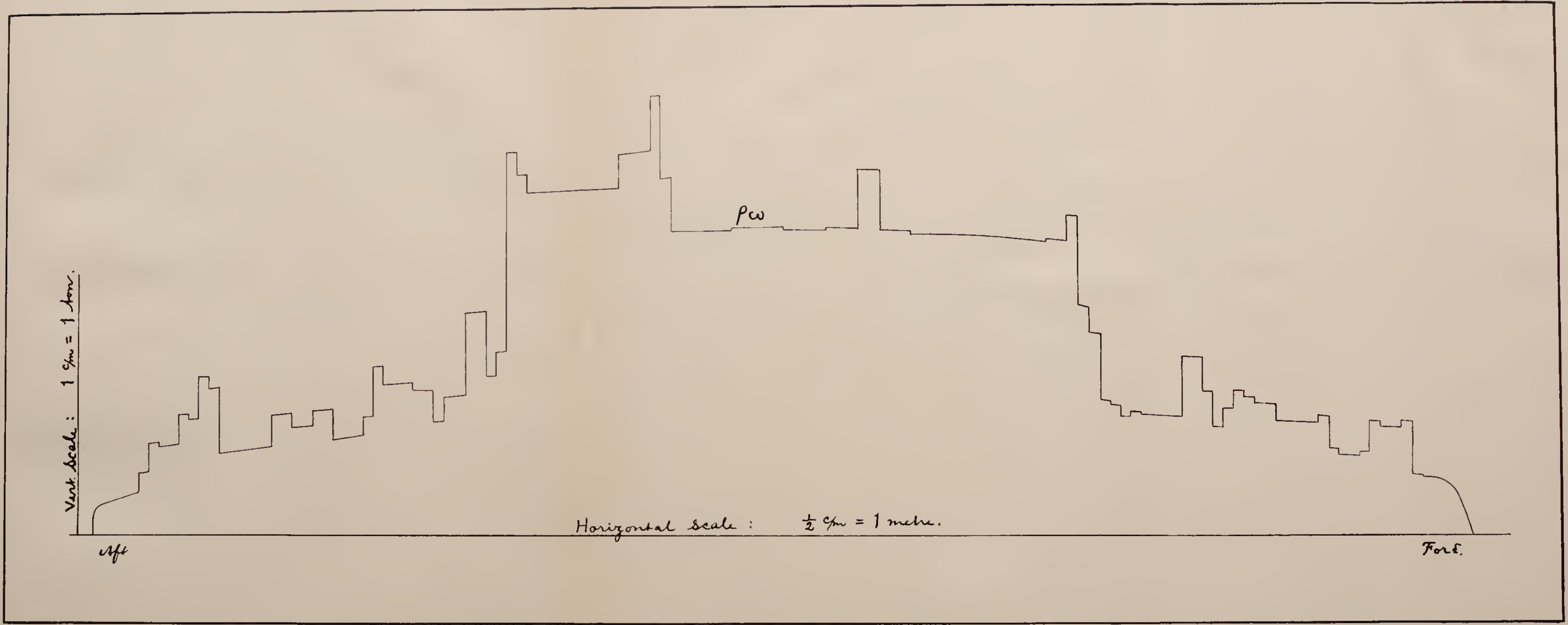


PLATE I.



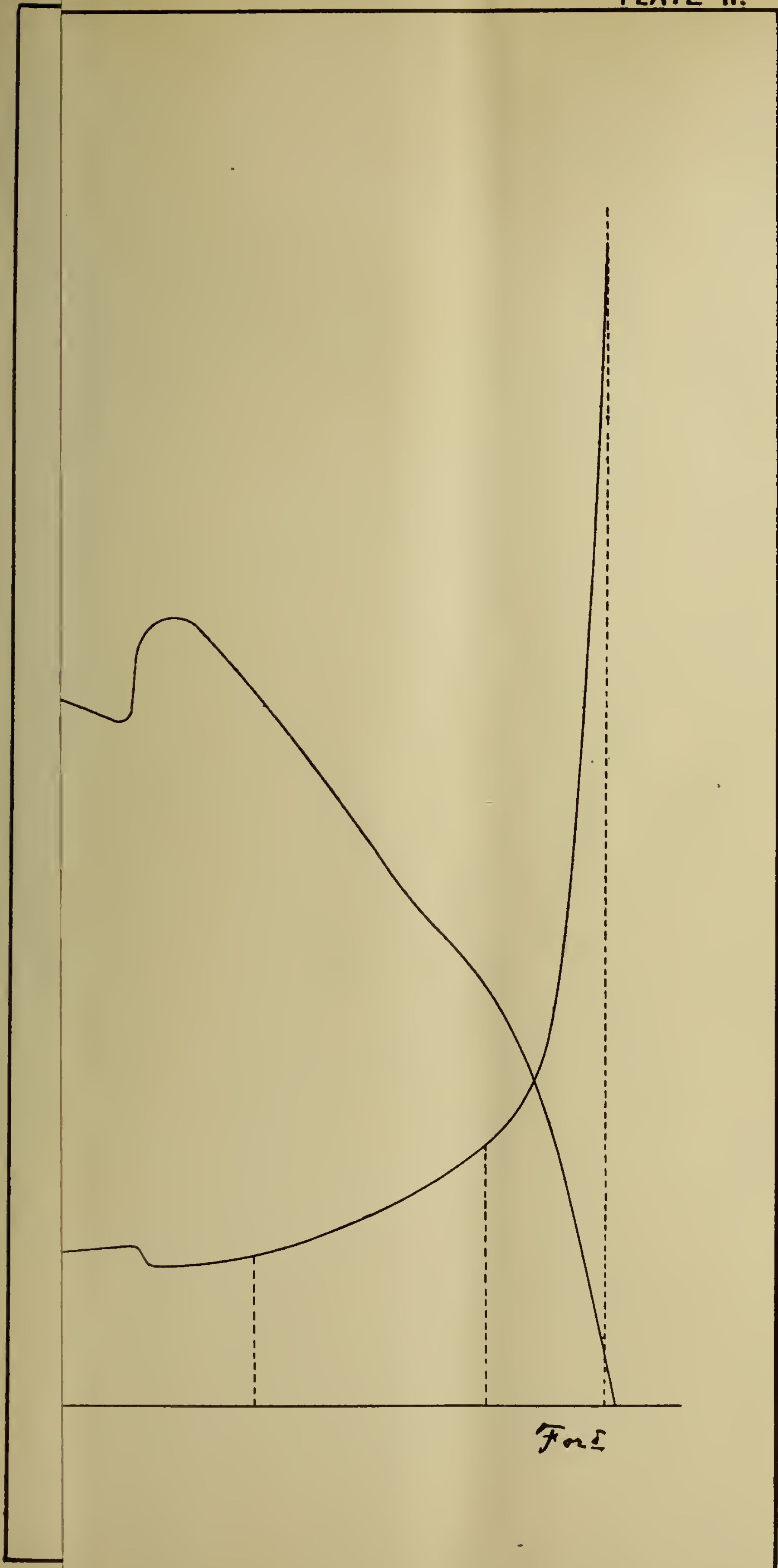
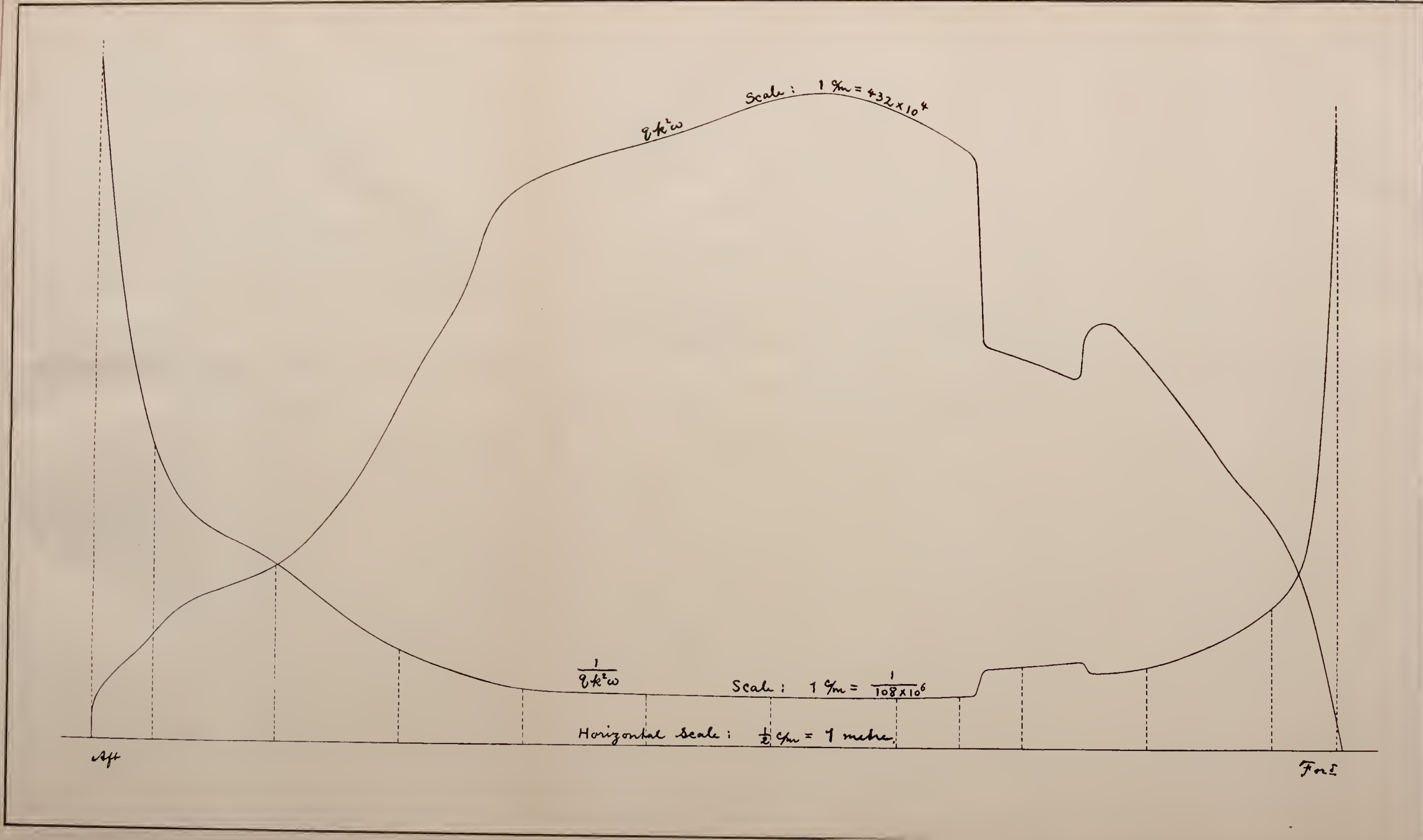
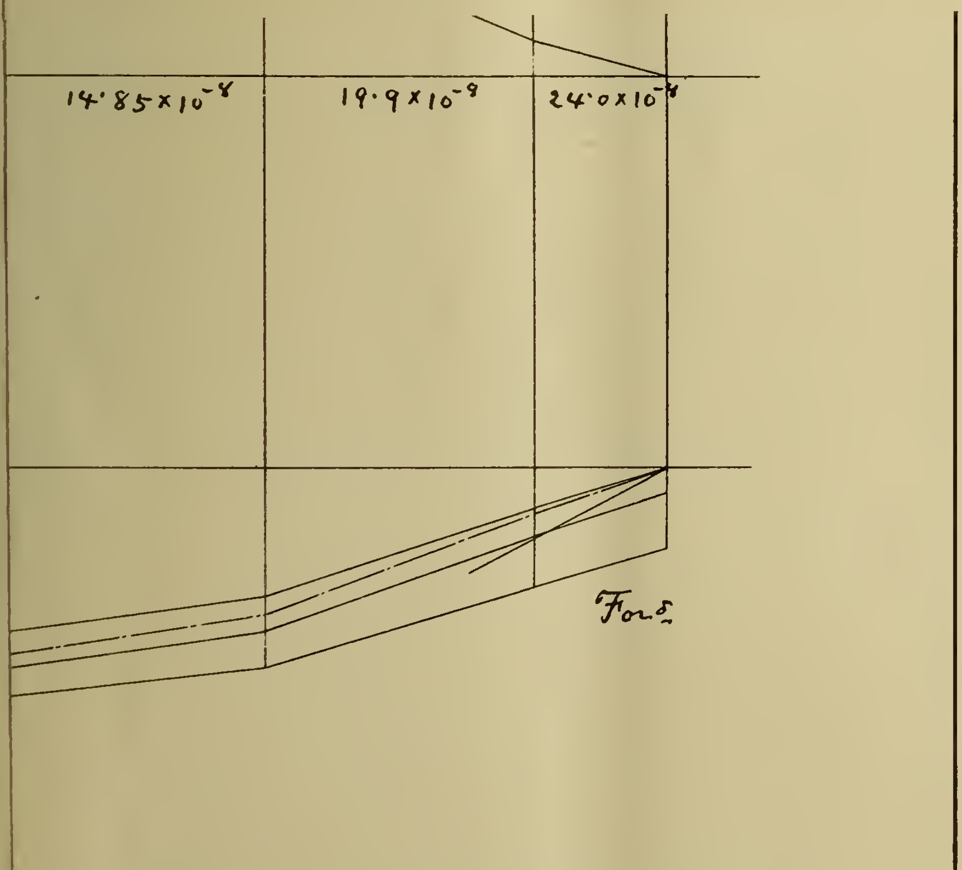
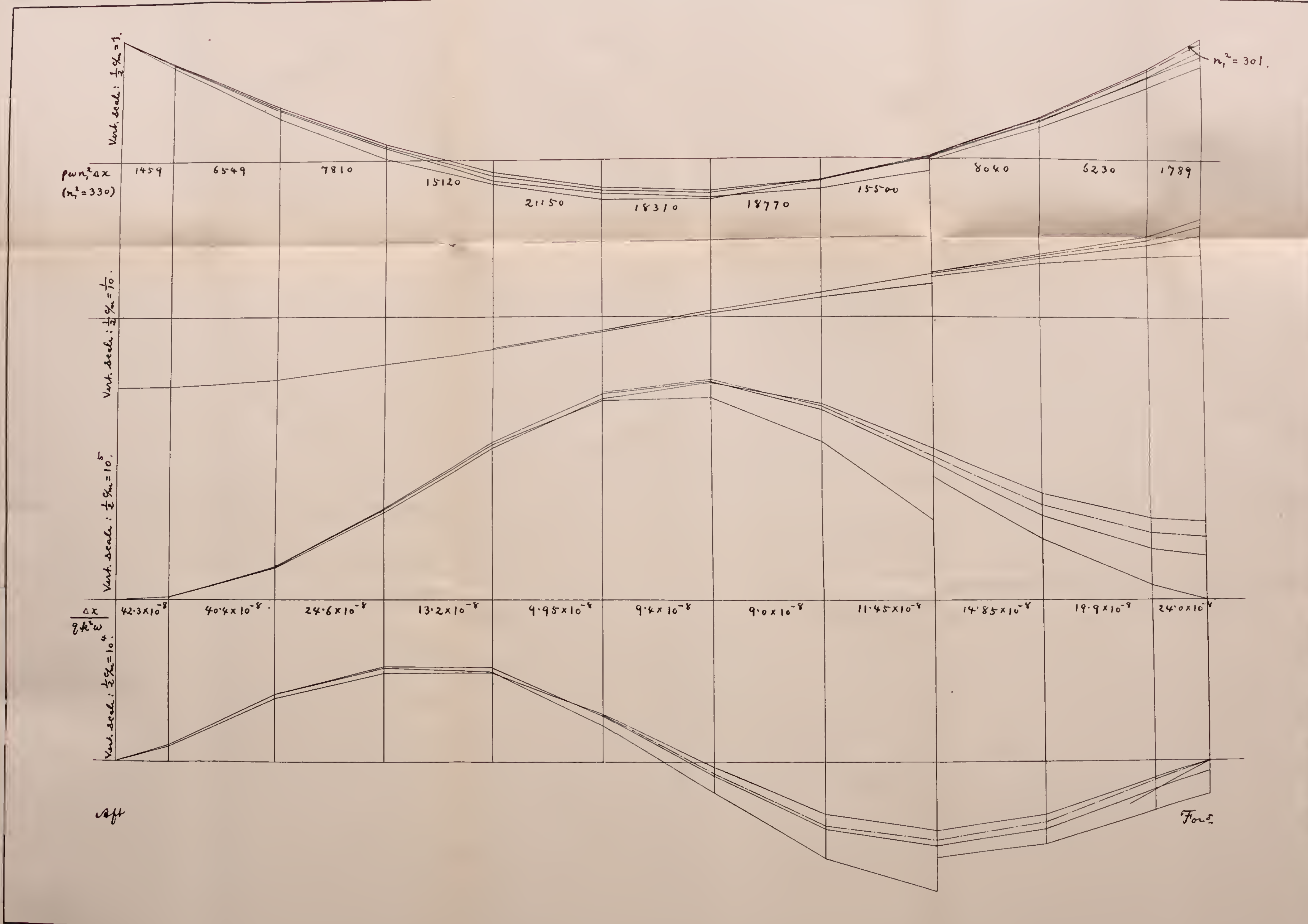
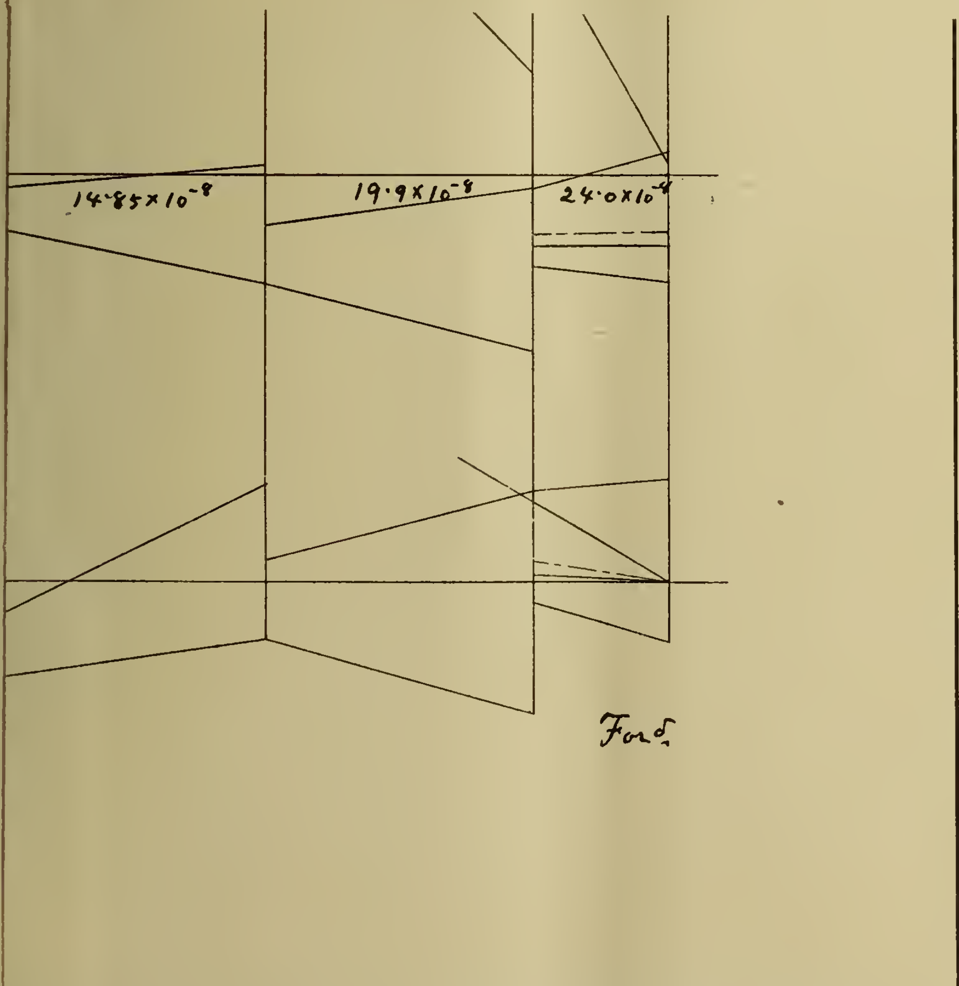


PLATE II.


$\varrho k^2 \omega$
Scale: 1 c/m = 432 × 10^4
$\frac{1}{\varrho k^2 \omega}$
Scale: 1 c/m = $\frac{1}{108 \times 10^6}$
Horizontal Scale: ½ c/m = 1 metre.
Aft
Ford.

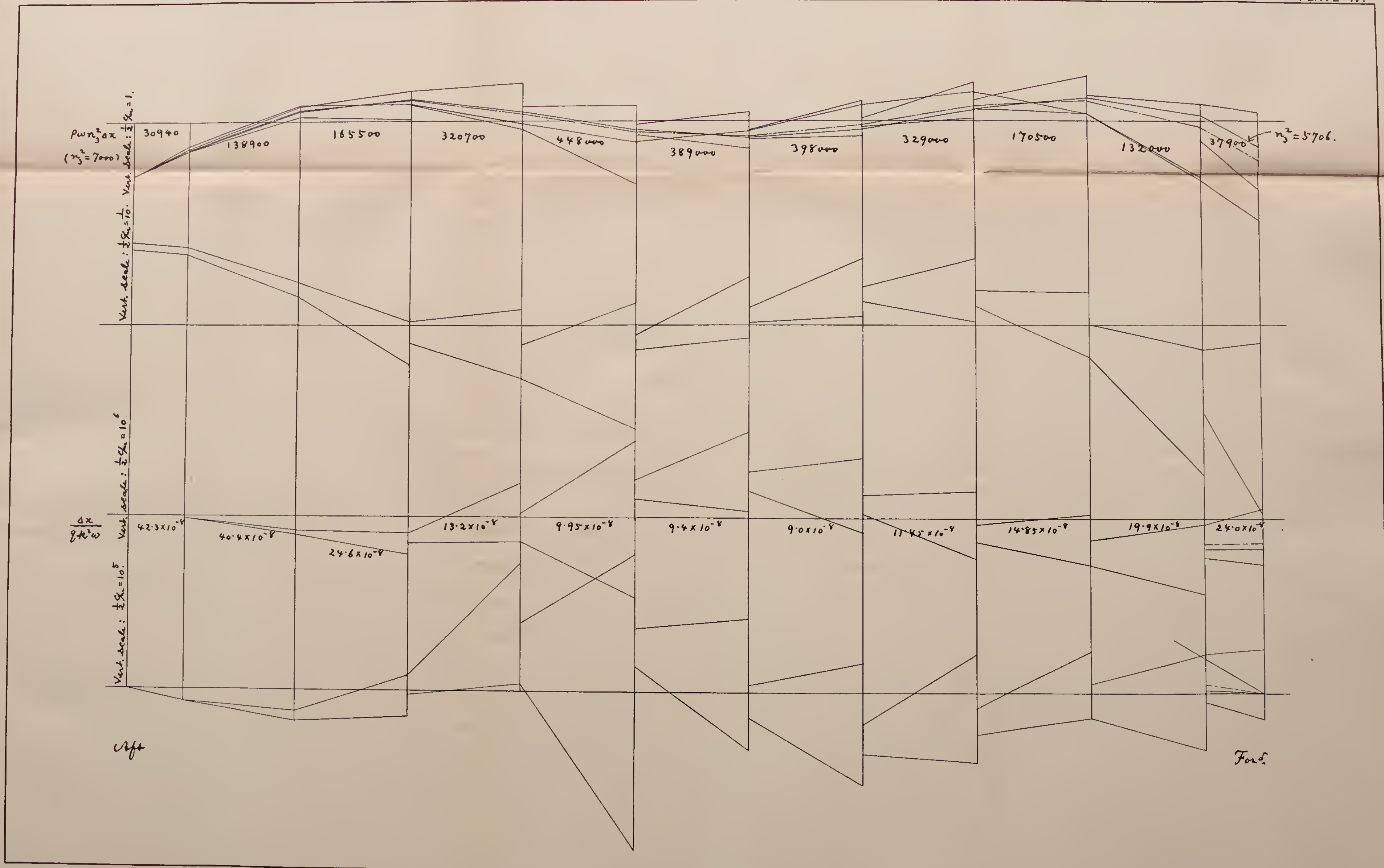

14·85×10⁻⁸
19·9×10⁻⁸
24·0×10⁻⁸
Fon^d


$n_1^2 = 301$.
Vert. scale: $\frac{1}{2}\%_m = 1$.
$\rho w n_1^2 \Delta x$
($n_1^2 = 330$)
1459
6549
7810
15120
21150
18310
18770
15500
8040
6230
1789
Vert. scale: $\frac{1}{2}\%_m = \frac{1}{10}$.
Vert. scale: $\frac{1}{2}\%_m = 10^5$.
$\frac{\Delta x}{g k^2 \omega}$
$42 \cdot 3 \times 10^{-8}$
$40 \cdot 4 \times 10^{-8}$
$24 \cdot 6 \times 10^{-8}$
$13 \cdot 2 \times 10^{-8}$
$9 \cdot 95 \times 10^{-8}$
$9 \cdot 4 \times 10^{-8}$
$9 \cdot 0 \times 10^{-8}$
$11 \cdot 45 \times 10^{-8}$
$14 \cdot 85 \times 10^{-8}$
$19 \cdot 9 \times 10^{-8}$
$24 \cdot 0 \times 10^{-8}$
Vert. scale: $\frac{1}{2}\%_m = 10^4$.
Aft
Ford.



14·85×10⁻⁸
19·9×10⁻⁸
24·0×10⁻⁸
Ford.

PLATE IV.



ρwn₃²Δx
(n₃² = 7000)
30940
138900
165500
320700
448000
389000
398000
329000
170500
132000
37900
n₃² = 5706.
Δx / gk²w
42·3×10⁻⁸
40·4×10⁻⁸
24·6×10⁻⁸
13·2×10⁻⁸
9·95×10⁻⁸
9·4×10⁻⁸
9·0×10⁻⁸
11·45×10⁻⁸
14·85×10⁻⁸
19·9×10⁻⁸
24·0×10⁻⁸
Aft
Ford.

PLATE V.

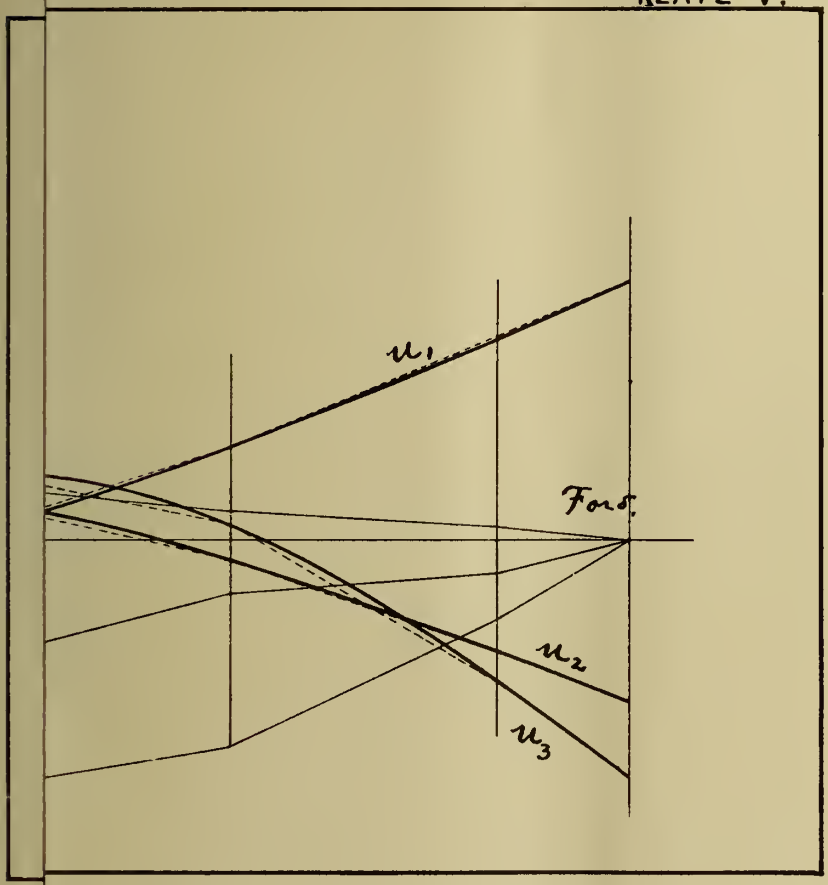


PLATE V.

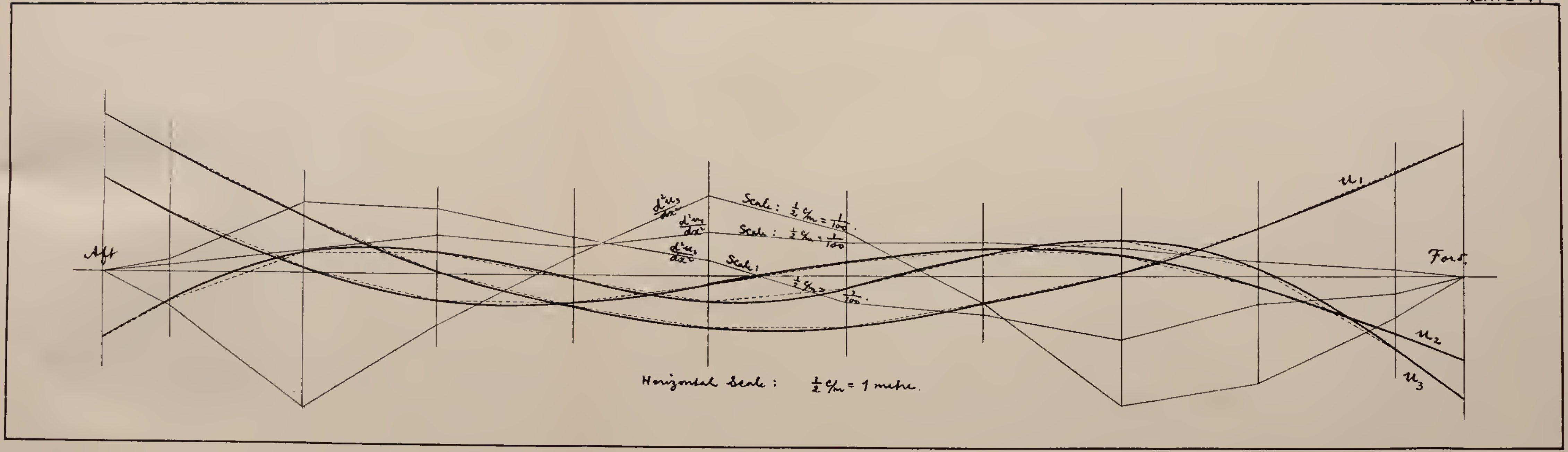
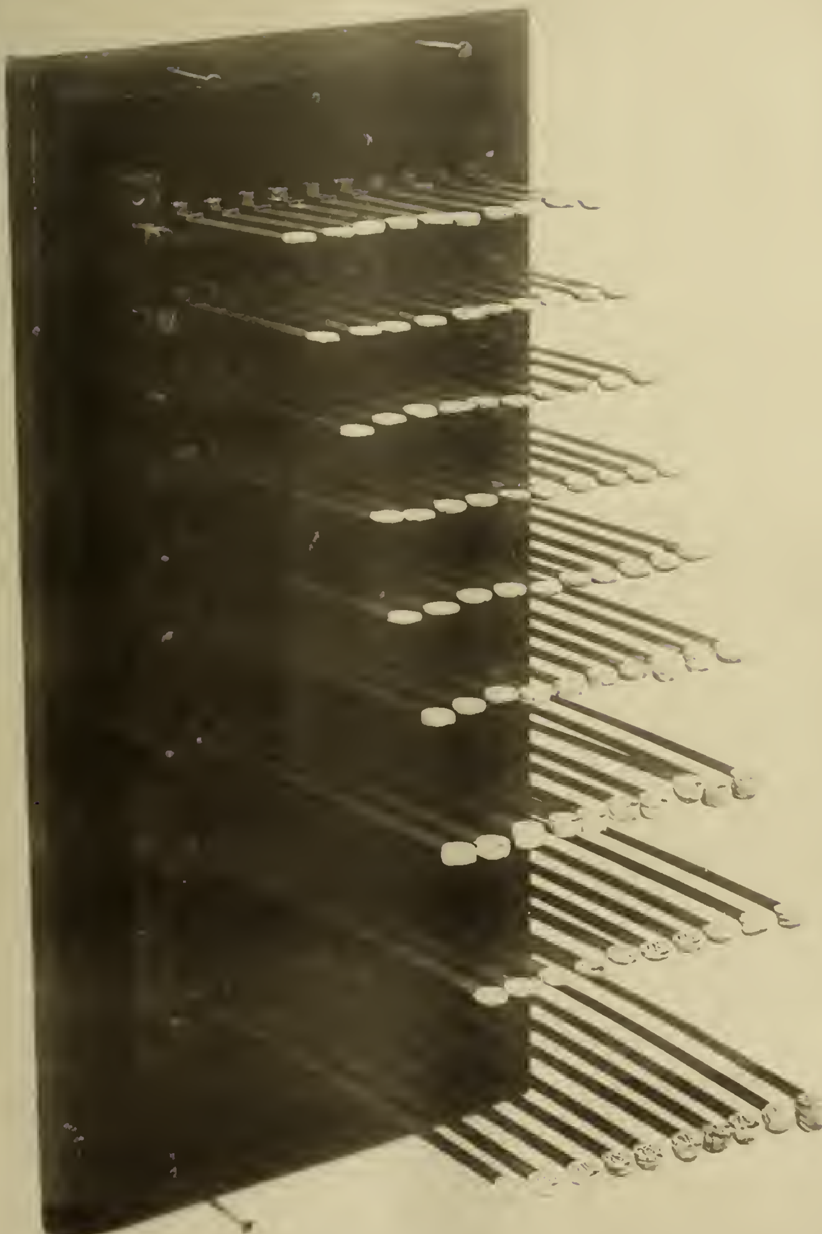
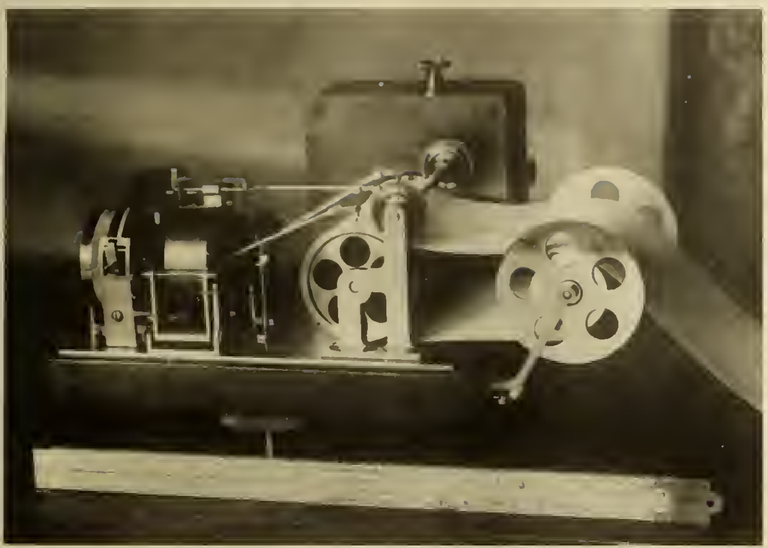


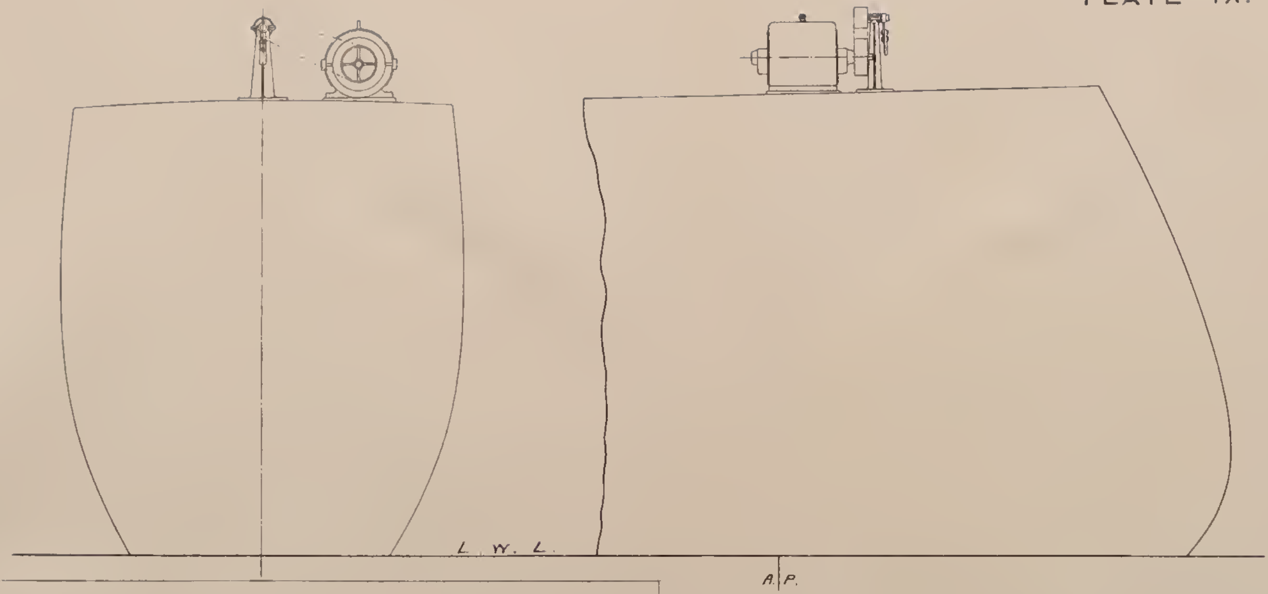
PLATE VI.

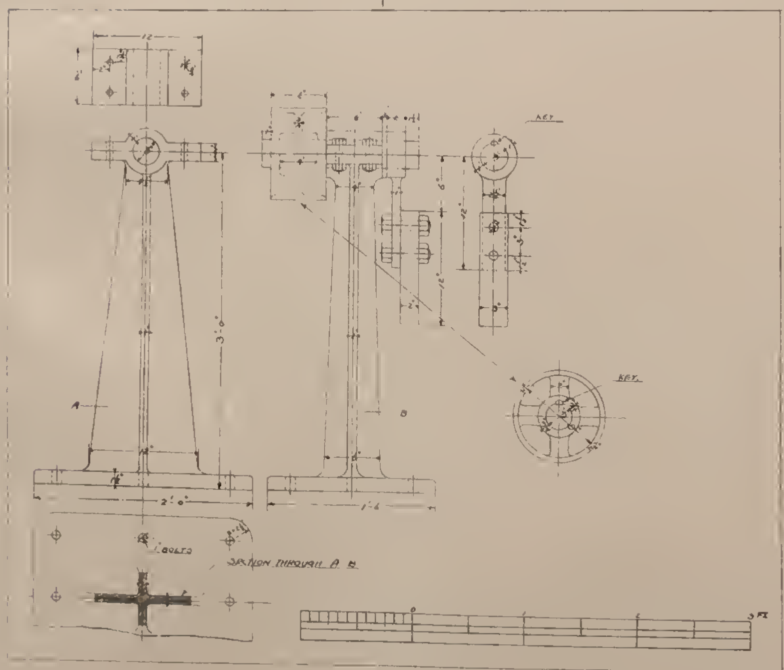


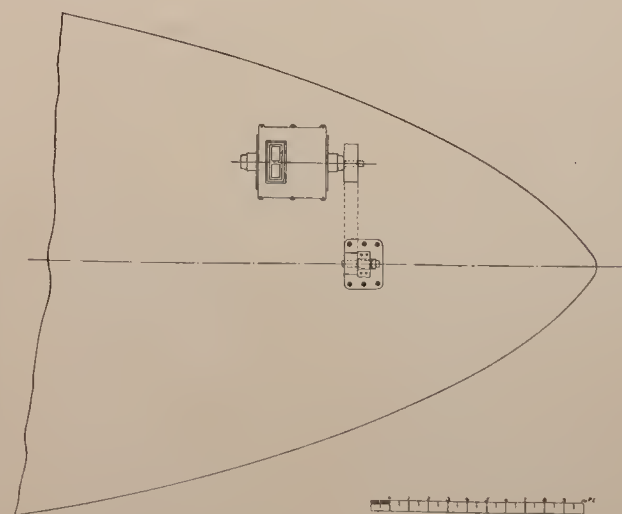



6"
2"
A

PLATE IX.







AGENT FOR THE SALE OF THIS JOURNAL.

Z. P. MARUYA & Co., Ltd.

Tôri Sanchome, Nihonbashi,

TOKYO.

明治四十三年五月二十四日印刷
明治四十三年五月二十七日發行

編纂兼發行者　東京帝國大學

東京市日本橋區兜町二番地
印刷者　神谷岩次郎

東京市日本橋區兜町二番地
印刷所　東京印刷株式會社

東京市日本橋區通三丁目十四番地
賣捌所　丸善株式會社書店

CONTENTS.

Vol. V. No. I.

Page.
On Vibration of steamers. By Seinen Yokota, *Kogakushi, Kogakuhakushi....* 1

Contents of Volume IV. (Latest Volume.)

Nos. Page.

1. On a New Unsaturated Fatty Acid in Japanese Sardine Oil. By M. Tsujimoto, *Kōgakushi* ... 1
   The occurrence of Clupanodonic Acid in Herring and Whale Oils. By M. Tsujimoto, *Kōgakushi* ... 11
2. On the Starch of Glutinous Rice. By Y. Tanaka, *Kōgakushi* ... 15
   New Sulphide Colours: Fukugi Brown and Fukugi Brown G. By E. Itō, *Kōgakushi* ... 31
   New Azo-Colours produced from Fukugetin. By E. Itō, *Kōgakushi* ... 35
3. On the Hydrolytic Enzyme in the Resting Seeds of Some Gramineæ. By Y. Tanaka, *Kōgakushi* ... 39
   Nitro-colouring Matters derived from Fukugiflavin. By E. Itō, *Kōgakushi.* 53
   On the Japanese Dyewood "Doss." By E. Itō, *Kōgakushi*... 57
   On the Composition of Chrysalis Oil. By M. Tsujimoto, *Kōgakushi* ... 63
   On a Few Japanese Vegetable Oils. By M. Tsujimoto, *Kōgakushi* ... 75
   On Japanese Lacquer. By K. Miyama, *Kōgakushi* ... 89
4. The Fundamental Principles of the Loading of the Active Materials of Alternating Current Transformers and Polyphase Induction Motors. By Hidetaro Hō, *Kōgakushi* ... 111
   Wie ändert sich die Kapacität des Bleiaccumulators? Von Y. Furuya, *Kōgakushi* ... 143
5. The Occurrence of Clupanodouic Acid in Dab and Green Turtle Oils. By M. Tsujimoto, *Kōgakushi* ... 177
   On the Cause of Odours of Oils and Fats, especially of Marine Animal Oils. By M. Tsujimoto, *Kōgakushi* ... 181
   On the Resinous Substance deposited from the Mixed Fatty Acids of Marine Animal Oils on Keeping. By M. Tsujimoto, *Kōgakushi*... 193
   On the Oxidised Fatty Acids of Marine Animal Oils. By M. Tsujimoto, *Kōgakushi* ... 197
   Drying of Japanese Lacquer at a High Temperature. By K. Miyama, *Kōgakuhakushi, Kōgakushi*... 201
   Application of Diastases present in Unmalted Cereals to the Preparation of Alcohol and "Amé," By Y. Tanaka, *Kōgakushi*... 205
6. Notes on the Determination of the Physical Properties of Steel. By M. Kaichiro Imaizumi, *Kōgakushi* ... 219

These publications are issued at irregular intervals. When about 300 pages are reached, they are formed into one volume.

PRINTED BY THE TOKYO PRINTING CO. L'D.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
VOL. V. No. 2.

Published May, 1910.

The Action of Acids in the Enzymic Decomposition of Oil by Castor Seeds.

By

Yoshio Tanaka, *Kōgakushi.*

The existence of a fat-splitting enzyme in oily seeds was suggested by Schützenberger as long ago as 1876. Reynolds Green[1] in 1889 definitely proved the presence, in the germinating seeds of *Ricinus*, the castor-oil plant, of an enzyme capable of splitting up fat. According to him this lipase, to adopt the recent term, works most advantageously in a neutral solution and also in the presence of dilute alkalies, but he did not prove the similar fact with the lipase from resting seeds. He found only that the resting seeds of castor-oil plant, which are themselves inactive, developed a lipolytic activity on being kept in dilute acid.

In 1902, Connstein, Hoyer, and Wartenberg[2] published the results of some very careful researches upon this subject. In contrast to Green they found that the activity of the lipase of castor seeds does not manifest itself at all in an alkaline medium but is most active in a strong acid one, the best concentration of the acid being between N/10 and N/3 strength. This enzyme has since been made the subject of detailed studies by more recent investigators and it has been shown that the addition of acid is essential for lipolytic hydrolysis by castor seeds.

---

[1] R. Green's Soluble Ferments and Fermentation, 2nd ed., 1901, p 243

[2] Connstein, etc., Berichte, 1902, p. 3968–4006.

But scarcely any careful consideration has been given to the rôle played by the acid that is used in the hydrolysis of oil by castor seeds. Green presumed that the acid liberated the active lipase from the zymogen which exists in the castor seeds. But this was not completely established on a sound experimental basis. H. E. Armstrong[1] found that hydrolysis by Ricinus lipase could be brought about only in the presence of acid and that the acid does not act by liberating the enzyme. Therefore a further justification and a more accurate knowledge of the entire subject is needed. The object of the present paper is to record a number of results which have been obtained chiefly with reference to the rôle of the acid in the lipolytic hydrolysis of oil by castor seeds.

Experimental.

Experiments were made with the pressed cake of decorticated castor seeds. As the oil to be hydrolysed by the lipase, the author used that of the soja bean.

Action of mineral acids.

It is well known that the lipase of castor-oil seeds is rendered capable of splitting up fatty oils by the addition of a soluble acid to the medium in which it is at work. But the quantity of acid to be added seems to stand in close relationship to the amount of the castor-oil seeds.

Experiment 1.

Two samples of 50 grams each of soja bean oil were mixed with different quantities of powdered castor seed cake, equal quantities of water, and N/10 sulphuric acid; the mixture was allowed to stand for five hours at 35°C. The result was as follows:

---

1 H. E. Armstrong, Jour. Chem. Soc., 1906, Abstr. i, p. 126.

	Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 sulphuric acid added.	C. c. of water added.	Per cent. of soja bean oil hydrolysed in five hours.
(*a*)	50	5	10	10	85·56
(*b*)	50	2	10	10	16·41

In this experiment the quantity of the acid added to each sample and the concentration of acid in the media were respectively equal. If the concentration of the acid were the sole factor of the lipolytic action of castor seed, the amount of change would be approximately proportional to the quantity of acting enzyme present, at least at the beginning of the action. On referring to the above table it will be noticed that the oil hydrolysed in (a) amounts to more than five times that of (b) for an amount of castor seeds only 2·5 times that of the latter. The results suggest that the quantity of acid to be added has a direct relation to the amount of castor seeds.

Experiment 2.

50 grams each of soja bean oil were hydrolysed at 40°C. by an equal amount of pressed castor seeds, with different quantities of N/10 sulphuric acid; the amounts of oil decomposed in one and two hours were as follows:

	Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 sulphuric acid.	C. c. of Water.	Per cent. of soja bean oil hydrolysed in	
					one hour.	two hours.
(*a*)	50	2	0	10	0·0	0·0
(*b*)	50	2	2	8	11·72	20·82
(*c*)	50	2	4	6	28·42	49·81
(*d*)	50	2	5	5	29·35	51·86
(*e*)	50	2	6	4	29·01	51·57
(*f*)	50	2	7	3	23·44	41·31
(*g*)	50	2	8	2	16·11	32·51
(*h*)	50	2	10	0	9·08	18·45

It seems that, in the hydrolysis of the oil by the lipase of castor seeds, as the quantity of the acid added increases, the activity of the lipase increases up to an optimum which is reached when 5—6 c.c. of N/10 sulphuric acid for every two grams of the castor seed cake are required; it then falls off. That the maximum lipolytic action obtained in the case of (d) or (e) was not due to the optimum concentration of soluble acid in the media (viz. N/16—N/20 sulphuric acid) is proved by the following experiment.

Experiment 3.

To 50 grams of soja bean oil were added varying amounts of N/20 sulphuric acid, the concentration of the acid being the same as the optimum in the last experiment.

The results obtained with equal amounts of castor seeds are given below:

Grams of oil.	Grams of pressed castor seeds.	C. c. of N/20 sulphuric acid.	Per cent. of oil decomposed in	
			one hour.	two hours.
50	2	10	28·7	51·3
50	2	20	10·3	19·0
50	2	30	2·9	5·8
50	2	40	1·5	3·2

These figures show that different quantities of acid of the same concentration do not produce equal effects in the enzymic hydrolysis of oil. It follows that the absolute amount of acid to be added is the essential factor of lipolytic action and that its concentration has no conspicuous effect upon the activity of castor seeds. These results are confirmed by the following experiment.

Experiment 4.

Equal amounts of soja bean oil were hydrolysed at 40°C. with the same quantity of pressed castor seeds, adding equal amounts of N/10 sulphuric acid and varying the amount of water.

Grams of soja been oil.	Grams of pressed castor seeds.	C. c. of N/10 sulphuric acid.	C. c. of water.	Per cent. of oil decomposed in	
				one hour.	two hours.
50	2	5	0	27·3	49·8
50	2	5	5	28·7	51·5
50	2	5	7·5	29·3	51·6
[illegible]	[illegible]	[illegible]	10·0	30·1	52·4

With the Compliments of the Director of the Engineering College.

Experiment 5.

Different quantities of soja bean oil were hydrolysed at 38°C. by equal amounts of pressed castor seeds and acid water of equal concentration; the amounts of oil decomposed in one hour were as follows:

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 sulphuric acid.	C. c. of water.	Grams of oil decomposed in one hour.
25	1	3	5	7·79
30	1	3	5	7·81
35	1	3	5	7·85

It seems that, in the hydrolysis of the oil by the lipase of castor seeds, as the quantity of the acid added increases, the activity of the lipase increases up to an optimum which is reached when 5—6 c.c. of N/10 sulphuric acid for every two grams of the castor seed cake are required; it then falls off. That the maximum lipolytic action obtained in the case of (d) or (e) was not due to the optimum concentration of soluble acid in the media (viz. N/16—N/20 sulphuric acid) is proved by the following experiment.

Experiment 3.

	castor seeds.	sulphuric acid.	one hour.	two hours.
50	2	10	28·7	51·3
50	2	20	10·3	19·0
50	2	30	2·9	5·8
50	2	40	1·5	3·2

These figures show that different quantities of acid of the same concentration do not produce equal effects in the enzymic hydrolysis of oil. It follows that the absolute amount of acid to be added is the essential factor of lipolytic action and that its concentration has no conspicuous effect upon the activity of castor seeds. These results are confirmed by the following experiment.

Experiment 4.

Equal amounts of soja bean oil were hydrolysed at 40°C. with the same quantity of pressed castor seeds, adding equal amounts of N/10 sulphuric acid and varying the amount of water.

Grams of soja been oil.	Grams of pressed castor seeds.	C. c. of N/10 sulphuric acid.	C. c. of water.	Per cent. of oil decomposed in	
				one hour.	two hours.
50	2	5	0	27·3	49·8
50	2	5	5	28·7	51·5
50	2	5	7·5	29·3	51·6
50	2	5	10·0	30·1	52·4
50	2	5	15·0	28·4	49·7

Thus, there is no appreciable difference in the results when the absolute quantities of the acid added are equal, in spite of the varying concentration of the acid in medium, that is, the absolute quantity of acid added, but not its concentration, is the factor essential for the activity of Ricinus lipase.

Experiment 5.

Different quantities of soja bean oil were hydrolysed at 38°C. by equal amounts of pressed castor seeds and acid water of equal concentration; the amounts of oil decomposed in one hour were as follows:

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 sulphuric acid.	C. c. of water.	Grams of oil decomposed in one hour.
25	1	3	5	7·79
30	1	3	5	7·81
35	1	3	5	7·85

Thus, the same quantity of pressed castor seeds, disintegrated in the different quantities of oil, hydrolyses the same amount of oil at equal intervals, after the addition of the same quantity of acid. These results show that the amount of acid required for a definite quantity of castor seeds to obtain the maximum of hydrolysis does not stand in relation to the amounts of the fatty oil, but to the quantity of the castor seed alone. One gram of pressed castor seeds used in the present investigation, required 2·5—3·0 c.c. of N/10 sulphuric acid for the maximum hydrolysis.

The same things are true of nitric acid, as well as of hydrochloric, as may be seen in the following experiment:

Experiment 6.

25 grams of soja bean oil were hydrolysed at 40°C. by the same amount of pressed castor seeds, adding different quantities of N/10 acids; the amount of oil decomposed was as follows:

Nitric acid.

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 nitric acid.	C. c. of water.	Per cent. of oil decomposed in one hour.
25	1	1·5	4·5	18·3
25	1	2·0	4·0	28·7
25	1	2·5	3·5	31·6
25	1	3·0	3·0	30·8
25	1	3·5	2·5	26·4

Hydrochloric acid.

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 hydrochloric acid.	C. c. of water.	Per cent. of oil decomposed in	
				one hour.	two hours.
25	1	1·5	3·5	21·2	——
25	1	2·0	3·0	30·0	49·5
25	1	2·5	2·5	32·5	52·9
25	1	3·0	2·0	31·5	51·3
25	1	4·0	1·0	19·0	35·2

These figures show that the optimum amounts of the acids for one gram of the pressed castor seeds are 2·5—3 c.c. of N/10 strength.

Non-necessity of free mineral acid in lipolytic medium.

As has been pointed out above, the addition of a definite quantity of mineral acid for a definite quantity of castor seeds is necessary and its concentration is not the main factor of the lipolytic action. From these results, it is highly probable that the added acid does not act merely by making the medium acidic, but must act chemically upon the constituents of the castor-oil seeds. Experiments were then made as to the acidity of the medium in which the enzyme is at work. But it is somewhat difficult to determine by any simple method whether the added acid remains in a free state or not. The author undertook the following experiment to determine the acidity of the extract prepared by steeping the pressed castor seeds in the optimum quantity of mineral acid.

Experiment 7.

The extract which was made from the ground castor seed cake by triturating it with the optimum guantity of N/10 sulphuric acid, was used in place of the acidified water required in the lipolytic hydrolysis of oil; the amount of oil hydrolysed in one hour was as follows:

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of extract obtained by digesting 5 grs. of pressed castor seeds with 12·5 c.c. of N/10 sulphuric acid and 20 c.c. of water for 20 minutes.	Per cent. of oil decomposed in one hour.
25	1	6·5	1·1

If the added acid be in the free state, then 6·5 c.c. of the above extract will contain 2·5 c.c. of N/10 sulphuric acid, and hence the enzymic hydrolysis of oil by castor seeds must be considerably promoted in the above experiment. The result, as may be seen in the above table, shows no marked promotion of the hydrolysis.

In the present experiment, a further addition of acid produced a high yield of fatty acid as follows:

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 sulphuric acid.	C.c. of the extract as above.	Per cent. of oil hydrolysed in one hour.
25	1	2·5	6·5	44·5

From these results, it is almost certain that the acid added in the optimum quantity does not exist in a free state in the medium. I found also that an extract obtained by digesting pressed castor seeds with the optimum quantity of mineral acid was indifferent to methyl-orange but slightly acidic to litmas. This acidity to litmas seems to be due to a weak acid liberated in the decomposition of certain salts by the added mineral acid. To determine this acidity the following experiment was undertaken.

Experiment 8.

6 grams of pressed castor seeds were triturated with 36 c.c. of N/20 sulphuric acid and filtered. The filtrate was treated with pure alcohol and the precipitates produced were filtered off. The clear filtrate was titrated with standard alkali solution, using phenolphthalein as indicator. By comparison with the control experiment, in which 6 grams of pressed castor seeds had been treated with 36 c.c. of water, the following result was obtained:

	C. c. of N/10 caustic potash required.
Chief experiment	2·2
Control experiment	0·3

By subtracting the natural acidity, an acid corresponding to 1·9 c.c. of N/10 alkali solution remained, that is, 0·31 c.c. of N/10 acid for each gram of pressed castor seeds. This acid is neutral to methyl-orange. Thus, the mineral acid added in the lipolytic hydrolysis of oil by castor seeds does not remain in a free state and a small amount of an unknown weak acid which is not enough to promote the lipolytic power of castor seeds is liberated. From these results, it is, in all probability, to be deduced that the acidity in lipolytic medium is not necessary for the hydrolysis, as will be also proved in subsequent experiments

Action of Organic Acids.

In the lipolytic hydrolysis by castor seeds, the actions of organic acids are somewhat different from those of mineral acids.

Experiment 9.

The optimum amounts of several organic acids may be seen in the following tables, the hydrolysing temperature being 38°C.

(i) Formic Acid.

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/5 formic acid.	C. c. of water.	Per cent. of oil decomposed in one hour.
25	1	1·0	6·0	34·7
25	1	1·25	5·75	37·5
25	1	1·5	5·5	38·2
25	1	2·0	5·0	35·3
25	1	2·5	4·5	29·8
25	1	3·0	4·0	22·0

(ii) Acetic Acid.

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/5 acetic acid.	C. c. of water.	Per cent. of oil decomposed in one hour.
25	1	1·5	5·5	36·6
25	1	2·0	5·0	39·9
25	1	2·5	4·5	38·8
25	1	3·0	4·0	37·5
25	1	4·0	3·0	35·2
25	1	5·0	2 0	31·1

(iii) Propionic Acid.

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/5 propionic acid.	C. c. of water.	Per cent. of oil decomposed in one hour.
25	1	1·5	5·5	37·5
25	1	2·0	5·0	40·8
25	1	2·5	4·5	41·0
25	1	3·0	4·0	40·0
25	1	4·0	3·0	39·3

(iv) Butylic Acid.

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/5 Butylic acid.	C. c. of water.	Per cent. of oil decomposed in one hour.
25	1	1·5	5·5	29·6
25	1	2·0	5·0	39·3
25	1	2·5	4·5	43·5
25	1	3·0	4·0	44·0
25	1	4·0	3·0	44·0
25	1	5·0	2 0	43·6
25	1	6·0	1·0	42·5
25	1	7·0	0·0	41·0

(v) Oxalic Acid.

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 oxalic acid.	C. c. of water.	Per cent of oil decomposed in one hour.
25	1	2·0	5·0	21·7
25	1	2·5	4·5	27·5
25	1	3·0	4·0	26·7
25	1	4·0	3·0	19 0
25	1	5·0	2·0	10·2

(vi) Lactic Acid.

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of N/10 lactic acid.	C. c. of water.	Per cent. of oil decomposed in 90 minutes.
25	1	3·0	4·0	44·5
25	1	4·0	3·0	41·6
25	1	5·0	2·0	37·6
25	1	6·0	1·0	29·3
25	1	8·0	0·0	17·5

As can be seen from the table, the optimum acidity of an acid in the acetic series is relatively greater the higher the homologue is and the inhibitory action of an acid due to its excess is greater the lower this homologue is. The retarding effect exerted by an excess of oxalic and lactic acids, which are stronger than acetic or propionic acid is more remarkable than the latter is. Thus, the optimum quantity of a weaker organic acid is larger than that of a stronger one, and the degree of retarding action due to an excessive quantity of the former is less than that of the latter.

It has been remarked in experiments 7 and 8 that an inorganic acid which had been added to the pressed castor seeds in the optimum quantity did not exist in the free state. In the case of organic acids the same may be also seen.

Experiment 10.

The present experiment was carried out like experiment 7; the acid extract of pressed castor seeds was used instead of acidified water in the hydrolysis of oil by castor seeds:

Grams of soja bean oil.	Grams of pressed castor seeds.	C. c. of acid extracts.	Per cent. of oil decomposed in 60 minutes.
25	1	Extract A. 7·0	1·0
25	1	,, B. 7·0	1·7
25	1	,, C. 7·0	44·5

Extract A was prepared by triturating 5 grs. of pressed castor seeds with 15 c.c. of N/10 oxalic acid and 20 c.c. of water for 10 minutes.

Extract B was prepared by triturating 5 grs. of pressed castor seeds with 15 c.c. of N/10 butylic acid and 20 c.c. of water for 10 minutes.

Extract C was prepared by triturating 5 grs. of pressed castor seeds with 25 c.c. of N/5 butylic acid and 10 c.c. of water for 10 minutes.

The results prove indirectly that the extracts A and B do not contain an appreciable amount of free acid, whereas extract C contains a marked amount of free acid.

This experiment, taken in conjunction with those recorded in experiment 9, (iv) and (v), indicates that castor seed lipase acts

well in the absence of free acid and also in the presence of weak organic acids such as butylic acid.

Experiment 11.

The addition of a weak organic acid in a slight excess above the optimum of the mineral acid retards the lipolytic action of castor seeds in a very slight degree as may be seen in the following table, the hydrolysing temperature being 38°C.

Grams of oil.	Grams of pressed castor seeds.	C. c. of N/10 sulphuric acid.	C. c. of N/10 butylic acid.	C. c. of water.	Per cent. of oil decomposed in 60 minutes.
25	1	3	0	4	29·3
25	1	3	1	3	29·0
25	1	3	2	2	28·4

The Optimum Quantity of Acid to be added.

The optimum amounts of mineral and comparatively easily dissociable organic acids required for the greatest activity of castor seeds are proportional to an acidity equivalent to NaOH. For example, the optimum of sulphuric or hydrochloric acid for one gram of pressed castor seeds which is used in the present investigation is 2·5—3·0 c.c. of N/10 strength. In the case of the less dissociable organic acids, such as acetic, propionic, and butylic acids, the optimum amounts are greater.

The following formulæ will be found convenient for calculating the optimum quantity of an acid for a definite amount of castor seeds:

$$Q = E.\ c.q.$$

Where Q is the optimum quantity of added acid required for an certain amount of castor seeds or seed cake, q; E is the equivalent weight of the acid neutralizable by NaOH, c a constant for the castor seeds or seed cake.

As an example, the values of c for the pressed castor seeds used in the present investigation are as follows:

Acid.	Value of c.
Sulphuric acid	0·00025 — 0·0003
Hydrochloric acid	0·00025 — 0·0003
Nitric acid	0·00025 — 0·0003
Oxalic acid	0·00025 — 0·0003
Lactic acid	0·00025 — 0·0003
Formic acid	0·00025 — 0·0003
Acetic acid	0·0004 — 0·0005
Propionic acid	0 0004 — 0·0006
Butylic acid	0·0005 — 0·0010

Any acid is effective to activate the castor seed. According S. Fokin,[1] oxalic acid considerably weakens the enzymic action of castor seed, and phosphoric and nitric acids destroy the enzyme. But the writer does not recognise the inhibitory effect of the acids named, provided a proper amount be used, as already stated.

Zymogen of Lipase.

J. R. Green[2] suggests that the lipase exists as an inactive zymogen in the resting seeds of Ricinus which can be activated by weak acids. H. E. Armstrong[3] states that, in the hydrolysis of fatty oils by the lipase of castor seeds, no action takes place unless

1 S. Fokin, Chem. Rev. Fett u. Harz Ind., 1906, S. 193.
2 J. R. Green, Jour. Soc. Chem. Ind., 1890, Abst. p. 649.
3 H. E. Armstrong, Jour. Chem. Soc., 1906, Abstr. i, p. 126.

an acid is present, and that the added acid does not act merely by liberating the enzyme.

I was now interested to reach a more definite understanding of the matter.

Experiment 12.

5 grams of the pressed cake of castor seeds were triturated with 30 c.c. of N/20 sulphuric acid for 30 minutes and the milky liquid was filtered. The insoluble residue was washed completely free from acid or other soluble matters. The final product which was now in a pasty condition was triturated with 50 grams of soja bean oil and about 10 c.c. of water in a porcelain mortar and allowed to stand for about 18 hours at the room temperature. Analysis showed that about 76·5% of the oil had been decomposed, though no free acid was added in this case.

The fact that the lipase preparation obtained by the above method has a strong power of decomposing fatty oil without the addition of any acid, leads the writer to draw the conclusion that, as suggested by Green, lipase exists in the state of inactive zymogen in castor seeds and the latter can be readily developed by dilute acids into the active enzyme. The writer also concludes that castor seed lipase exerts its power in a neutral medium. These conclusions agree well with the previously mentioned observation that, in spite of the absence of free acid which was added in the optimum quantity to the pressed castor seeds, the lipolytic hydrolysis is considerably promoted.

This opinion of the writer is contrary to the generally accepted one held by the present investigators, such as Connstein, Hoyer, Braun, Behrendt, Fokin, Taylor, and Armstrong, who

maintain that the castor seed lipase acts only in presence of acid.

It is also noticed that, contrary to the opinion of Green[1], the liberated enzyme is not soluble in water and that the presence of a slight excess of weak organic acid in the lipolytic medium does not exibit a marked inhibitory action towards the enzyme, whilst weak alkalies have a deleterious influence.

It is also proved by experiment, that the added acid also combines with certain constituents of the castor seed, especially the nitrogenous matters, and passes into solution.

Experiment 13.

The present experiment demonstrates a difference between lipase and its zymogen with regard to their behaviour towards dilute alkali.

(a) 2 grams of pressed castor seeds were triturated with 10 c.c. of N/50 sodium carbonate for 10 minutes.

(b) 6 c.c. of N/10 sulphuric acid were added to 2 grams of pressed castor seeds and the mixture was then triturated with 4 c.c. of N/5 sodium carbonate solution for 10 minutes.

The two samples differed only in that (a) had beed treated with the alkali, while (b) had been acted on by the acid before being treated with the alkali, the resulting alkalinity being equal. At the end of the time stated, they were triturated with 8 c.c. of N/10 sulphuric acid and 50 grams of soja bean oil at 35°C. respectively.

The amounts of the oil decomposed in 60 minutes were as follows:

---

1 R. Green's Soluble Ferments and Fermentation, 2nd ed., 1901, p. 243.

	Grams of oil decomposed in 60 minutes.
(a)	11·7
(b)	4·4

These figures show that lipase is more rapidly destroyed by a dilute alkali solution than its zymogen is, as is also the case with pepsin and its antecedent.

Summary.

1. The addition of the proper quantity of acid to the castor seeds is necessary in hydrolysing fats by the latter. The absolute quantity of the added acid is the main factor of the lipolytic activity of castor seeds and its concentration has no marked influence.

2. The optimum amount of acid required for a definite amount of castor seeds in order to obtain the maximum hydrolysis does not stand in relationship to the quantity of oil, but to the amount of castor seeds alone. It is not the same in every acid. In the case of mineral and comparatively strong organic acids, their optimum quantities are proportional to their equivalent weights corresponding to NaOH, while, in the less dissociable organic acids they are larger. It is also found that the optimum quantity of acid in the acetic series increases as the homologue ascends.

3. The effect of a highly dissociable acid on the castor seeds is, in general, more remarkable than that of a less dissociable one, that is, the higher dissociable acid increases the lipolytic action of the castor seed more, and subsequently retards it more when the optimum is passed.

4. Castor seeds contain lipase in the form of an insoluble zymogen, which can be readily converted by dilute acid into the insoluble actual enzyme.

5. The acid added to the castor reeds, in its optimum quantity does not act by acidifying the medium in which the lipase of castor seeds acts, but chiefly by developing the enzyme from its zymogen. The acid also combines wtih certain basic constituents in castor seeds, especially nitrogenous matters, to produce soluble, neutral compounds.

6. A lipase preparation which is obtained by treating pressed castor seeds with the optimum quantity of any acid and by completely washing out all the soluble matters with water, has a strong power of hydrolying fatty oils without the use of any acid. From this, it follows necessarily that lipase exists in castor seeds as zymogen and that lipase is absolutely insoluble in water and active in a neutral medium.

7. Active lipase works most energetically in a neutral medium and it is less active in the presence of free acid, especially of mineral acid; it is inactive in an alkaline medium.

8. Lipase is less stable towards dilute alkalies than is its zymogen.

January 15, 1910.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERCITY, TOKYO, JAPAN.
Vol. V. No. 3.

larch, 1912.

ERRATA.

page	line	
10	16	' $A\,T_d = m\,w \frac{0.57}{\frac{\tau}{b}} \sin\left(\frac{\tau}{b}\,\frac{\pi}{2}\right) I \sin\psi$ ' should read $A\,T_d = m\,w \frac{0.57}{\frac{b}{\tau}} \sin\left(\frac{b}{\tau}\,\frac{\pi}{2}\right) I \sin\psi$
14	19	' total mumber of turns ' should read total number of turns
16	14	' $W_a = E\,I \cos\phi$ ' should read $W_a = E\,I \cos\psi$
16	16	' $= \frac{\omega}{P} D$ ' should read $= \frac{\omega}{p} D$
17	20	' Equstion ' should read Equation
21	12	' $W'_{a\,max} = \frac{P^2\,\varepsilon^2}{r[\ +\alpha^2)}$ ' should read $W'_{a\,max} = \frac{P^2\,\varepsilon^2}{r(1+\alpha^2)}$
27	4	' x_l'' is not equal to x_l ' should read x_l'' is not equal to x_l'
39	Table II_a	' E ' should read E_P
41	18	' ollows ' should read follows

tors

one of the
or charac-
from the
nt is, to a
Prof. E.

the motor
the open
are given
notor.
sake of
respective
graphical
ing a clear
e result is
problems.
parameter

which leads us to a simpler process so far as its application to practical problems is concerned.

4. Castor seeds contain lipase in the form of an insoluble
zymogen, w
the insolubl
5. The aci
does not ac
of castor se
from its zy
basic const
matters, to
6. A lipas
castor seeds
completely
has a stro
of any aci
exists in ca
insoluble in
7. Active
medium an
especially
medium.
8. Lipase
zymogen.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERCITY, TOKYO, JAPAN.
Vol. V. No. 3.

Published March, 1912.

The Characteristics of Synchronous Motors

By

M. Shibusawa, *Kōgakushi.*

Introduction.

The armature reaction in a synchronous motor is one of the leading items in the solution of the problem of motor characteristics. In Chapter I therefore it is fully discussed from the theoretical point of view, though the method of treatment is, to a certain extent, a modification of those investigated by Prof. E. Arnold.

Chapter II deals with the no-load characteristics of the motor and shows the meaning of the constants, namely, the open circuit coefficient and the short circuit coefficient, which are given by the author in explanation of the performance of the motor.

In Chapter III, the motor is assumed, for the sake of simplicity, to have a constant internal reactance irrespective of the load. Such a motor is treated, in general, by the graphical method. Although this method is convenient for getting a clear idea of the properties of the machine, yet frequently the result is not accurate enough to admit of its use in actual problems. Keeping this in mind, the author introduces a new parameter which leads us to a simpler process so far as its application to practical problems is concerned.

The author works out in Chapter IV a series of equations and curves as regards the general behavior of the loaded motor. Thus we can design a motor in accordance with any characteristics required, and also calculate the size of the conductors of a transmission line suitable for any given synchronous motor.

Finally in Chapter V experimental results are given for the purpose of enabling a comparison with those calculated.

Chapter I. Armature Reactions.

The armature reactions of synchronous machines have been fully discussed by many authorities. We therefore attempt to treat them according to the theories introduced by Prof. E. Arnold (Die Wechselstromtechnik IV.) revised to a certain extent.

The armature current of a synchronous machine produces two varieties of magnetic flux. The first opposes or assists and distorts the magnetic flux produced by the exciting field, but the second is quite independent of them, not passing through the magnetic pole pieces. The former induces magnetic reaction and the latter self induction.

§ 1. Effective Armature Resistance.

The effective resistance of armature conductors is, as a general rule, larger than their ohmic resistance, owing to the eddy current in the armature iron and conductors, and to the induced current in the exciting field circuit. Therefore we may put

$$r_a = c\, r_g \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where r_a = effective resistance

r_g = ohmic resistance

c = a constant

The value of c lies between 1.5 and 2.5 for single-phase, and 1. 3 and 2 for poly-phase machines.

§ 2. Self Inductance.

Let s_n = number of armature conductors per slot in series,

q = number of slots per phase per pole,

p = number of pairs of poles,

w = $p\,q\,s_n$ = number of turns in series per phase,

λ_x = magnetic conductivity of the circuit surrounding the conductors per cm not passing pole pieces,

l_x = length of conductors in cm where λ_x is concerned,

L = coefficient of self induction,

ϕ_x = magnetic flux interlinking the conductors s_n when one ampere of current is flowing in them.

$$= s_n l_x \lambda_x$$

Then
$$L = \Sigma(s_n \phi_x)10^{-8} = 2pqs_n^2 \Sigma l_x \lambda_x 10^{-8} \text{ Henry.}$$

where the summation is to be done over a half length of the armature coil.

We find thus the reactance due to self induction is

$$x_a = 2\pi n L = 4\pi n p q s_n^2 \Sigma l_x \lambda_x 10^{-8}$$

$$= \frac{4\pi n w^2}{pq} \Sigma(l_x \lambda_x) 10^{-8} \quad \text{.........................} \quad (2)$$

To calculate the value $\Sigma\,(l_x\,\lambda_x)$ it is convenient to distinguish three classes of magnetic circuits:—

(a) The circuit surrounding each slot,

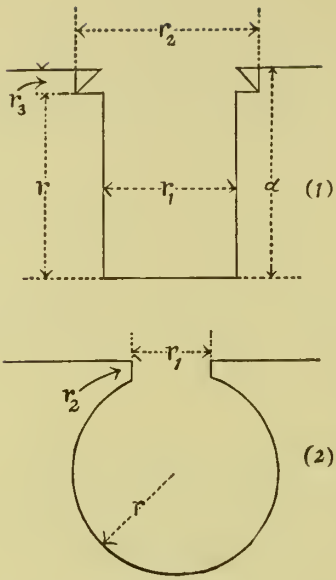


Fig. 1.

(b) The circuit surrounding one or more slots and passing from one end of a tooth to the other through the air,

(c) The circuit surrounding the coils on the outside of the iron.

(*a*) $\Sigma(l_x\lambda_x)$ of the circuit '*a.*'

Here l_x is the same for all the magnetic circuits and is equal to the length of the coil inside of the iron $= l_i$.

Put $\lambda_n =$ magnetic conductivity of the circuit '*a.*'

Then for the slot (1), Fig. 1.

$$\lambda_n = 0.4\pi\left(\frac{r}{3r_1} + \frac{2r_3}{r_1+r_2}\right) \qquad (3)$$

and for (2)

$$\lambda_n = 0.4\pi\left(0.623 + \frac{r_2}{r_1}\right) \qquad (4)$$

(See P. 44, E. Arnold, Die Wechselstromtechnik IV.)

(*b*) $\Sigma(l_x\lambda_x)$ of the circuit '*b.*'

Put $\lambda_k =$ magnetic conductivity of the circuit '*b,*'

or, $\lambda_k l_i = \Sigma(l_x\lambda_x)$.

Let us first assume the number of the slots per phase per pole to be one or q = 1. Consider the coils in the shaded slot in Fig. 2, and put $\lambda_k^{I}, \lambda_k^{II}, \lambda_k^{III}$......for the conductivity of the magnetic circuits passing through the 1st, 2nd, 3rd......teeth on both sides of the slot respectively and $\lambda_l^{I}, \lambda_l^{II}, \lambda_l^{III}$......for those passing through the 1st, 2nd, 3rd......slots. Put $z=r_1+t_1$, then

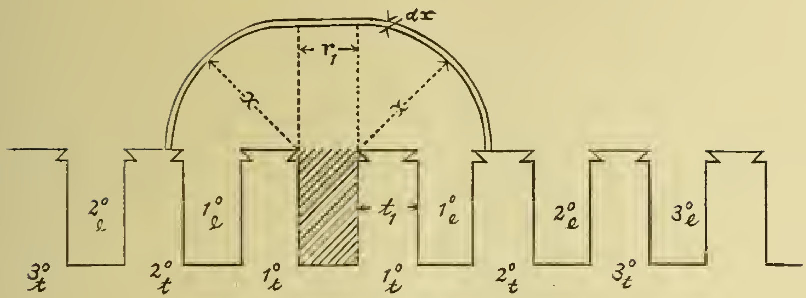


Fig. 2.

$$\lambda_k^{I} = 0.4\pi\int_{x=0}^{x=t_1}\frac{dx}{r_1+\pi x} = 0.4\times 2.3\log\left(1+\frac{\pi t_1}{r_1}\right) = 0.92\log\left(1+\frac{\pi t_1}{r_1}\right)$$

$$\lambda_k^{II} = 0.4\pi\int_{x=z}^{x=z+t_1}\frac{dx}{r_1+\pi x} \qquad = 0.92\log\left(1+\frac{\pi t_1}{r_1+\pi z}\right)$$

$$\lambda_k^{III} = 0.4\pi\int_{x=2z}^{x=2z+t_1}\frac{dx}{r_1+\pi x} = \qquad = 0.92\log\left(1+\frac{\pi t_1}{r_1+2\pi z}\right)$$

etc. etc.

$$\lambda_l^{I} = 0.4\pi\int_{x=t_1}^{x=z}\frac{dx}{2d+r_1+\pi x} = 0.4\times 2.3\log\left(1+\frac{\pi r_1}{2d+r_1+\pi t_1}\right)$$

$$= 0.92\log\left(1+\frac{\pi r_1}{2d+r_1+\pi t_1}\right)\text{nearly}$$

$$\lambda_l^{II} = 0.4\pi\int_{x=z+t_1}^{x=2z}\frac{dx}{2d+r_1+\pi x} = \qquad = 0.92\log\left(1+\frac{\pi r_1}{2d+r_1+\pi(z+t_1)}\right)$$

etc. etc.

Put $\quad \lambda_k^{(n)} = \lambda_k^{N}+\lambda_k^{N+1}+\ldots\ldots+\lambda_l^{N}+\lambda_l^{N+1}+\ldots\ldots$

The values of $\lambda_k^{N}, \lambda_k^{N+1}, \ldots\ldots \lambda_l^{N}, \lambda^{N+1}\ldots\ldots$ are very small if N is large enough and therefore we may assume that

$$\lambda_k^{(n)} = 0.4\pi\int_{x=z(n-1)}^{x=\tau}\frac{dx}{\pi x} = 0.92\log\frac{\tau}{z\,(n-1)} \quad \text{nearly}$$

where τ = pole pitch.

Then we have

$$\lambda_k = \lambda_k^I + \lambda_k^{II} + \ldots\ldots + \lambda_l^I + \lambda_l^{II} + \ldots\ldots + \lambda_k^{(n)} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)_1$$

If the number of slots per phase per pole is two, or q = 2, the flux produced by one ampere of current in the coils of a slot passing through the circuit whose magnetic conductivity is

	λ_k^I	surrounds the conductors	S_n	
,,	λ_k^{II}	,,	$2\ S_n$	
,,	λ_l^I	,,	$1.5\ S_n$	approximately
,,	λ_k^{III}	,,	$2\ S_n$	
,,	λ_l^{II}	,,	$2\ S_n$	approximately.

$$\therefore \quad \lambda_k = \lambda_k^I + 1.5\lambda_l^I + 2(\lambda_k^{II} + \lambda_k^{III} + \ldots\ldots + \lambda_k^{(n)}) \ldots\ldots\ldots\ldots\ldots \quad (5)_2$$

If the mumber of slots per phase per pole is three, or $q = 3$,

$$\lambda_k = \lambda_k^I + \tfrac{1}{3}(2+3+2)\lambda_k^{II} + 3(\lambda_k^{III} + \ldots\ldots) + \tfrac{1}{3}(1.5+2+1.5)\lambda_l^{II}$$
$$+ \tfrac{1}{3}(2.5+3+2.5)\lambda_l^{III} + 3(\lambda_l^{IV} + \ldots\ldots) + 3\lambda_k^{(n)}.$$

$$= \lambda_k^I + \frac{7}{3}\lambda_k^{II} + \frac{5}{3}\lambda_l^{II} + \frac{8}{3}\lambda_l^{III} + 3(\lambda_k^{III} + \ldots\ldots + \lambda_l^{IV} + \ldots\ldots + \lambda_k^{(n)}) \quad (5)_3$$

If q = 4,

$$\lambda_k = \lambda_n^I + \frac{10}{4}\lambda_k^{II} + \frac{14}{4}\lambda_k^{III} + \frac{7}{4}\lambda_l^{II} + \frac{12}{4}\lambda_l^{III} + \frac{15}{4}\lambda_l^{IV}$$

$$+ 4(\lambda_k^{IV} + \ldots\ldots + \lambda_l^{V} + \ldots\ldots + \lambda_k^{(n)}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)_4$$

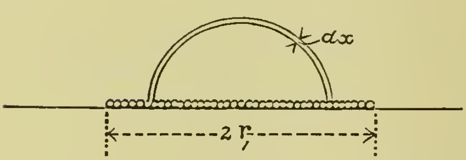


Fig. 3.

If the conductors are distributed over the surface of the armature iron,

$$S_n^2 \Sigma(l_x \lambda_x) = \Sigma\left(\frac{S_x^2}{R_x}\right)$$

where R_x is the magnetic reluctance of a tube of force interlinking S_x.

$$S_x = S_n \frac{x}{r_1}$$

$$\frac{1}{R_x} = 0.4\pi\frac{l_i d_x}{\pi x} = 0.4 l_i \frac{dx}{x}$$

$$\frac{S_x^2}{R_x} = S_n^2 l_i 0.4 \frac{x\,dx}{r_1^2}$$

$$\lambda_k = \int_{x=0}^{x=r_1} 0.4\frac{x\,dx}{r_1^2} + \int_{x=r_1}^{x=\tau} 0.4\pi\frac{dx}{\pi x}$$

$$= 0.2 + 0.92 \log\left(\frac{\tau}{r_1}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

(c) $\Sigma(l_x\lambda_x)$ of the circuit 'c.'

Put $\lambda\ l_s = \Sigma\,(l_x\lambda_x)$,

where l_s = length of the coil on the outside of the slot.

We shall give here its approximate value found by Prof. E. Arnold (Wechselstromtechnik IV. P. 49).

$$\lambda_s = 0.46 q_s (\log\frac{l_s}{d_s} - 0.2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where q_s = number of coils in the same phase approaching one another,

d_s = diameter of a circle whose circumference is equal to the peripheral length of the total conductors $s_n q_s$ including the insulating materials and air spaces.

§ 3. Magnetic Reactions.

(a). Effective M. M. F. due to Magnetic Reaction.

It is a well known fact that the magnetic field produced by a single phase current in the armature of a synchronous machine can be considered as two equal fields rotating in opposite directions. The amplitude of these fields is one half of the maximum value of their resultant field. Therefore, if the magnetic pole is rotating in synchronism with the current, one of the fields is rotating in

the same speed and direction with the magnetic pole, and the other in the opposite direction inducing E. M. F. in the exciting field circuit with double frequency. The former is generally called the synchronous, and the latter the inverse field. The inverse field is, as a general rule, strongly damped by the induced alternating current in the exciting field circuit.

The curve of the M. M. F. due to the armature current I of a single phase one slot machine along the surface of the armature is rectangular in shape having its amplitude $\sqrt{2}\,I\,w_n=\sqrt{2}\,I\,\frac{S_n}{2}$ per pole if the current is in a sine wave. Now we may decompose this rectangular wave into sine waves by Fourier's theorem and the amplitude of the fundamental wave is found to be,

$$\sqrt{2}\,\frac{I\,S_n}{2}\times\frac{4}{\pi} = 0.9\,I\,S_n$$

3rd harmonic $= \frac{1}{3}0.9\,I\,S_n$

5th harmonic $= \frac{1}{5}0.9\,I\,S_n$

etc. etc. (See Die Wechselstromtechnik I, P. 148)

This fundamental wave may be regarded as consisting of two fields rotating in opposite directions, each having an amplitude of $0.45\ I\ S_n$.

In a machine which has two or more slots per phase per pole, the wave of M. M. F. produced by the current in the conductors of each slot differs in phase from every other wave on account of the difference of their positions. Hence the arithmetical sum of the M. M. F. does not show their resultant, but a certain correcting factor f_w should be introduced into the equation:—

$$f_w = \frac{\sin\frac{q}{Q}\frac{\pi}{2}}{q\sin\frac{1}{Q}\frac{\pi}{2}}$$

where Q = number of slots per pole

q = number of slots per phase per pole.

Thus we find the amplitude of the synchronously rotating M. M. F. of a single phase machine is $0.45 f_w I S_n q$, neglecting the effect of higher harmonics.

In a poly-phase machine, each phase produces two M. M. F.s rotating in opposite directions. Their synchronous M. M. F.s being in the same phase are added together, while the inverse M. M. F.s cancel each other and become nil. Therefore the amplitude of the resultant synchronous M. M. F.s is:

$$A\,T_m = 0.45\, f_w\, m\, S_n\, q\, I \quad \text{per pole} \;\ldots\ldots\ldots\ldots \quad (8)_a$$

$$A\,T_m = 0.9\;\; f_w\, m\, w\, I \quad \text{total} \;\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)_b$$

where m is number of phases.

These are the M. M. F. s of the effective magnetic reaction due to the armature current of the alternator.

(b). Decomposition of Magnetic Reaction.

Let the equation of E. M. F. induced in an armature be $E \sin \omega t$ and that of current be $I \sin (\omega t - \psi)$ then

$$I \sin (\omega t - \psi) = I \cos \psi \sin \omega t - I \sin \psi \sin \left(\omega t - \frac{\pi}{2} \right)$$

The component $I \sin \psi$ is in the quadrature with the main E. M. F. and therefore strengthens or weakens the main magnetic field, while the component $I \cos \psi$ is in the same phase and therefore distorts the main field. The former is here called the direct magnetic reaction and the latter the cross magnetic reaction.

(c). Direct Magnetic Reaction.

The M. M. F. of the direct magnetic reaction is in the same

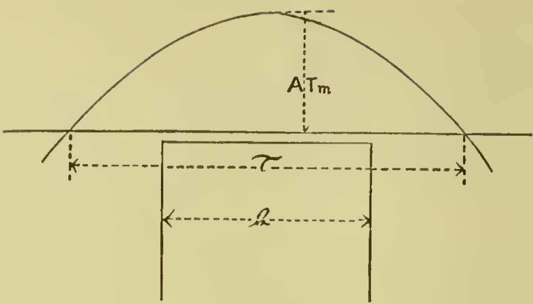


Fig. 4.

phase with the main exciting field and hence the mean value of the M. M. F. over the magnetic pole face is the effective reaction. Let K_α be the coefficient of the direct magnetic reaction,

then,
$$K_\alpha = \frac{1}{\frac{b}{\tau}\pi} \int_{x=-\frac{b}{\tau}\frac{\pi}{2}}^{x=+\frac{b}{\tau}\frac{\pi}{2}} \cos x \, dx = \frac{\sin\left(\frac{b}{\tau}\frac{\pi}{2}\right)}{\frac{b}{\tau}\frac{\pi}{2}} \quad \ldots\ldots\ldots\ldots \quad (9)$$

$$\therefore \quad A\,T_d = 0.45\, m f_w K_\alpha S_n q I \sin\psi \quad \text{per pole} \ldots\ldots\ldots \quad (10)_a$$

$$A\,T_d = 0.9\, m f_w K_\alpha w I \sin\psi \quad \text{total} \ldots\ldots\ldots\ldots \quad (10)_b$$

where $A\,T_d$ = effective M. M. F. of direct magnetic reaction. The above result is given by Prof. E. Arnold (Wechselstromtechnik IV. P. 53—P. 63). Mr. A. Russell gives also the same result (A Treatise on the Theory of Alternating Currents II. P. 76). Dr. G. Kapp gives the following values:

$$A\,T_d = m\,w \frac{0.57}{\frac{\tau}{b}} \sin\left(\frac{\tau}{b}\frac{\pi}{2}\right) I \sin\psi$$

$$= \cdot 895\, m K_\alpha w I \sin\psi,$$

and he adds that the difference of the coefficients for single-slot and multiple-slot machines is insignificant so that they can be assumed to be practically the same, (Dynamomaschinen. 4. Auflage, P. 430).

(d) Cross Magnetic Reaction.

The maximum M. M. F. of the cross magnetic reaction is:

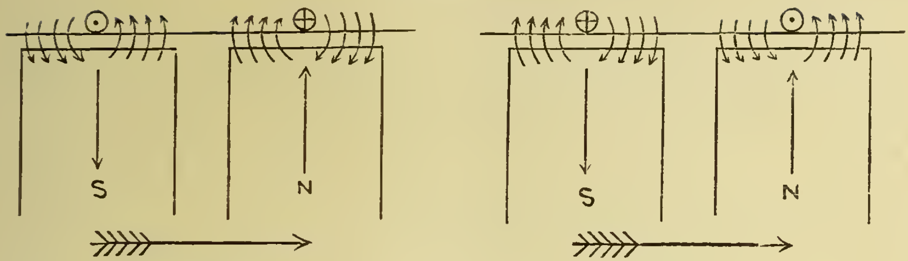


Fig. 5.

$$A\,T_{cm} = 0.45 f_w\, m\, q\, S_n\, I \cos\psi \quad \text{per pole}$$

$$= 0.9 f_w\, m\, w\, I \cos\psi \quad \text{total}$$

This M. M. F. produces a magnetic field in the air gap, as shown in Fig. 5. In the generator, the leading tips of the pole pieces are weakened and the trailing tips strengthened by this field; while in the motor, on the contrary, the former is strengthened and the latter weakened. Neglecting the effect of the magnetic saturation we may assume that the total magnetic flux remains unvaried by the cross magnetic reaction, although it is distorted.

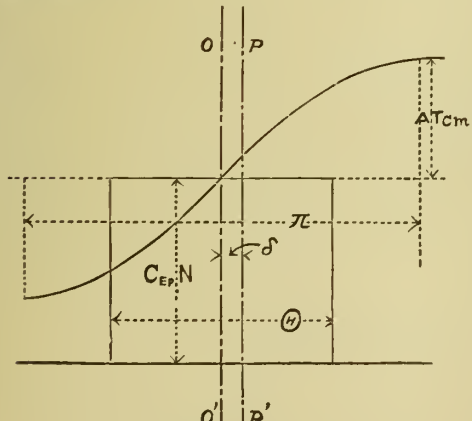


Fig. 6.

Let $00'$, Fig. 6, be the center line of the resultant M. M. F. $C_{Ep}\,N$, of the main exciting field and the effective direct magnetic reaction, and PP' be the new center line of the resultant M. M. F. taking the cross magnetizing M. M. F. into consideration,

and put $$\theta = \frac{b}{\tau}\pi$$

$$\delta = \overline{oo'} \wedge \overline{pp'}$$

then $$C_{Ep} N\left(\frac{\Theta}{2}-\delta\right)+\int_{x=\delta}^{x=\frac{\Theta}{2}} A\, T_{cm} \sin x\, dx = C_{Ep} N\left(\frac{\Theta}{2}+\delta\right)-\int_{x=\delta}^{x=\frac{\Theta}{2}} A\, T_{cm} \sin x\, dx$$

$$-2\, C_{Ep} N \delta + 2\, A\, T_{cm}\left(-\cos x\right)_{\delta}^{\frac{\Theta}{2}} = 0$$

$$\delta = \frac{A\, T_{cm}\left(\cos \delta - \cos \frac{\Theta}{2}\right)}{C_{Ep} N}$$

$$\delta^{0} = \frac{0.9 f_{w}\, m\, w\, I \cos \varphi\left(\cos \delta - \cos \frac{\Theta}{2}\right)}{C_{Ep} N} \frac{180}{\pi} \text{ in degrees} \qquad (11)_a$$

If δ^0 is very small, we may assume $\cos \delta = 1$,

$$\delta^{0} = \frac{0.9 f_{w}\, m\, w\, I \cos \varphi\left(1-\cos \frac{\Theta}{2}\right)}{C_{Ep} N} \frac{180}{\pi} \cdots\cdots\cdots\cdots \qquad (11)_b$$

The equations 11_a and 11_b have thus been deduced from the above described assumptions; that is to say, the weakened magnetic strength of one side of the pole piece is exactly equal to the strengthened magnetic strength of the other side, and the E. M. F. induced in the armature conductor is not affected by the deformation of the magnetic field which is caused by the cross magnetic reaction, but is proportional to the total magnetic flux. These assumptions may be safely made if the M. M. F. of the cross magnetic reaction is small, compared with the total resultant field.

Chapter II. No-Load Characteristics.

§ 4. Open Circuit Characteristics.

Although it is impossible to express the open circuit characteristic by an exact mathematical equation, as it depends entirely upon the quality of the iron, yet it is quite useful in

practice to know the degree of magnetic saturation. For this purpose the American Institute of Electrical Engineers gives a method in the sections 57 and 58 of the Standardization Rules issued in 1907. With this method the difficult point is to draw accurately a line tangent to the curve and a small deviation makes a large error. Therefore the author proposes the following method:—

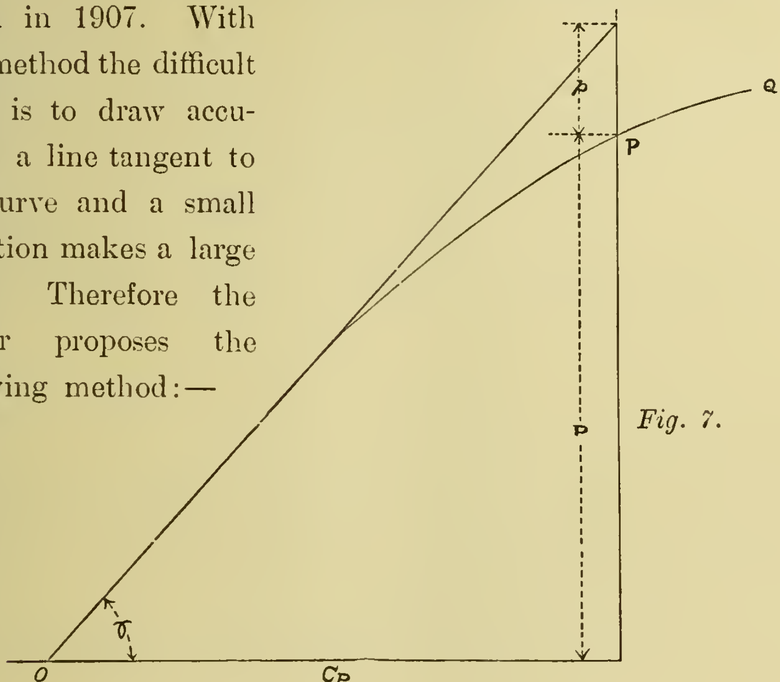


Fig. 7.

We draw a curve with a percentage of the normal voltage as ordinate and with a percentage of the exciting current at the normal voltage as abscissa, which we shall hereafter call the percentage open circuit characteristic. Then the inclination of this curve at the origin shows the degree of the magnetic saturation.

Let the curve OPQ in Fig. 7. be the open circuit characteristic, then

$$\tan\gamma = \frac{P+p}{C_p} = v \text{ ......voltage per unit exciting current when the iron is not saturated.}$$

If the curve is the percentage open circuit characteristic, then

$$\tan \gamma = \frac{100 + \frac{p}{P}100}{100} = 1 + \frac{p}{P} = e \dots\dots\dots \quad (12)$$

$$v = e\frac{P}{C_P} \dots\dots\dots\dots\dots\dots\dots\dots \quad (13)$$

Thus we see the value 'e' determines the open circuit characteristic and therefore 'e' is called the open circuit coefficient.

We give three curves I, II, III in Fig. 8. as examples of the percentage open circuit characteristics with weak, moderate, and strong magnetic densities respectively, for the sake of clearness in the explanation of the following chapters.

§ 5. Short Circuit Characteristics.

The exciting ampere-turns and the corresponding armature current of an alternator, when the machine is short circuited, give the short circuit characteristic.

Let P = normal voltage,

C_p = Exciting current which would generate P at open circuit,

I_0 = normal current,

C_{I_0} = Exciting current which would generate I_o at short circuit,

N = total mumber of turns of the field coil,

K_d = Ka 0.9 m f_w = coefficient of direct armature reaction,

z_a = impedance of armature conductors per phase.

Then

$$C_{I_0} - K_d\frac{w}{N}I_0 \sin\psi = \frac{1}{v}I_0 z_a$$

We may put $\sin\psi = 1$

$$C_{I_0} = I_0\left\{K_a\frac{w}{N}+\frac{z_a}{v}\right\} = I_0\left\{K_a\frac{w}{N}+\frac{z_a\,C_P}{e\,P}\right\}\ldots\ldots\ 14)$$

Therefore, as long as the inclination of the open circuit characteristic is constant and equal to v, C_{I_0} varies proportionally with I_0, or, in other words, the short circuit characteristic is a straight line. We may express the above equation as follows:—

$$\frac{K_a\,w\,I_0}{C_p\,N} = \left(\frac{K_a I_0}{C_P}\right)\left(\frac{w}{N}\right) = \frac{C_{I0}}{C_P} - \frac{I_0\,z_a}{e\,P} = \frac{1}{a} - \frac{d_z}{e}\ \ldots\ (15)$$

where $d_z = \dfrac{I_0\,z_a}{P}$ = relative drop due to the armature impedance,

and $a = \dfrac{C_P}{C_{I_0}}$

This shows that the values $\dfrac{K_a\,I_o\,w}{C_p\,N}$ or $\dfrac{\text{A. T. of direct armature reaction}}{\text{Exciting A. T. at normal voltage}}$ and $\dfrac{1}{a}$ vary in a straight line, as $\dfrac{d_z}{e}$ is usually small compared with $\dfrac{1}{a}$ in ordinary machines. Thus the value 'a' determines the short circuit characteristic and therefore we call 'a' the short circuit coefficient.

Chapter III. Ideal Synchronous Motor.

The armature reactions and the self inductance of a synchronous motor are usually combined in one, and represented by what is called the synchronous reactance of the motor. We here call this fictitious motor, which has a constant armature reactance, 'the ideal synchronous motor.'

§ 6. General Equation of Ideal Synchronous Motor.

Suppose a generator is running a motor through a transmission line, and let

P = terminal E. M. F. of the generator,

E = generated E. M. F. of the motor,

r = effective resistance of the motor armature and line,

x = synchronous reactance of the motor armature and line reactance,

z = $\sqrt{r^2+x^2}$ = impedance of the motor armature and line,

I = current in armature and line,

ϕ = angle between P and I,

ψ = angle between $-E$ and I,

θ = $\phi-\psi$ = angle between P and $-E$,

ψ_0 = $\tan^{-1}\dfrac{x}{r}$ = angle between I and $I\,z$,

W_I = $P\,I\cos\phi$ = Power supplied to the motor from the generator,

W_a = $E\,I\cos\phi$ = electric power transformed into mechanical power by the motor,

$$= \frac{\omega}{P}D$$

where $\omega = 2\pi n$ = angular velocity,

p = number of pairs of poles,

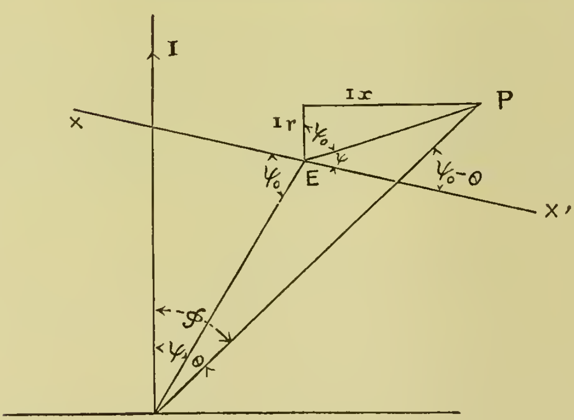


Fig. 9.

D = torque.

The vectorial diagram of the E. M. F. s of an ideal synchronous motor is shown in Fig. 9, where OI shows the direction of the current, OP the magnitude and direction of P; EP, those

of Iz; OE the magnitude of E, but in the opposite direction. We have then,

$$P^2 = (E + I\,r\cos\psi + I\,x\sin\psi)^2 + (I\,x\cos\psi - I\,r\sin\psi)^2$$

$$P^2 = E^2 + I^2z^2 + 2\,E\,I\,(r\cos\psi + x\sin\psi)$$

$$P^2 - E^2 - I^2z^2 - 2\,W_a\,r = 2x\sqrt{(EI)^2 - W_a^2} \quad \ldots\ldots \quad (16)$$

This is the well-known fundamental equation of the ideal synchronous motor. In this equation P, E, I, r, x and W_a are variables, and if any four of them are known, we have a quadratic equation of the other two. But to solve practical problems it is far simpler to use the following method proposed by the author:—

Draw a line XX' with an angle $\psi = \angle X'EP$, then

$$I\,z\cos\psi = P\cos(\psi_0 - \theta) - E\cos\psi_0$$

$$E\,I\cos\psi = W_a = \frac{E}{z}(P\cos(\psi_0 - \theta) - E\cos\psi_0) \ldots\ldots \quad (17)$$

put $\qquad \dfrac{x}{r} = a \qquad\qquad z = r\sqrt{1 + a^2}$

$$\cos\psi_0 = \frac{r}{z} = \frac{1}{\sqrt{1+a^2}} \qquad \sin\psi_0 = \frac{x}{z} = \frac{a}{\sqrt{1+a^2}}$$

$$W_a = \frac{1}{r(1+a^2)}(E\,P\cos\theta + a\,E\,P\sin\theta - E^2)$$

put $\qquad k = \cos\theta + a\sin\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)_a$

$$W_a = \frac{E^2}{r(1+a^2)}\left(\frac{P}{E}k - 1\right) \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

Equstion 18_a may be transformed as follows:—

$$\left.\begin{aligned} k &= \sqrt{1+a^2}\cos(\theta - \psi_0) \\ \tan\psi_0 &= a. \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)_b$$

when $\qquad \theta = 0$

$$k = \sqrt{1+a^2}\cos(-\psi_0) = 1$$

$$\theta = \pi$$

$$k = \sqrt{1+a^2}\cos(\pi - \psi_0) = -1.$$

The maximum value of k occurs when

$$\theta = \psi_0 = \tan^{-1} a.$$

$$k = \sqrt{1+a^2}.$$

The values of k for various values of θ and a are shown in Fig. 10_{a-d}.

We next get the equation of the current from the diagram:—

$$P^2z^2 = P^2 + E^2 - 2PE\cos\theta$$

$$I^2 = \frac{1}{r^2(1+a^2)}(P^2 + E^2 - 2EP\cos\theta)$$

$$= \frac{E^2}{r^2(1+a^2)}\left(1 + \frac{P^2}{E^2} - 2\frac{P}{E}\cos\theta\right) \quad \text{............} \quad (20)$$

Equations 18, 19, and 20 are the fundamental equations for determining the motor characteristics. In the above equations the angle θ, which is the supplement of the angle between the generator terminal E. M. F. and the counter E. M. F. of the motor, is introduced as a parameter.

To further simplify, we shall divide the problems into two classes:

a. Generator terminal E. M. F. constant,

b. Generated E. M. F. of motor constant.

If the generator terminal E. M. F. P is constant, put $E = \varepsilon P$, then the equations become:—

$$W_a = \frac{P^2}{r(1+a^2)}(\varepsilon k - \varepsilon^2) \quad \text{..........................} \quad (21)_a$$

or

$$k = \frac{W_a r(1+a^2)}{P^2 \varepsilon} + \varepsilon \quad \text{..........................} \quad (21)_b$$

$$I^2 = \frac{P^2}{r^2(1+a^2)}(1+\varepsilon^2-2\varepsilon\cos\theta)\ldots\ldots\ldots\ldots\quad(22)$$

If the generated E. M. F. of motor, E, is constant, put $P=\rho E$, then the equations become:

$$W_a = \frac{E^2}{r(1+a^2)}(\rho k-1)\ldots\ldots\ldots\ldots\ldots\ldots\quad(23)_a$$

or

$$k = \frac{1}{\rho}\left\{\frac{W_a r(1+a^2)}{E^2}+1\right\}\ldots\ldots\ldots\ldots\quad(23)_b$$

$$I^2 = \frac{E^2}{r^2(1+a^2)}(1+\rho^2-2\rho\cos\theta)\ \ldots\ldots\ldots\quad(24)$$

§ 7. Generator Terminal E. M. F. P, constant.

The general equations in this case are given in the preceding paragraph, but we shall here discuss particular cases.

(a) Minimum and Maximum Values of E at a given Power.

We have from equation 21_b

$$\varepsilon^2 - k\varepsilon + \frac{W_a\, r(1+a^2)}{P^2} = 0$$

$$\varepsilon = \frac{k \pm \sqrt{k^2 - 4\dfrac{W_a\, r(1+a^2)}{P^2}}}{2}$$

From this equation, it follows that for a given W_a and a given P, the maximum value of k gives the maximum and minimum values of ε. From equation 18_b we have the conditions of the maximum value of k as follows:—

$$\left.\begin{aligned}\tan\theta &= a\\ \cos\theta &= \frac{1}{\sqrt{1+a^2}}\\ \sin\theta &= \frac{a}{\sqrt{1+a^2}}\end{aligned}\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\quad(25)$$

$$k = \frac{1}{\sqrt{1+a^2}} + \frac{a^2}{\sqrt{1+a^2}} = \sqrt{1+a^2} \qquad \text{............} \quad (26)$$

Substituting this value in above equation,

$$\varepsilon = \frac{\sqrt{1+a^2} \pm \sqrt{1+a^2-4\left\{\frac{W_a\, r(1+a^2)}{P^2}\right\}}}{2} \qquad \text{.........} \quad (27)$$

The roots of this equation give the minimum and maximum values of ε; in other words, with the values of ε outside of these limiting values the motor goes out of step for the given load, W_a.

The current in this case.

$$\left.\begin{aligned} I_1 &= \sqrt{\frac{P^2}{r^2(1+a^2)}\left(1+\varepsilon_1^2-2\varepsilon_1\frac{1}{\sqrt{1+a^2}}\right)} \\ I_2 &= \sqrt{\frac{P^2}{r^2(1+a^2)}\left(1+\varepsilon_2^2-2\varepsilon_2\frac{1}{\sqrt{1+a^2}}\right)} \end{aligned}\right\} \text{.........} \quad (28)$$

where ε_1 and ε_2 are the roots of equation 27.

(b) Minimum Current at a given Power.

The minimum current I_o at a given power W_a occurs when $\hat{P I} = \phi = 0$. then,

$$W_a = P I_0 - r I_0^2$$

$$I_0 = \frac{P \pm \sqrt{P^2-4r\, W_a}}{2r} \qquad \text{.....................} \quad (29)$$

$$E^2 = (P - I_0 r)^2 + I_0 x^2$$

$$\varepsilon = \sqrt{\frac{(P-I_0 r)^2+I_0^2 r^2 a^2}{P^2}} \qquad \text{...............} \quad (30)$$

(c) Running Light.

In this case $W_a = o$, then

$$k = \varepsilon \qquad \text{..............................} \quad (31)$$

If a is large enough, we may assume $\cos\theta = 1$, then

$$I = \pm\frac{P}{Z}(1-\varepsilon) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

Therefore, the curve giving I as function of ε is two straight lines meeting at the horizontal axis.

(d) Maximum Power.

The general equation of the power is:

$$W_a = \frac{P^2}{r(1+a^2)}\{\varepsilon\cos\theta + \varepsilon a \sin\theta - \varepsilon^2\}$$

Firstly, if ε only is variable, the maximum power occurs when $\frac{d\,W_a}{d\,\varepsilon} = 0$, or

$$\frac{d\,W_a}{d\,\varepsilon} = \frac{P^2}{r(1+a^2)}\{\cos\theta + a\sin\theta - 2\varepsilon\} = 0$$

$$\varepsilon = \tfrac{1}{2}(\cos\theta + a\sin\theta) = \tfrac{1}{2}k$$

$$W'_{a\,m\,x} = \frac{P^2\,\varepsilon^2}{r(1+a^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (33)$$

Secondly, if θ only is variable, the maximum power occurs when $\frac{d\,W_a}{d\,\theta} = 0$, or

$$\frac{d\,W_a}{d\,\theta} = \frac{P^2}{r(1+a^2)}\{-\varepsilon\sin\theta + \varepsilon a\cos\theta\} = 0$$

$$\tan\theta = a$$

$$\sin\theta = \frac{a}{\sqrt{1+a^2}}$$

$$\cos\theta = \frac{1}{\sqrt{1+a^2}}$$

$$W_a''_{max} = \frac{P^2 \varepsilon}{r(1+\alpha^2)}\{\sqrt{1+\alpha^2}-\varepsilon\} \quad \text{......} \quad (34)$$

Thirdly, the maximum power, when both ε and θ are variable, occurs when the above two conditions are fulfilled at the same time:

$$\varepsilon = \tfrac{1}{2}(\cos\theta + \alpha\sin\theta) = \tfrac{1}{2}\sqrt{1+\alpha^2}$$

$$W_{a\,max} = \frac{P^2}{4r} \quad \text{......} \quad (35)$$

The current in this case

$$I = \frac{P}{2r} \quad \text{......} \quad (36)$$

§ 8. Generated E. M. F. of motor, E, Constant.

The general equations in this case are given in equations $23_{a,b}$, and 24. We shall here discuss the particular cases.

(a) Minimum Value of P at a given Power.

The equation 23_b shows that ρ is inversely proportional to k, therefore it has minimum value when k is maximum, that is:

$$\tan\theta = \alpha$$

$$\cos\theta = \frac{1}{\sqrt{1+\alpha^2}}$$

$$\sin\theta = \frac{\alpha}{\sqrt{1+\alpha^2}}$$

$$k = \sqrt{1+\alpha^2}$$

$$\therefore \qquad \rho_{min} = \frac{1}{\sqrt{1+\alpha^2}}\left\{\frac{W_a r(1+\alpha^2)}{E^2}+1\right\}$$

Hence the minimum value of the generator terminal E. M. F. at the given motor out-put W_a is:

$$P_{min} = \frac{E}{\sqrt{1+a^2}}\left\{\frac{W_a r(1+a^2)}{E^2}+1\right\} \quad \ldots\ldots\ldots\ldots \quad (37)$$

The current in this case.

$$I^2 = \frac{E^2}{r^2(1+a^2)}\left\{1+\frac{1}{(1+a^2)}\left[\frac{W_a r(1+a^2)}{E^2}+1\right]^2 - \frac{2}{1+a^2}\left[\frac{W_a r(1+a^2)}{E^2}+1\right]\right\} \quad (38)$$

(b) Minimum Current at a given Power.

The minimum current I_o at a given power W_a occurs when

$$\hat{EI} = \psi = 0,$$

then

$$W_a = EI_0$$

$$I_0 = \frac{W_a}{E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

$$P = \sqrt{(E+I_0 r)^2+(I_0 r)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (40)$$

(c) Running Light.

If the machine is running light, $W_a = O$, and

$$k = \frac{1}{\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (41)$$

If a is large enough, we may assume $\cos\theta = 1$, then

$$I = \pm\frac{E}{Z}(1-\rho) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (42)$$

which shows two straight lines meeting at the axis.

(d). Maximum Power at a given Resistance.

The general equation of the power, when E is constant, is

$$W_a = \frac{E^2}{r(1+a^2)}\{\rho\cos\theta + a\rho\sin\theta - 1\}$$

Firstly, if ρ only is variable, the equation becomes:

$$W_a = \rho A - B$$

where A and B are constants. This is an equation of a straight line, or, in other words, W_a varies proportionally with ρ and has no maximum value.

Secondly, if θ only is variable, the maximum W_a occurs when

$$\tan\theta = a$$

$$\cos\theta = \frac{1}{\sqrt{1+a^2}}$$

$$\sin\theta = \frac{a}{\sqrt{1+a^2}}$$

$$k = \sqrt{1+a^2}$$

$$W'_{a\,max} = \frac{E^2}{r}\left\{\frac{\rho}{\sqrt{1+a^2}} - \frac{1}{1+a^2}\right\} \quad \text{.............. (43)}$$

put

$$y = \frac{\rho}{\sqrt{1+a^2}} - \frac{1}{1+a^2} \quad \text{........................ (44)}$$

$$W'_{a\,max} = \frac{E^2}{r}y \quad \text{.................................... (45)}$$

The curves showing the relations between the variables y, ρ and a are given in Fig. 11.

Thirdly, if a and θ are variable, $W_{a\,max}$ is found by differentiating equation 43 with respect to a and equating to zero:

$$\frac{dW_a}{da} = \frac{E^2}{r}\,\frac{a}{(1+a^2)^{\frac{3}{2}}}\left\{\frac{2}{\sqrt{1+a^2}} - \rho\right\} = 0$$

$$\left.\begin{aligned} \rho &= \frac{2}{\sqrt{1+a^2}} \\ a &= \sqrt{\frac{4-\rho^2}{\rho^2}} \end{aligned}\right\} \quad \text{.................................. (46)}$$

$$y_{max} = \frac{1}{1+a^2} = \frac{\rho^2}{4} \quad \ldots\ldots\ldots\ldots \quad (47)$$

The curve of y_{max} is shown in Fig. 11.

The equation of current in this case is:

$$I^2 = \frac{E^2}{r^2(1+a^2)}\left\{1+\rho^2-2\rho\frac{1}{\sqrt{1+a^2}}\right\} \quad \ldots\ldots \quad (48)$$

$$\therefore \qquad I = \frac{E}{r}m \quad \ldots\ldots\ldots\ldots \quad (49)$$

where

$$m = \frac{1}{\sqrt{1+a^2}}\sqrt{1+\rho^2-2\rho\frac{1}{\sqrt{1+a^2}}} \quad \ldots\ldots \quad (50)$$

The curves showing the relations between the variables m, ρ and a are given in Fig. 12. When W_a is maximum and equal to $\frac{E^2}{r}\frac{1}{1+a^2} = \frac{E^2}{r}\frac{\rho^2}{4,}$ the current is:

$$I_m = \frac{E}{r}\frac{\rho}{2} = \frac{E}{r}\frac{1}{\sqrt{1+a^2}} \quad \ldots\ldots \quad (51)$$

$$m_m = \frac{\rho}{2} = \frac{1}{\sqrt{1+a^2}}$$

This is shown in Fig. 12.

(e) Maximum Power at a given Reactance.

If the reactance x, instead of r, is given, the equation 45 becomes

$$W_a = \frac{E^2}{r}y = \frac{E^2}{x}ay$$

put $\qquad ay = y'$, or

$$y' = \frac{a\rho}{\sqrt{1+a^2}} - \frac{a}{1+a^2} \quad \ldots\ldots\ldots\ldots \quad (52)$$

$$W_a = \frac{E^2}{x}y' \quad \ldots\ldots\ldots\ldots \quad (53)$$

The curves showing the relations between y', a and ρ are given in Fig. 13.

The current in the above case is:

$$I = \frac{E}{x} a\, m$$

put $\quad a m = m'$ or

$$m' = \frac{a}{\sqrt{1+a^2}}\sqrt{1+\rho^2-2\rho\frac{1}{\sqrt{1+a^2}}} \quad \text{.........} \quad (54)$$

$$I = \frac{E}{x} m' \quad \text{.........} \quad (55)$$

The curves showing the relations between the variables m', ρ and a are given in Fig. 14.

§ 9. Examples.

(i) To find the maximum line drop of a transmission line for a synchronous motor which can stand for a certain load, even when the generator E. M. F. is dropped to a certain percentage of the normal E. M. F.

Let $\quad x_a$, r_a = reactance and resistance of the motor,

$\quad x_l$, r_l = reactance and resistance of the line.

The value of x_l is generally small compared with x_a and therefore we may first assume $x_a + x_l = x_a$ and constant, then from equation 53 we have

$$y' = \frac{x_a}{E^2} W_a$$

Now, for the given value of ρ, we may find a from Fig. 13 which corresponds to the value of y' above found. Then

$$r_l = \frac{x_a}{a} - r_a$$

If x_l is not negligible, assume first $x_a+x_l = x_a$ as before, and find $r_l' = \frac{x_a}{a} - r_a$, then calculate x_l' from r_l'. Assume again $x_a+x_l = x_a+x_l'$, and find a'' and r_l''. Then calculate again x_l'' from r_l''. If $x_l'' = x_l'$ the assumption was right, if x_l'' is not equal to x_l, calculate again assuming a suitable value of x_l until they coincide.

Example:—

$E = 3000$ volts $\qquad P_{min} = 2100$ volts

or $\rho = .7$

$r_a = 3$ ohms $\qquad x_a = 75$ ohms

$W_a = 60.000$ watts

$$y' = \frac{75 \times 60000}{3000^2} = .5$$

From Fig. 13, we get $a = 5.2$

$$r_l = 11.4 \text{ ohms}$$

or percentage drop of the line is 7.6%

From Fig. 12, we get $m = .21$

or current in this case $= 42.1$ amp.

(ii) To find the maximum power in the above example when the exciting current of the motor is kept constant.

Assume $\rho = 1$, then from equation 43

$$W_{a\,max} = \frac{E^2}{r}\left\{\frac{\rho}{\sqrt{1+a^2}} - \frac{1}{1+a^2}\right\} = \frac{3000^2}{14.4} \times .1528 = 95.5 \times 10^3 \text{ watts}$$

$$= 1.59\, W_a$$

This may be directly found from Fig. 11.

(iii) To find a suitable reactance of a synchronous motor which can stand for a certain load, even when the generator voltage is dropped to a certain percentage of the normal voltage, assuming the drop due to the line is given.

From equation 45

$$y=\frac{(r_a+r_l)\,W_a}{E^2}$$

from Fig. 11, we find α for the given ρ then

$$x_a=\alpha(r_a+r_l)-x_l.$$

(iv) To find a suitable reactance of a synchronous motor which can stand for a certain load, within an allowable limit of current, even when the generator voltage is dropped to a certain percentage of the normal voltage.

Let $$E=3000 \text{ volts}$$

$$W_a=60.000 \text{ watts}$$

$$\rho=.7$$

$$I_0 r=300 \text{ volts} \quad \text{or} \quad 10\,\%$$

assume $$I_{max}=2\,I_0=40 \text{ amp}$$

from equation 49 $$m=\frac{I_m r}{E}=.2$$

from Fig. 12. $$\alpha=5.45$$

$$I_0 x=1635 \quad \text{or} \quad 54.5\,\%.$$

(v) To find how the power will be distributed, when two or more synchronous motors with different resistances and reactances are driving one and the same shaft.

Let

no of mach.	P	r_a	x_a	a	E	W_a
1	100	0.2Ω	4Ω	20	100	1000
2	100	0.2	3	15	100	?
3	100	0.3	3	10	100	?

Then $$k_1=\frac{W_{a_1}\,r_1(1+\alpha_1^2)}{P^2}+1=9.02$$

from Fig. 10 we get $$\theta_1=24^\circ$$

the current in this case

$$I_1 = \frac{P}{r_1}\sqrt{\frac{1+\varepsilon^2-2\varepsilon\cos\theta}{1+a_1^2}} = 10.375 \text{ amp.}$$

Now, as the three machines are driving one and the same shaft, it follows:—

$$\theta_1 = \theta_2 = \theta_3,$$

$\therefore$ $k_2 = 7$ from Fig. 10, $a = 15$,

$k_3 = 4.98$,, ,, , $a = 10$,

and then we have

$$W_{a_2} = \frac{(k_2-1)P^2}{r_2(1+a_2^2)} = 1328 \qquad I_2 = \frac{P}{r_2}\sqrt{\frac{1+\varepsilon^2-2\varepsilon\cos\theta}{1+a_2^2}} = 13.81$$

$$W_{a_3} = \frac{(k_3-1)P^2}{r_3(1+a_3^2)} = 1312 \qquad I_3 \qquad = 13.78$$

This shows that the power is distributed proportionally to the reactances, and is almost independent of the resistances if the reactances are large enough.

(vi) To find the excitation, in the above example, with which the power may be equally distributed.

$$k_2 = \frac{W_{a_2} r_2(1+a_2^2)}{P^2\varepsilon_2} + \varepsilon_2$$

put $W_{a_2} = 1000$ and $k_2 = 7$, then we get

$$\varepsilon_2^2 - 7\varepsilon_2 + \frac{1000\times 0.2\times 226}{10000} = 0$$

$$\varepsilon_2 = 6.28 \quad \text{or} \quad .72$$

in the same way $\varepsilon_3 = 4.27$ or $.71$

$\therefore$ $E_2 = 72$ $I_2 = 15.15$

$E_3 = 71$ $I_3 = 15.12$

Chapter IV. Synchronous Motor.

§ 10. Motor Characteristics.

In Chapter I, we have stated that the reactance of a synchronous machine can not be considered to be constant, but consists of armature reactions and armature self inductance. Therefore the real characteristics of a synchronous motor can not be found by so simple a process as that used in the preceding chapter.

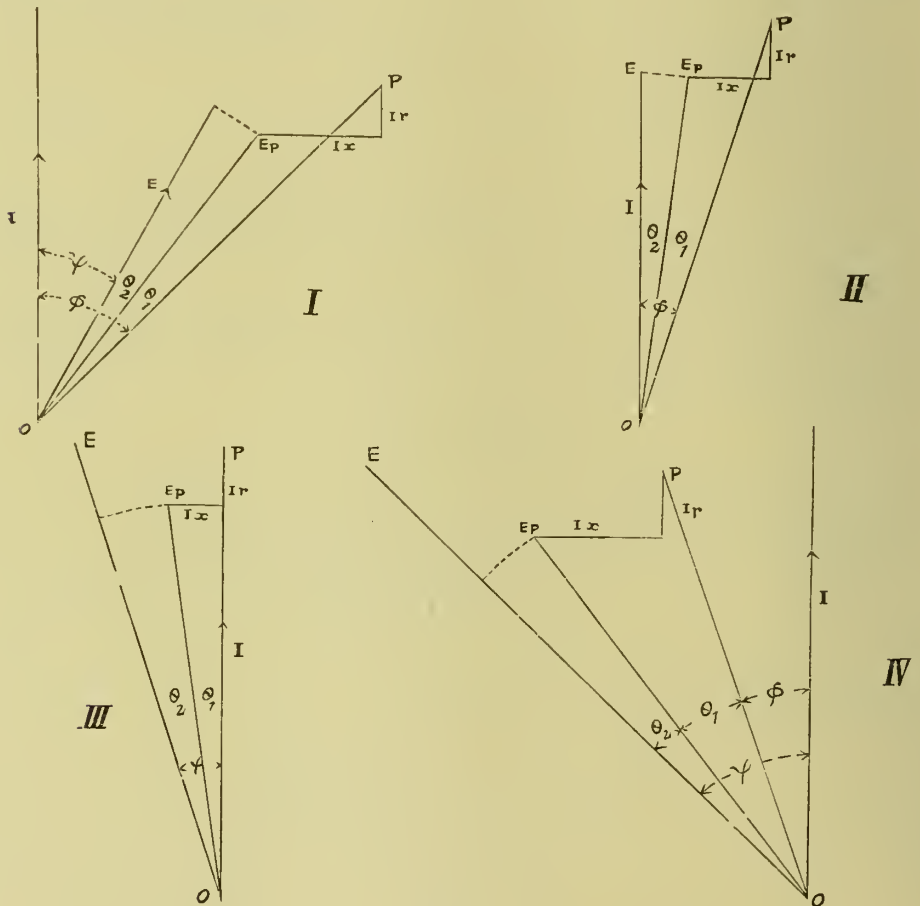


Fig. 15.

When a synchronous motor is running, there are four different cases as shown in Fig. 15, I, II, III, IV, where

I = current,

Ir = resistance drop of the line and the motor armature,

Ix = self inductive drop of the line and the motor armature,

P = generator terminal E. M. F.,

E_p = virtual induced E. M. F. of the motor,

E = nominal induced E. M. F. of the motor.

Here I shows the case when the current lags behind P and E,

II shows the case when the current is in phase with E, or $\psi = 0$,

III shows the case when the current is in phase with P, or $\phi = 0$,

IV shows the case when the current leads Pand E.

If, in the above diagram, Ir and Ix indicate the resistance and reactance drops of the armature, P shows the impressed E. M. F. of the motor.

Considering an ideal synchronous motor having generated E. M. F. E_p, driven by a generator of which the terminal E. M. F. is P, the relation between P, E_p, θ_1, and I can be found by the method stated in the preceding chapter.

Suppose P, r, x and W_a are given and let $\varepsilon = \frac{E_n}{P}$; then from equation 21_b,

$$k = \frac{W_a\, r(1+a^2)}{P^2\,\varepsilon} + \varepsilon$$

Thus θ_1 is found from Fig. 10, and we have

$$I^2 = \frac{P^2}{r^2(1+a^2)}(1+\varepsilon^2-2\varepsilon\cos\theta_1)$$

Put C_{Ep} = exciting current which would generate E_p at open circuit,

C_E = exciting current which would generate E at open circuit.

We have also

$$\cos\phi = \frac{W_a + I^2 r}{PI}$$

$$\theta_2 = \frac{A\,T_{cm}\left(\cos\theta_2 - \cos\frac{\Theta}{2}\right)}{C_{Ep}\,N}\,\frac{180}{\pi} = \frac{.9\,f_w\,m\,w\,I\cos\psi}{C_{Ep}\,N}\,\frac{180}{\pi}\left(\cos\theta_2 - \cos\frac{\theta}{2}\right)$$

$$\psi = \phi - \theta_1 - \theta_2$$

$$C_E = C_{Ep} - K_d\,I\,\frac{w}{N}\sin\psi.$$

where ψ is considered positive when I lags behind E and negative when I leads E. Thus we may find the curve giving I as the function of C_E or the motor characteristics.

If, in the above digram, Ir and Ix are small compared with P, we can find E_p and θ_1 as follows:—

$$W_a = P\,I\cos\phi - I^2 r$$

$$E_p = P - I\,r\cos\phi - I\,x\sin\phi \quad\ldots\ldots\ldots\ldots\ldots \quad (56)$$

$$\theta_1 = \frac{I\,x\cos\phi - I\,r\sin\phi}{E_p}\,\frac{180}{\pi}\ldots\ldots\ldots\ldots\ldots\ldots \quad (57)$$

where ϕ is considered positive when I lags behind P and negative when I leads P. The further calculation of C_E can be done as before mentioned.

§ 11. Generator Terminal E. M. F. constant.

(a) Minimum Current at a given Power.

The minimum current I_n at a given power W_a and a given generator E. M. F. P occurs when there exists no phase displacement between P and I or $\phi = o$ as shown in Fig. 15, III.

$$W_a = P\,I_n - r I_n^2$$

$$I_n = \frac{P-\sqrt{P^2-4\,r\,W_n}}{2\,r} \quad \text{.........................} \quad (58)$$

$$E_p = \sqrt{(P-I_n r)^2+(I_n x)^2}$$

$$\theta_1 = \tan^{-1}\frac{I_n\,x}{P-I_n\,r}$$

Then the value C_E in this case can be calculated by the method stated in the above paragraph.

(b) Running Light.

When the motor is running light, the vectors E and P are almost in the same plase, and I and P are in quadrature. In this case, then,

$$E_p = P-I\,x \quad \text{...........................} \quad (59)$$

$$C_E = C_{Ep} \mp K_d\,I\frac{w}{N} \quad \text{.....................} \quad (60)$$

where the sign $(-)$ is used when I lags behind E and the sign $(+)$ when I leads E. If $C_E = 0$,

$$C_{Ep} = K_d\,I\frac{w}{N} \quad \text{........................} \quad (61)$$

which occurs only when I lags behind E.

§ 12. Exciting Current Constant.

(a) Minimum Generator Voltage at a given Power.

The vector diagram of E. M. F. in this case is as shown in Fig. 15, IV. Now, we have

$$W_a = E_p\,I\cos(\phi-\theta_2)$$

If E_p is smaller than about 70 % of P, we may assume:

$$E_p = v(C_E - K_d\,I\frac{w}{N}\sin\phi) \quad \text{..................} \quad (62)$$

then $$W_a = v(C_E - K_d I \frac{w}{N} \sin\psi) I \cos(\psi - \theta_2)$$

Solving the above equation, we get

$$I = \frac{v\, C_E \cos(\psi - \theta_2) \pm \sqrt{v^2 C_E^2 \cos^2(\psi - \theta_2) - 4 W_a v K_d \frac{w}{N} \sin\psi \cos(\psi - \theta_2)}}{2 v K_d \frac{w}{N} \sin\psi \cos(\psi - \theta_2)} \quad (63)$$

The term under the radical sign may be transformed as follows:

$$\sin\psi \cos(\psi - \theta_2)\left\{v^2 C_E^2(\cot\psi \cos\theta_2 + \sin\theta_2) - 4 W_a v K_d \frac{w}{N}\right\}$$

This value decreases if ψ increases and finally becomes zero when ψ reaches a certain value which is maximum and beyond that value I becomes imaginary. On the other hand E_p decreases as ψ increases and becomes minimum when ψ reaches its maximum. Therefore minimum E_p occurs when the above term becomes zero or I has only one value. This gives minimum P. Thus the condition of minimum P is:

$$r^2 C_E^2(\cot\psi \cos\theta_2 + \sin\theta_2) - 4 W_a v K_d \frac{w}{N} = 0 \quad (64)$$

then $$I = \frac{C_E}{2 K_d \frac{w}{N} \sin\psi} \quad (65)_a$$

Put $\quad I = i I_0$

$$i = \frac{C_E}{2 K_d I_0 \frac{w}{N} \sin\psi} \quad (65)_b$$

$$E_p = v K_d I \frac{w}{N} \sin\psi = v \frac{C_E}{2} \quad (66)$$

$$C_{Ep} = K_d I \frac{w}{N} \sin\psi = \frac{C_E}{2} \quad (67)$$

from epuation 64,

$$\cot\psi \cos\theta_2 + \sin\theta_2 = \frac{4 W_a v K_d \frac{w}{N}}{v^2 C_E^2} = M \quad (68)$$

$$\theta_2 = \frac{A\,T_{cm}\left(\cos\theta_2 - \cos\frac{\Theta}{2}\right)}{C_{Ep}\,N}\,\frac{180}{\pi}$$

$$= \frac{\cdot 9\,f_w\,m\,w\,I\cos\psi}{\cdot 9 f_w\,m\,K_\alpha\,w\,I\sin\psi}\left(\cos\theta_2 - \cos\frac{\Theta}{2}\right)\times 57.3$$

$$= 57.3\,\frac{\cot\psi}{K_\alpha}\left(\cos\theta_2 - \cos\frac{\Theta}{2}\right) \quad \cdots\cdots\cdots\cdots \quad (69)$$

and thus we find

$$P_{min} = \sqrt{\left\{v\frac{C_E}{2}\cos(\psi-\theta_2)+I\,r\right\}^2 + \left\{v\frac{C_E}{2}\sin(\psi-\theta_2)-I\,x\right\}^2} \quad (70)$$

(b) Minimum Current at a given Power.

The minimum current at a given Power and a given exciting current occurs when there exists no phase displacement between E and I.—Fig. 15, II.

Here $$E_p = E$$

$$W_a = E\,I\cos\theta_2 \quad \cdots\cdots\cdots\cdots \quad (71)$$

$$\theta_2 = \frac{A\,T_{cm}\left(\cos\theta_2 - \cos\frac{\Theta}{2}\right)}{C_{Ep}\,N}\,57.3$$

$$= \frac{0.9\,f_w\,m\,w\,W_a}{C_E\,N\,E}\left(1 - \frac{\cos\frac{\Theta}{2}}{\cos\theta_2}\right)57.3 \cdots\cdots\cdots \quad (72)$$

$$P = \sqrt{(E\cos\theta_2 + I\,r)^2 + (E\sin\theta_2 + I\,x)^2} \quad (73)$$

(c) Running Light.

In this case,

$$C_{Ep} = C_E \pm K_d\,I\frac{w}{N} \cdots\cdots\cdots\cdots \quad (74)_a$$

where the sign (+) is used when I lags behind E and the sign (−) when I leads E.

$$P = E_p + Ix \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (75)$$

If $C_E = 0$, we have

$$C_{Ep} = K_d I \frac{w}{N} \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad (74)_b$$

§ 13. Maximum Power for a given Generator E. M. F. and a given Exciting Current, as well as

Minimum Exciting Current for a given Generator E. M. F. and a given Power.

We have found in equation 63,

$$I = \frac{v\, C_E \cos(\psi-\theta_2) \pm \sqrt{v^2 C_E^2 \cos^2(\psi-\theta_2) - 4\, W_a\, v\, K_d \frac{w}{N} \sin\psi \cos(\psi-\theta_2)}}{2\, v\, K_d \frac{w}{N} \sin\psi \cos(\psi-\theta_2)}$$

The term under the radical sign may be transformed as follows:—

$$4\, v\, K_d \frac{w}{N} \sin\psi \cos(\psi-\theta_2)\left\{\frac{v\, C_E^2}{4K_d \frac{w}{N}}(\cot\psi \cos\theta_2 + \sin\theta_2) - W_a\right\}$$

Suppose C_E is given, then this term decreases according as W_a increases and finally it becomes zero, in which case, W_a is maximum, because beyond that value I becomes imaginary.

$$\therefore \qquad W_{a\,max} = (\cot\psi \cos\theta_2 + \sin\theta_2)\frac{v\, C_E^2}{4K_d \frac{w}{N}} \quad \ldots\ldots\ldots\ldots \quad (76)$$

The term under the radical sign may also be transformed as follows:—

$$v^2 \cos^2(\psi-\theta_2)\left\{C_E^2 - \frac{4\, W_a\, K_d \frac{w}{N}}{v(\cot\psi \cos\theta_2 + \sin\theta_2)}\right\}$$

Suppose W_a is given, then this term decreases according as C_E decreases and finally it becomes zero, in which case, C_E is minimum.

$$\therefore \qquad C^2_{E\,min} = \frac{4\,W_a\,K_d\,\frac{w}{N}}{v(\cot\psi\cos\theta_2 + \sin\theta_2)} \qquad \text{(77)}$$

In both cases, we have equations the same as those shown in equations 65, 66, 67, 69 and 70.

To find the value $W_{a\,max}$ (or $C_{E\,min}$) at given P and C_E (or W_a) by direct calculation is too complex. We therefore draw a curve giving $W_{a\,max}$ (or $C_{E\,min}$) as the function of P at given C_E (or W_a) taking any value of ψ and then, from the curve thus drawn, we find the value $W_{a\,max}$ (or $C_{E\,min}$) corresponding to the given P.

§ 14. Examples of Motor Characteristics.

(a) Effect of Armature Self Inductance.

We shall firstly see how the armature self inductance of a synchronous motor affects its characteristics. As the synchronous motor is usually designed with a very weak magnetic density, we may assume it has open circuit characteristics as shown in Fig. 8, I or e = 1.1, and let us assume

$\frac{b}{\tau} = \cdot 667$ or $\cos\frac{\theta}{2} = .5$

$K_d = 2.15$

$I_0\,r_a$ = normal current × armature resistance = $.02P$

$I_0\,x_a$ = normal current × armature inductance = $.06P$ or $.12P$ taking both the extremes which would occur in an actual case.

$I_0\,z_a = .0633P$ or $.1218P$

$a = 2$

Then from equation 15 we have

$$K_d I_0 \frac{w}{N} = C_p\left\{\frac{1}{a} - \frac{I_0\,z_a}{P\,e}\right\} = 44.25$$

$$\text{or} \qquad = 38\,95$$

$$I_0 \frac{w}{N} = \qquad = 20.6$$

$$\text{or} \qquad = 18.1$$

Suppose the motor is driven without a transmission line, we then get the motor characteristics as follows:—

When the motor is running light,

put $\qquad I = i\, I_0$

$$E_p = P \mp i\, I_0\, x_a = P\left\{1 \mp i \frac{I_0\, x_a}{P}\right\}$$

$$C_E = C_{Ep} \mp K_d\, i\, I_0 \frac{w}{N}$$

when the motor is loaded,

$$W_a = P\, I_0 - r_a\, I_0^2 = P\, I_0 \left\{1 - \frac{r_a\, I_0}{P}\right\} = .98\, P\, I_0$$

$$\cos\phi = \frac{W_a + r_a\, I^2}{P\, I} = \frac{.98\, P\, I_0 + i^2 \times .02\, P\, I_0}{P\, i\, I_0} = \frac{.98 + .02 i^2}{i}$$

from (56) $\qquad E_p = P - r_a\, I_0\, i \cos\phi \mp x_a\, I_0\, i \sin\phi$

$$= P\left\{1 - \frac{r_a\, I_0}{P}\, i \cos\phi \mp \frac{x_a\, I_0}{P}\, i \sin\phi\right\}$$

,, (57) $\qquad \theta_1^0 = 57.3 \dfrac{i\, I_0\, x_a \cos\phi \mp i\, I_0\, r_a \sin\phi}{E_p}$

,, $(11)_a$ $\qquad \theta_2^0 = 57.3 \dfrac{.9\, m\, f_w\, I_0 \frac{w}{N} i \cos\psi (\cos\theta_2 - .5)}{C_{Ep}}$

put $\qquad m = 3, \qquad f_w = .966 \qquad$ for $q = 2., \qquad \psi = \pm\phi - \theta_1 - \theta_2$

$$\theta_2^0 = \frac{149.5\, I_0 \frac{w}{N}\, i \cos(\pm\phi - \theta_1 - \theta_2)(\cos\theta_2 - .5)}{C_{Ep}}$$

$$C_E = C_{Ep} \mp i\, K_d\, I_0 \frac{w}{N} \sin\psi$$

The following tables show the motor characteristics when numerical values are applied in the above equations.

TABLE I. (No Load)

		$I_0 x_a = .06\,P$				$I_0 x_a = .12\,P$			
i	P	E_P	C_{EP}	$i\,K_d I_0 \frac{w}{N}$	C_E	E_P	C_{EP}	$i\,K_d\,I_0 \frac{w}{N}$	C_E
1	100	94	92	44.25	47.75	88	84.8	38.95	45.85
.5	100	97	96	22.1	73.9	94	92	19.5	72.5
.2	100	98.8	98.2	8.85	89.35	97.6	96.8	7.8	89
0	100	100	100	0	100	100	100	0	100
.2	100	101.2	102	8.85	110.85	102.4	103.5	7.8	111.3
.5	100	103.	104.3	22.1	126.4	106	109	19.5	128.5
1	100	106	109	44.25	153.25	112	119	38.95	157.95

TABLE. II$_a$ (loaded)

$I_0 x_a = .06\,P$

i	$\cos\phi$	ϕ	$\sin\phi$	E	C_{Ep}	θ_1	θ_2	ψ	$\sin\psi$	$iK_d I_0 \frac{w}{N} \sin\psi$	C_E
2	.53	58°	.848	87.68	84	1.9°	25.3°	30.8°	.512	−45.3	38.7
1.5	.684	46.8	.729	91.3	88.5	2.5	21	23.3	.3955	−26.2	62.3
1.2	.84	32.9	.543	94.07	92	2.8	17.9	12.2	.2113	−11.2	80.8
1.1	.914	24	.4067	95.3	94	3.1	16.5	4.4	.0768	− 3.7	90.1
1	1	0	0	98	97	3.5	14.3	17.8	.3057	+13.5	110.5
1.1	.914	24	.4067	100.67	101	3.9	12.3	40.2	.6455	+31.4	132.4
1.2	.84	32.9	.543	101.89	102.5	4.1	11.4	48.4	.7477	+39.6	142.1
1.5	.684	46.8	.729	104.4	106.5	4.6	10	61.4	.8779	+58.3	164.8

TABLE. II$_b$ (loaded)

$$I_0\,x_a = .12\,P$$

i	$\cos\phi$	ϕ	$\sin\phi$	E_p	C_{Ep}	θ_1	θ_2	ψ	$\sin\psi$	$\frac{iK_d I_0 \frac{w}{N}}{\sin\psi}$	C_E
2	.53	58°	.848	77.53	73	6.9°	26.6°	24.5°	.4147	−32.3	40.7
1.5	.684	46.8	.729	84.85	80.9	6.9	20.6	19.3	.33	−19.3	61.6
1.2	.84	32.9	.543	90.16	87	6.9	16.8	9.2	.1598	− 7.5	79.5
1.1	.914	24	.4067	92.61	90	6.9	15.2	1.9	.033	− 1.4	88.6
1	1	0	0	98	97	7	12.4	19.4	.3322	+12.9	109.9
1.1	.914	24	.4067	103.37	105	7.1	10.3	41.4	.6613	+28.3	133.3
1.2	.84	32.9	.543	105.8	108.5	7.3	9.4	49.6	.7614	+35.6	144.1
1.5	.684	46.8	.729	111	117	7.5	8	62.3	.8853	+51.6	168.6

The above results are shown in Fig. 16. From these curves we see that the characteristics of the motors having the same open and short circuit coefficients but different self-inductances nearly coincide with each other, or, we may say that the armature self-inductance of a synchronous motor has little effect upon its characteristic.

(b). Effect of Magnetic Saturation

Let us assume

$$I_0\,x_a = .12\,P$$
$$a = 2$$
$$K_d = 2.15$$
$$e = 1.1 \qquad \text{Fig. 8, I.}$$
or $$= 1.4 \qquad \text{,, III.}$$

The results are shown in Fig. 17.

(c) Effect of Short Circuit Coefficient.

Let us assume

$$I_0 x_a = .12\,P$$
$$e = 1.1 \qquad \text{Fig. 8, I.}$$
$$K_d = 2.15$$
$$a = 1,\ 2 \quad \text{or} \quad 4.$$

The results are shown in Fig. 18.

From Figs. 17 and 18, we can find how the motor characteristics are affected by the open and short circuit coefficients, or e and a. If we draw curves taking sufficient numbers of a and e, we may find the particular values of a and e which give the required motor characteristics. Thus we find that:

'The load characteristics of a synchronous motor, are determined by its no-load characteristics.'

(d) Example of § 13.

As an example of § 13, we take a motor driven by a generator placed at a long distance. Let us assume the motor coefficients as ollows :—

$$\frac{b}{\tau} = .667 \quad \text{or} \quad \cos\frac{\theta}{2} = .5$$
$$K_\alpha = .825$$
$$m = 3$$
$$q = 2$$
$$f_w = .966$$
$$K_d = 2.15$$
$$e = 1.1 \quad \text{or} \quad \text{curve I in Fig. 8}$$
$$K_d\, I_0 \frac{w}{N} = \left(\frac{1}{a} - \frac{d_z}{e}\right)$$

From equation 76 we have.

$$W_{a\,max} = (\cot\psi\cos\theta_2 + \sin\theta_2)\frac{v\,C_E^2}{4K_d\frac{w}{N}}$$

and from (69)

$$\theta_2 = 57.3\frac{\cot\psi}{K\alpha}\left(\cos\theta_2 - \cos\frac{\Theta}{2}\right) = 69.5\cot\psi\,(\cos\theta_2 - .5)$$

put $$M = \cot\psi\cos\theta_2 + \sin\theta_2$$

The relation between M, θ_2 and ψ is shown in Fig. 19.

Now let us assume.

P = normal voltage at generator terminal = 100,

$I_0 r_a$ = normal current × armature resistance = ·02 P = 2,

$I_0 x_a$ = ,, × armature reactance = ·1 P = 10,

$I_0 r_l$ = ,, × line resistance = .08 P = 8,

$I_0 x_l$ = ,, × line reactance = .1 P = 10,

W_a = normal power of the motor = $[P - I_0(r_a + r_l)]\,I_0 = 90\,I_0$,

I = current $= i\,I_0$,

(i). Let the exciting current be given, or $C_E = 100$, then

maximum power $$w_m = \frac{W_{a\,max}}{W_a} = \frac{M\,v\,C_E^2}{4K_d\frac{w}{N}\times 90\,I_0} = 30.6\frac{M}{K_d I_0\frac{w}{N}}$$

Current $$i = \frac{C_E}{2K_d I_0\frac{w}{N}\sin\psi} = \frac{50}{K_d I_0\frac{w}{N}\sin\psi}$$

Terminal voltage

$$P' = \sqrt{\left[v\frac{C_E}{2}\cos(\psi-\theta_2) + I(r_a+r_l)\right]^2 + \left[v\frac{C_E}{2}\sin(\psi-\theta_2) - I(x_a+x_l)\right]^2}$$

$$= \sqrt{[55\cos(\psi-\theta_2) + 10\,i]^2 + [55\sin(\psi-\theta_2) - 20\,i]^2}$$

TABLE. III.

$K_d I_0 \frac{w}{N}$	w_m	M	ψ	θ_2	$\sin\psi$	i'	P'
$=.337\,M$			$a = 1$			$=\frac{.551}{\sin\psi}$	
90.73	1	2.97	17.2	45.2	.2957	1.86	92
,,	1.12	3.32	14.7	47.3	.2537	2.175	100
$=.533\,M$			$a = 1.5$			$=\frac{.871}{\sin\psi}$	
57.4	1	1.87	32	35.2	.5299	1.64	80
,,	1.35	2.53	21.6	42.2	.3681	2.36	100
$=.556M$			$a = 1.56$ (Minimum Current)			$=\frac{.91}{\sin\psi}$	
55	1	1.789	34	34	.5592	1.625	78.4
$=.75\,M$			$a = 2$			$=\frac{1.23}{\sin\psi}$	
40.73	1	1.333	45.2	26.8	.7095	1.733	71.6
,,	1.55	2.065	28.5	37.5	.4771	2.58	100
$=.997M$			$a = 2.5$			$=\frac{1.628}{\sin\psi}$	
30.73	1	1	55	21	.8192	1.99	66
,,	1.75	1.755	34.5	33.6	.5664	2.87	100
$=1.27\,M$			$a = 3$			$=\frac{2.08}{\sin\psi}$	
24.06	1	.788	62	16.8	.8829	2.36	62.8
,,	1.9	1.495	41	29.5	.6561	3.17	100
$=1.58\,M$			$a = 3.5$			$=\frac{2.58}{\sin\psi}$	
19.33	1	.633	67.5	13.5	.9239	2.8	61.2
,,	2.02	1.28	47.	26	.7314	3.53	100
$=1.94\,M$			$a = 4$			$=\frac{3.18}{\sin\psi}$	
15.73	1	.515	71.5	11	.9483	3.35	63.5
,,	2.06	1.06	53.5	22	.8038	3.96	100

The above results are shown in Fig. 20, where

Curve I shows P' as function of a when $W_a = .9\ P\ I_0$ the minimum P′ occurs when $a = 3.5$,

Curve II shows i' as function of a when $W_a = .9\ P\ I_0$ the minimum current occurs when $a = 1.56$,

Curve III shows w_m as function of a when $P' = 100 = P$,

Curve IV. shows $i'' =$ current at $W_{a\,max}$ as function of a when $P' = 100 = P$.

(ii) Let a be given and equal to 2, then

$$K_d I_0 \frac{w}{N} = 30.73$$

$$w_m = .997\left(\frac{C_E}{100}\right)^2 M$$

$$i = \frac{C_E}{61.46 \sin\psi}$$

TABLE IV.

C_E	w_m	M	ψ	θ_2	$\sin\psi$	i	P'
$w_m = .997\,M$						$i = \frac{1.628}{\sin\psi}$	
100	1	1	55	21	.8192	1.99	66
,,	1.75	1.755	34.5	33.6	.5664	2.87	100
$w_m = 1.205M$						$i = \frac{1.79}{\sin\psi}$	
110	1	.83	61	17.5	.8746	2.05	64.3
,,	1.87	1.55	39.5	30.5	.636	2.82	100
$w_m = 1.435M$						$i = \frac{1.95}{\sin\psi}$	
120	1	.697	65.5	14.7	.91	2.14	63.6
,,	2	1.392	43.5	28	.6884	2.84	100

C_E	w_m	M	ψ	θ_2	$\sin\psi$	i	P'
$w_m = 1.535$		Minimum P'				$i = \frac{2.02}{\sin\psi}$	
124	1	.652	67	13.8	.9205	2.2	63.5
$w = 1.685\,M$						$i = \frac{2.115}{\sin\psi}$	
130	1	.594	69	12.6	.9336	2.265	63.8
,,	2.12	1.258	47.5	25.6	.7373	2.87	100
$w_m = 1.955 M$						$i = \frac{2.275}{\sin\psi}$	
140	1	.512	72	10.8	.9511	2.39	64.5
,,	2.2	1.14	51	23.5	.7771	2.925	100
$w_m = 2.24\,M$						$i = \frac{2.44}{\sin\psi}$	
150	1	.446	74	9.6	.9613	2.52	65.5
,,	2.3	1.026	54.5	21.3	.8141	2.975	100

The above results are shown in Fig. 21, where

Curve I shows P' as function of C_E when $W_a = .9\ PI_0$

Curve II shows i' as function of C_E in the above case,

Curve III shows w_m as function of C_E when $P' = 100 = P$,

Thus we see the minimum terminal voltage occurs when $C_E = 124$ and the maximum output and the corresponding current do not vary much with the exciting current if the short circuit coefficient remains the same.

§ 15. Comparison of Motor Characteristics of an Ideal Motor and one having Armature Reactions.

Let us take the following examples:—

(a) The motor driven without transmission line.

Suppose we have.

$$a = 2.5$$
$$I_0 r_a = .02\,P$$
$$W_a = .98\,P I_0$$

Then

as ideal motor

$$I_0 x_a = .4\,P$$
$$a = \frac{.4\,P}{.02\,P} = 20$$
$$k = \frac{7.86}{\varepsilon} + \varepsilon \text{ (loaded)}$$
$$= \varepsilon \text{ (at no load)}$$
$$I = 2.5\,I_0 \sqrt{1+\varepsilon^2-2\,\varepsilon\cos\theta} \text{ (loaded)}$$
$$= 2.5\,I_0(1-\varepsilon) \text{ (at no load)}$$

Armature reactions considered

$$I_0 x_a = .12\,P \text{ (assumed)}$$
$$K_a I_0 \frac{w}{N} = 28.95$$
$$I_0 \frac{w}{N} = 13.45$$
$$e = 1.1 \text{ (assumed)}$$

The results are shown in Fig. 22.

(b) The motor driven through a long transmission line

Suppose we have

$$a = 2.5$$
$$I_0 r_a = \text{Armature resistance drop} = .02\,P$$
$$I_0 r_l = \text{line resistance drop} = .08\,P$$
$$I_0 x_l = \text{line reactance drop} = .1\,P$$
$$W_a = P I_0 - I_0^2(r_a + r_l) = .9\,P I_0$$

as ideal motor.

$$I_0 x_a = .4\,P$$
$$a = \frac{.4+.1}{.02+.08} = 5$$
$$k = \frac{2.34}{\varepsilon} + \varepsilon \text{ (loaded)}$$

Armature reaction considered

$$I_0 x_a = .1\,P \text{ (assumed)}$$
$$a = \frac{I_0(x_a + x_l)}{I_0(r_a + r_l)} = 2$$
$$\cos\phi = \frac{.9 + .1 i^2}{i} \left(i \text{ being equal to } \frac{I}{I_0} \right)$$

$k = \varepsilon$ (at no load) $\qquad k = \dfrac{.45}{\varepsilon} + \varepsilon$

$I = 2.5\, I_0 \sqrt{1 + \varepsilon^2 - 2\,\varepsilon \cos\theta}$ (loaded) $\qquad I = I_0 \sqrt{20(1 + \varepsilon^2 - 2\varepsilon\cos\theta)}$

$= 2.5\, I_0 (1 - \varepsilon)$ (at no load) $\qquad \theta_2 = \dfrac{2140\, i \cos(\phi - \theta_1 - \theta_2)(\cos\theta_2 - .5)}{C_{Ep}}$

The results are shown in Fig. 23. From Fig. 22 and 23 we see that both the characteristics agree fairly well for a weak magnetic density.

Chapter V. Numerical Examples.

§ 16 (a) 120 *KW.* Synchronous Motor.

Maker		Siemens-Schuckert Werke
KVA	$=$	$1.732 \times 3000 \times 29 = 150$ when $\cos\phi = .8$
P	$=$	3000 volts or 1732^v per phase
$\sim$	$=$	50
p	$=$	5
m	$=$	3
I_0	$=$	23.1

Effective Resistance, from equation 1,

ohmic resistance, (measured) $= r_g = 1.15^{\Omega}$ per phase

effective resistance $= r_a = 1.15 \times 1.74 = 2^{\text{ohm}}$

Reactance due to Self Induction, from equation 3,

$$\lambda_n = 0.4\,\pi \left\{ \frac{r}{3r_1} + \frac{2r_3}{r_1 + r_2} \right\}$$

$r_1 = 28{\cdot}5^{\,mm}$ $\qquad r = 51$

$r_2 = 35$ $\qquad r_3 = 6.35$

$\therefore \qquad \lambda_n = 1$

From equation 5_2, q being equal to 2

$$\lambda_z = \lambda_k^I + 1.5\lambda_l^I + 2(\lambda_k^{II} + \lambda_k^{III} + \ldots \lambda_l^{II} \ldots + \ldots\ldots + \lambda_k^{(n)})$$

$$D = 1050^{mm}$$

$$\tau = \frac{\pi D}{10} = 330$$

$$z = \frac{\pi D}{60} = 55$$

$$t_1 = z - r_1 = 26.5$$

$$d = r + r_3 = 57.35$$

$$\lambda_k^I = 0.92 \log\left(1+\frac{\pi t_1}{r_1}\right) = .92\times.594$$

$$\lambda_k^{II} = 0.92 \log\left(1+\frac{\pi t_1}{r_1+\pi z}\right) = .92\times.152$$

$$\lambda_l^I = 0.92 \log\left(1+\frac{\pi r_1}{2d+r_1+\pi t_1}\right) = .92\times.145$$

$$\lambda_l^{II} = \qquad = .92\times.0878$$

$$\lambda_k^{(n)} = 0.92 \log\frac{\tau}{z(n-1)} \qquad = .92\times.4771$$

$$\lambda_k = 2.066$$

From equation 7.

$$\lambda_s = 0.46\, q_s[\log\frac{l_s}{d_s} - .2]$$

$$d_s = \frac{2}{\pi}(65+50) = 73$$

$$l_s \text{ mean} \qquad = 55$$

$$\lambda_s = .62$$

$$\therefore \qquad x_a = \frac{4\pi\sim w^2}{pq}(l_i\,\lambda_n + l_i\,\lambda_k + l_s\,\lambda_s)\,10^{-8}$$

$$w = 330$$

$$l_i = 17.65$$

then we have

$$x_a = 5.95 \text{ ohms.}$$

We have on the other hand the open and short circuit characteristics in Fig. 24 and also:

$$b = 200$$

$$\frac{b}{\tau} = .606$$

$$f_w = .966$$

$$N = 5780$$

$v = 190^{\text{volt}}$ per ampere of exciting current

$$K_d = .9 f_w m \frac{\sin \frac{\pi}{2} \frac{b}{\tau}}{\frac{\pi}{2} \frac{b}{\tau}} = 2.23$$

$$K_d \frac{w}{N} = .1275$$

$$C_{I_0} - K_d I_0 \frac{w}{N} = \frac{I_0 z_a}{v}$$

$$z_a = \frac{v\left(C_{I_0} - K_d I_0 \frac{w}{N}\right)}{I_0} = 6.25$$

$$x_a = \sqrt{6.25^2 - 2^2} = 5.93$$

We have found, therefore, that the calculated reactance due to self inductance coincides with that measured.

The characteristics of this motor, measured and calculated, are shown in Fig. 25. We see that there occurs little difference between the results measured and calculated for a high exciting current.

(b) 50 *KW* Synchronous Motor.

Maker		General Electric Co.
P	=	500^{volt} or 288^{volt} per phase
$\sim$	=	60
p	=	4
m	=	3
I_0	=	57.7

The open and short circuit characteristics of this machine are shown in Fig. 26.

ohmic resistance of armature measured $= .1$ ohm per phase

effective resistance $= .15$

Suppose effective reactance $= x_a = .5$ or $I_0 x_a = .1 P$

$$K_a = 2.15$$

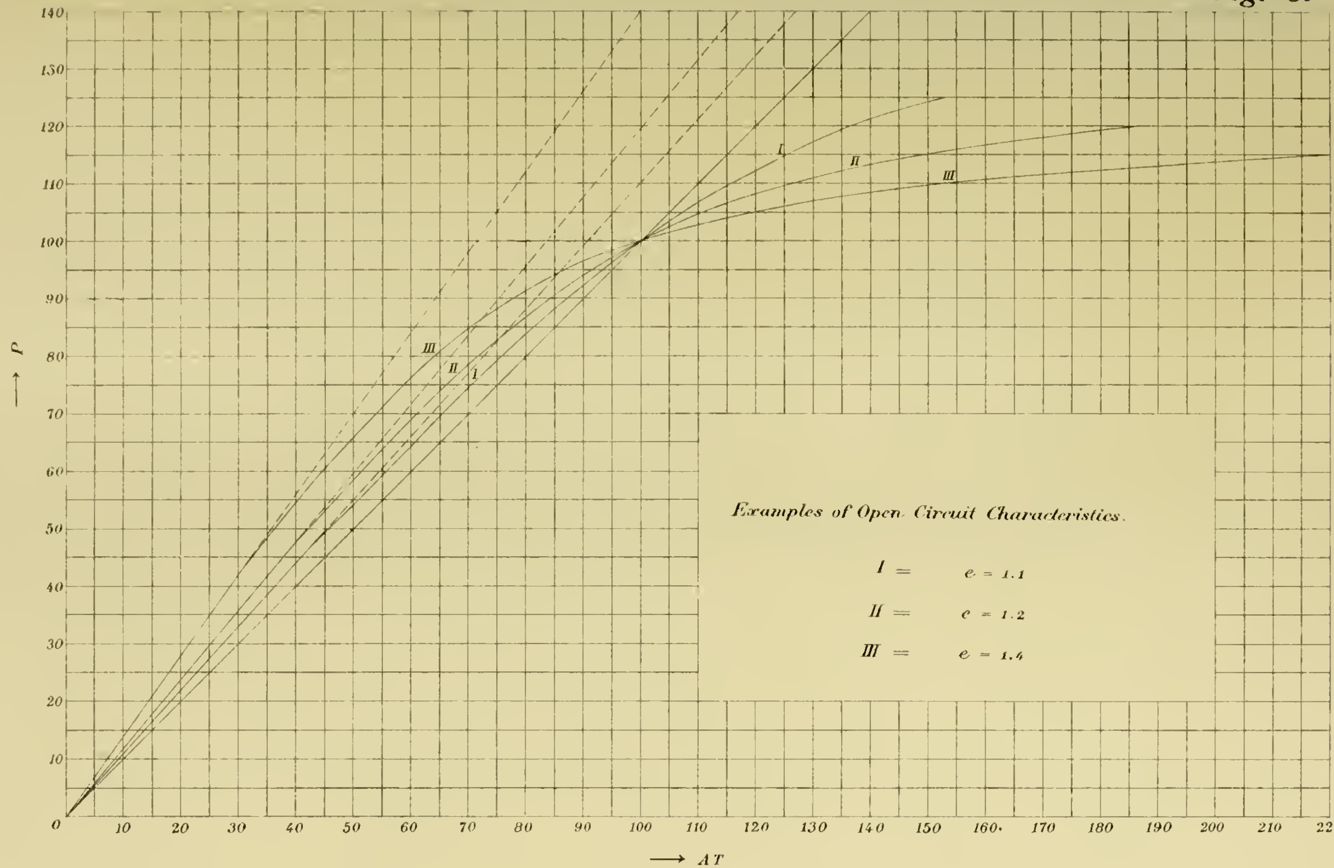
$$K_a \frac{w}{N} = .712$$

$$K_a I_0 \frac{w}{N} = 4.105$$

The motor characteristics, measured and calculated, are shown in Fig. 27. The difference which appears at no load arose from the fact that the actual measurements were made when the machine was little loaded. But they agree fairly well when loaded.

May, 1910.

Fig. 8.



Examples of Open Circuit Characteristics.
I = e = 1.1
II = e = 1.2
III = e = 1.4
P
AT

Fig. 10 a.

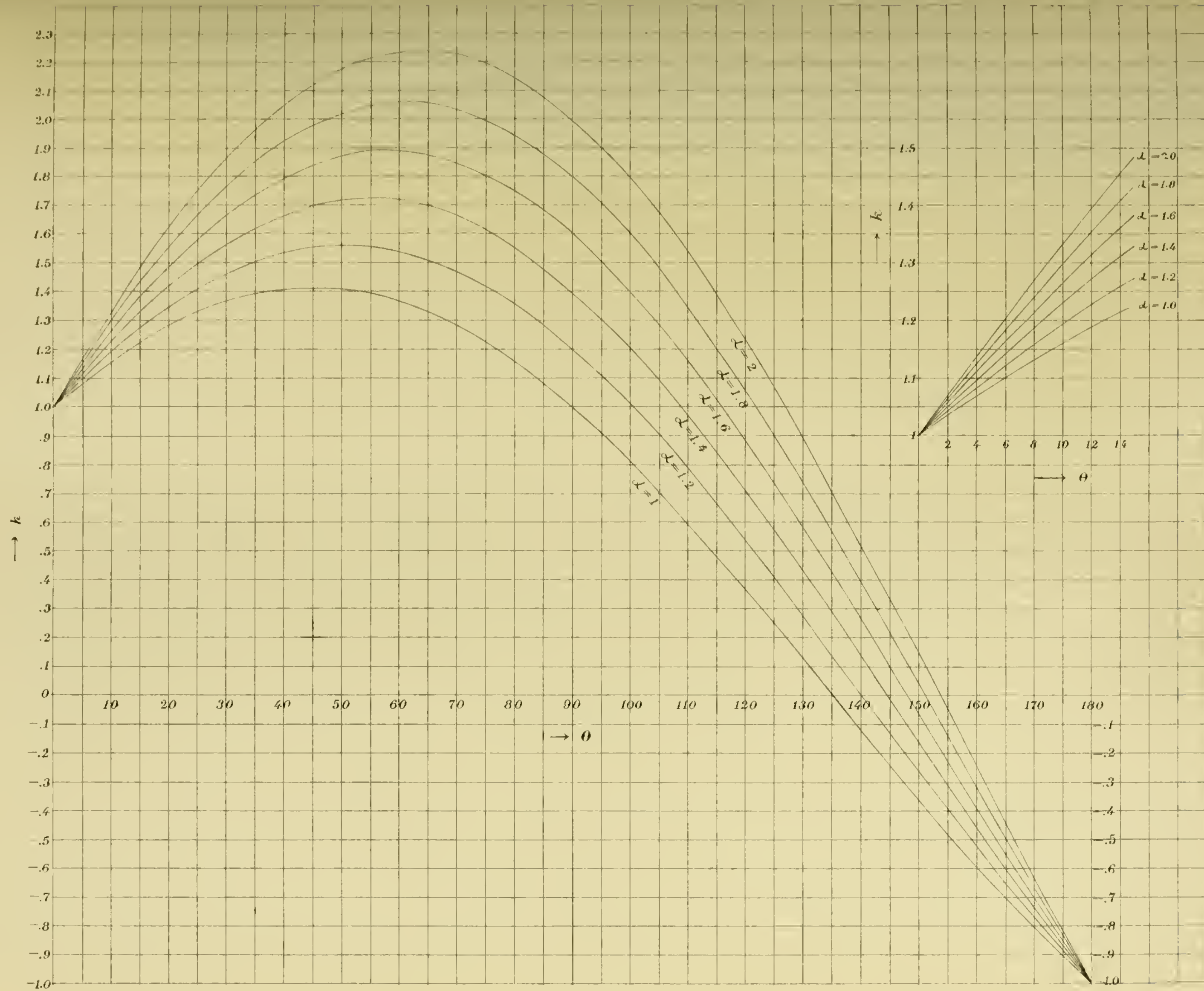


Fig. 10 b.

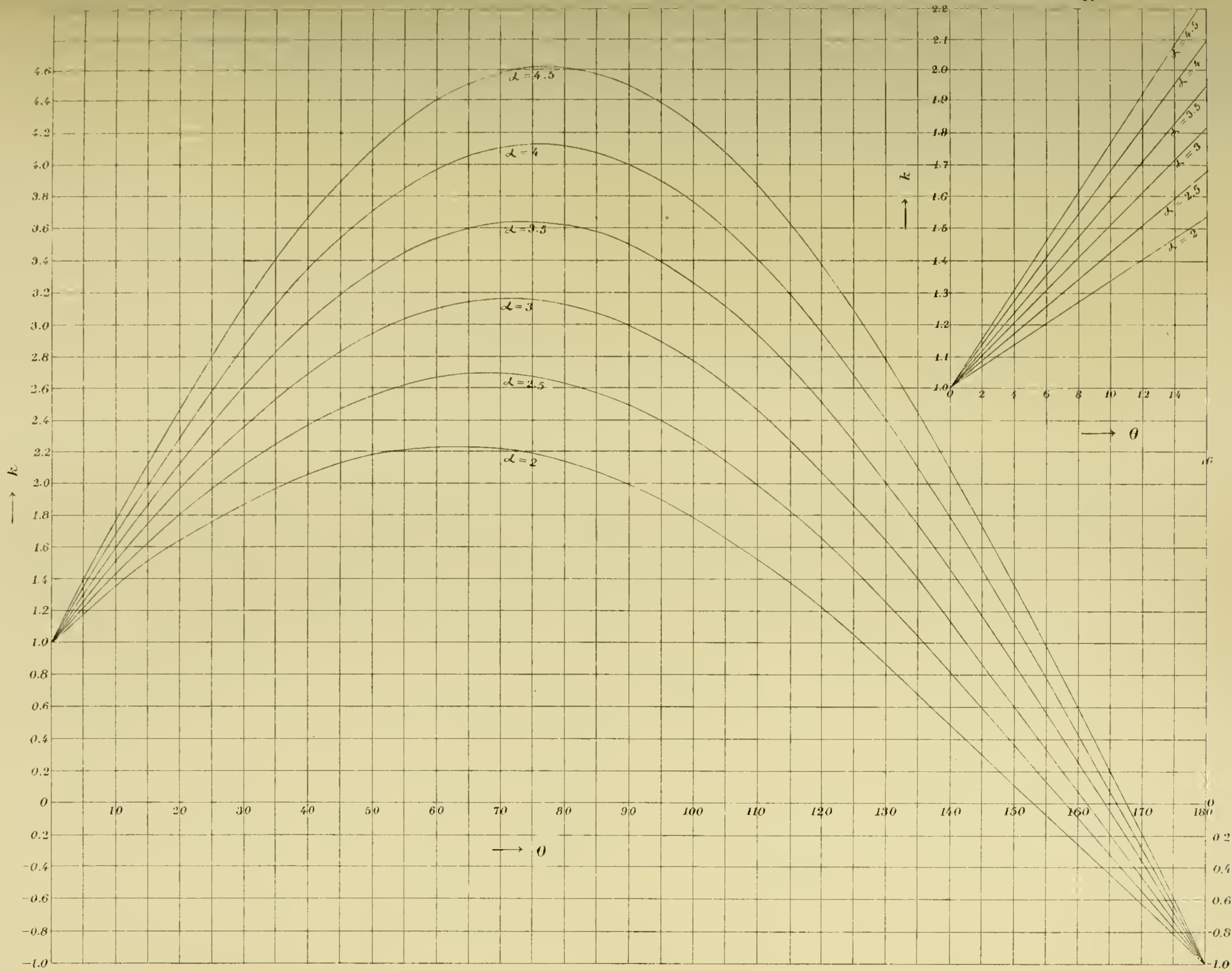
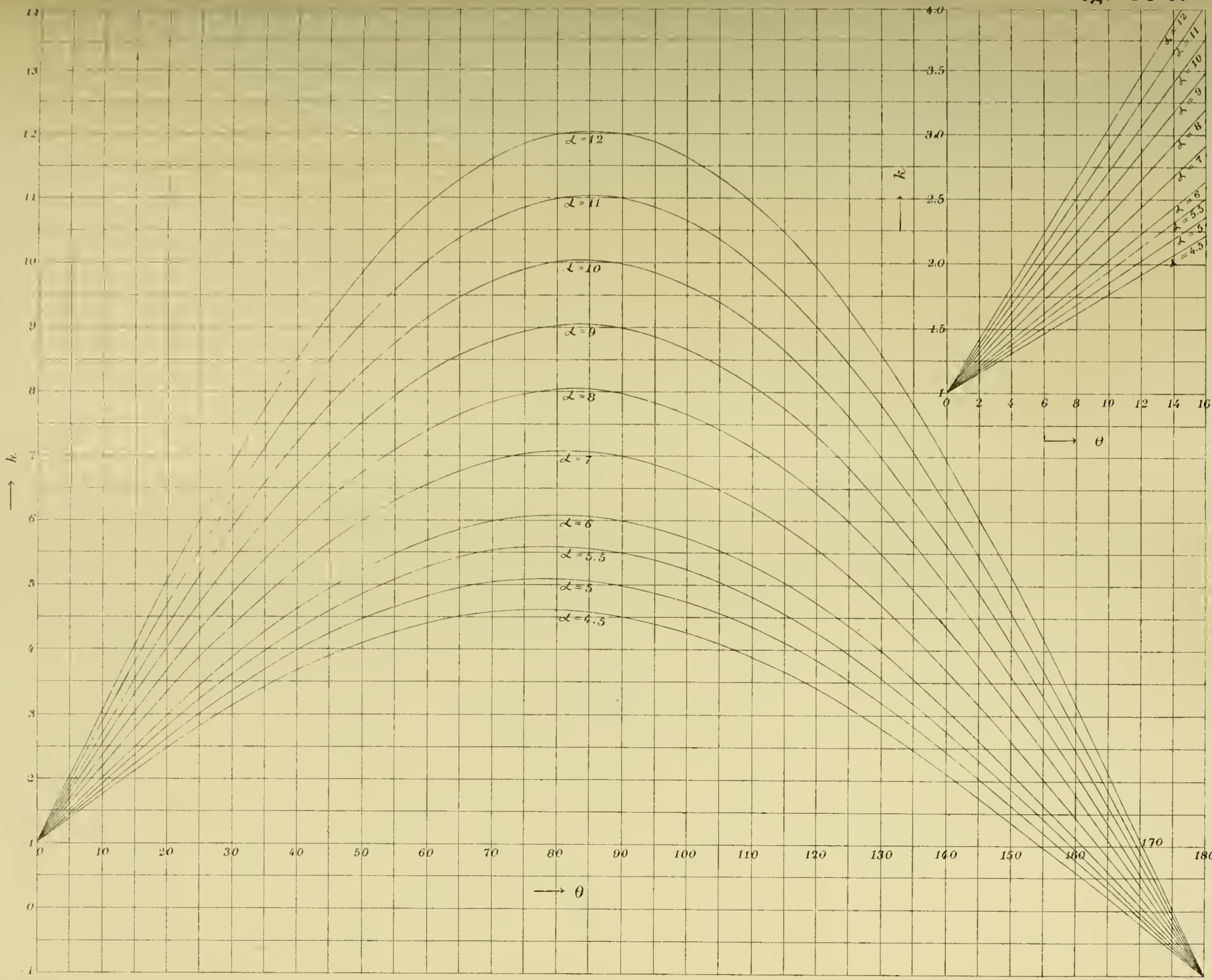
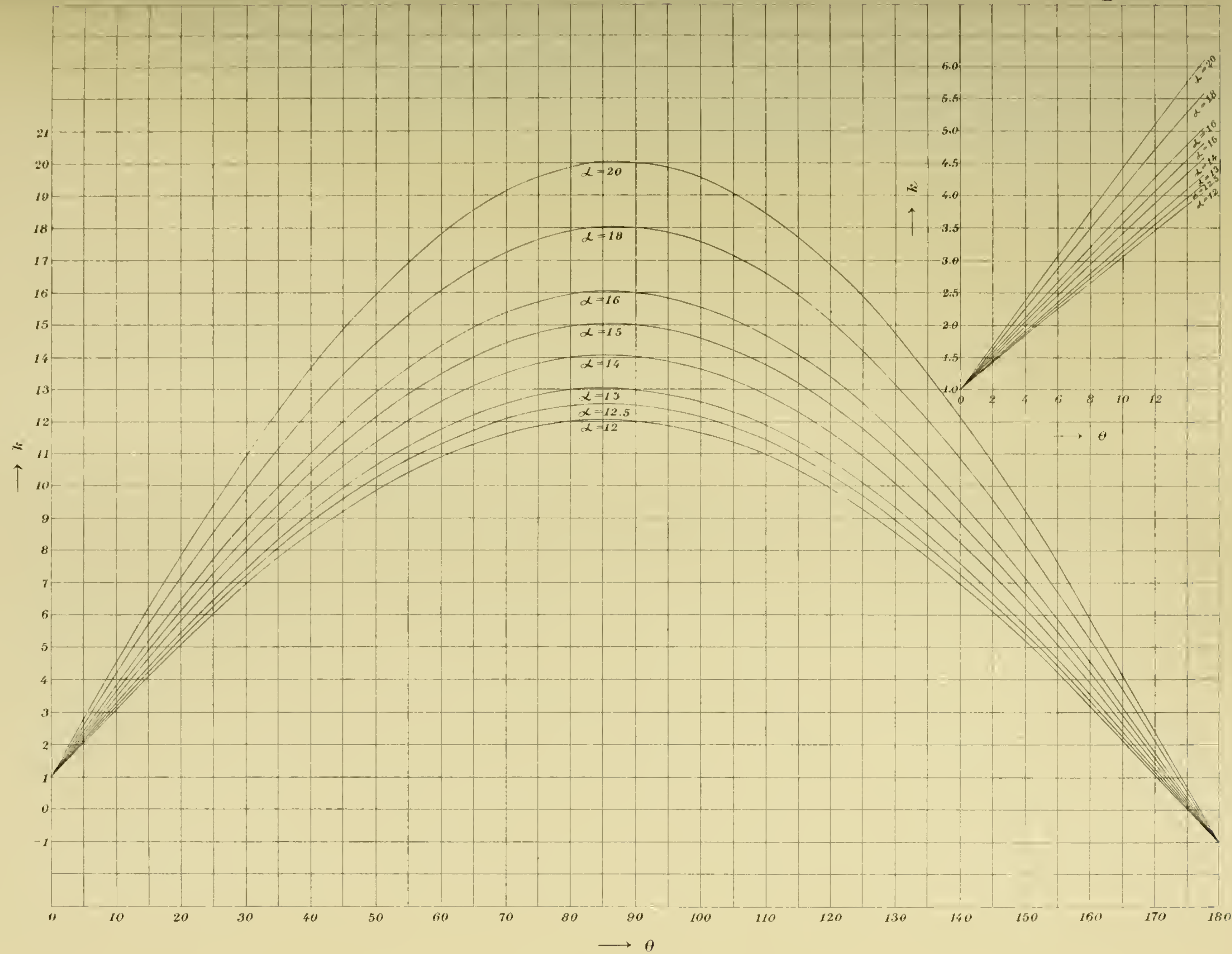


Fig. 10 c.



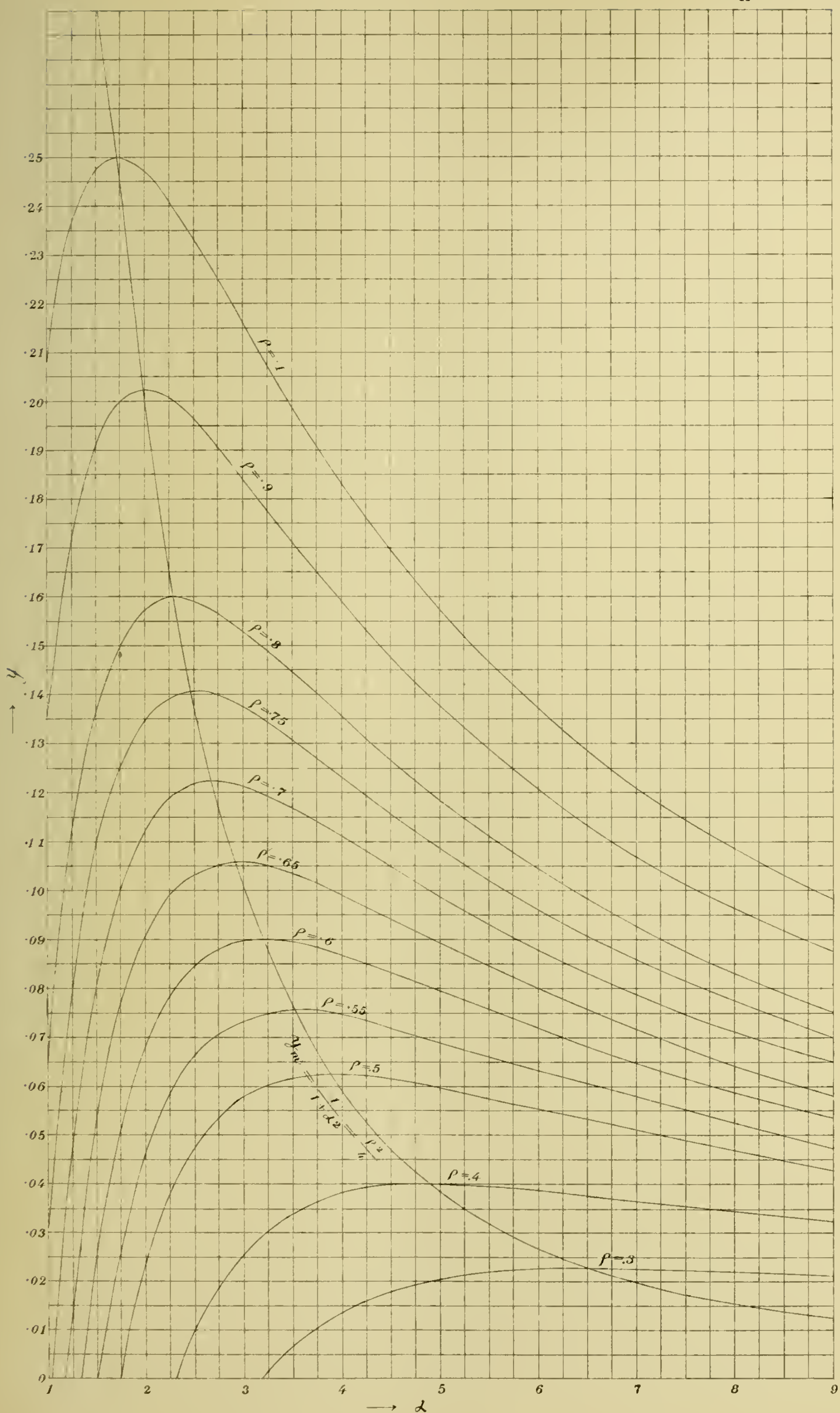
$\alpha = 12$
$\alpha = 11$
$\alpha = 10$
$\alpha = 9$
$\alpha = 8$
$\alpha = 7$
$\alpha = 6$
$\alpha = 5.5$
$\alpha = 5$
$\alpha = 4.5$
$\longrightarrow \theta$
$\longrightarrow h$
$\longrightarrow k$

Fig. 10 d.



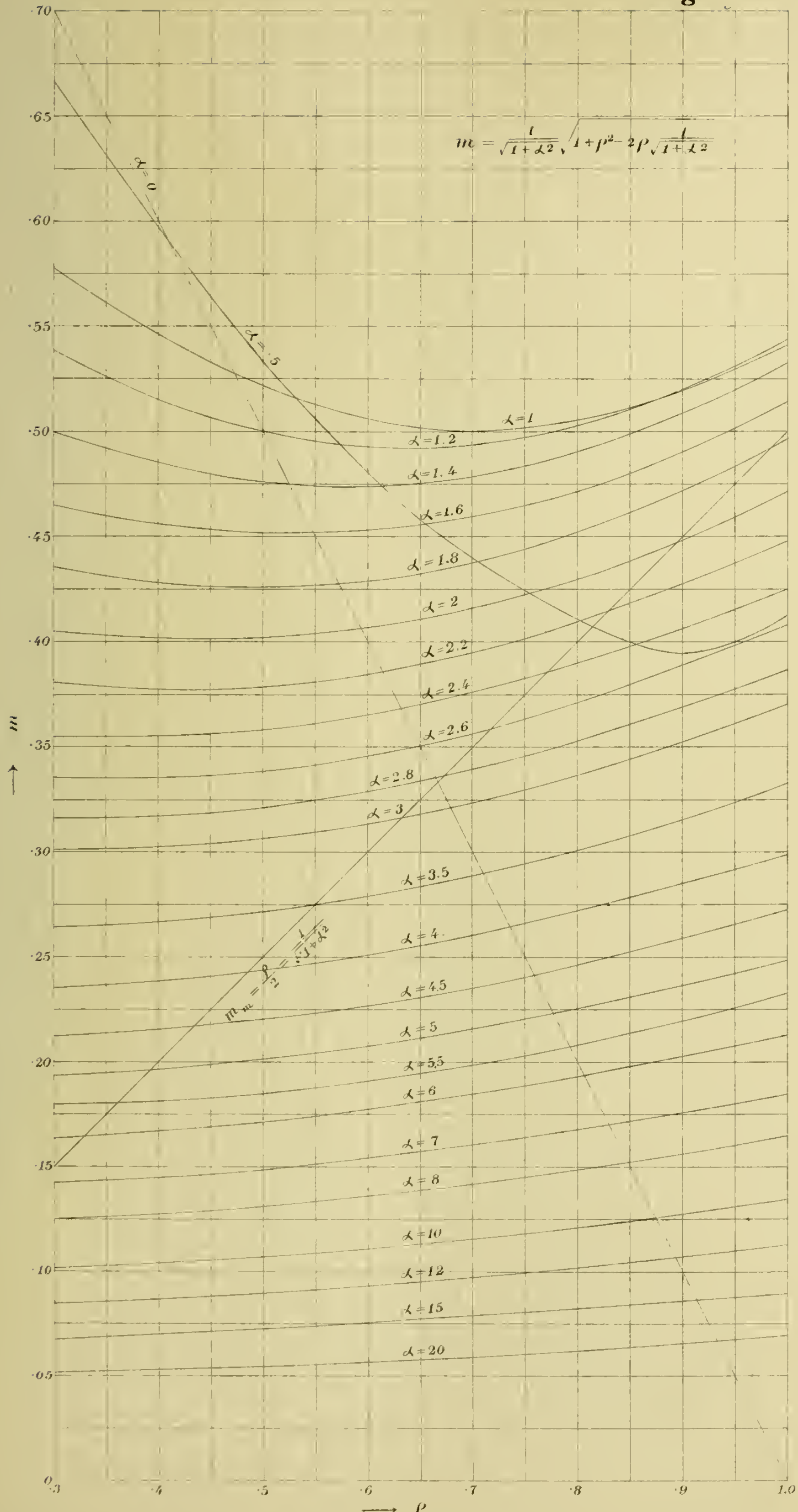
$\alpha = 20$
$\alpha = 18$
$\alpha = 16$
$\alpha = 15$
$\alpha = 14$
$\alpha = 13$
$\alpha = 12.5$
$\alpha = 12$
$\longrightarrow k$
$\longrightarrow \theta$
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
-1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0
0 2 4 6 8 10 12

Fig. 11.



·25
·24
·23
·22
·21
·20
·19
·18
·17
·16
·15
·14
·13
·12
·11
·10
·09
·08
·07
·06
·05
·04
·03
·02
·01
0
1
2
3
4
5
6
7
8
9
$\rho = \cdot 1$
$\rho = \cdot 9$
$\rho = \cdot 8$
$\rho = \cdot 75$
$\rho = \cdot 7$
$\rho = \cdot 65$
$\rho = \cdot 6$
$\rho = \cdot 55$
$\rho = \cdot 5$
$\rho = \cdot 4$
$\rho = \cdot 3$
α

Fig. 12.



$m = \frac{1}{\sqrt{1+\alpha^2}}\sqrt{1+\rho^2-2\rho\frac{1}{\sqrt{1+\alpha^2}}}$
$m_m = \frac{\rho}{m} = \frac{1}{\sqrt{1+\alpha^2}}$
α = 0
α = .5
α = 1
α = 1.2
α = 1.4
α = 1.6
α = 1.8
α = 2
α = 2.2
α = 2.4
α = 2.6
α = 2.8
α = 3
α = 3.5
α = 4
α = 4.5
α = 5
α = 5.5
α = 6
α = 7
α = 8
α = 10
α = 12
α = 15
α = 20
m
ρ
·70
·65
·60
·55
·50
·45
·40
·35
·30
·25
·20
·15
·10
·05
0
·3
·4
·5
·6
·7
·8
·9
1.0

Fig. 13.

$$y' = \frac{\alpha\rho}{\sqrt{1+\alpha_2}} - \frac{\alpha}{1+\alpha^2}$$

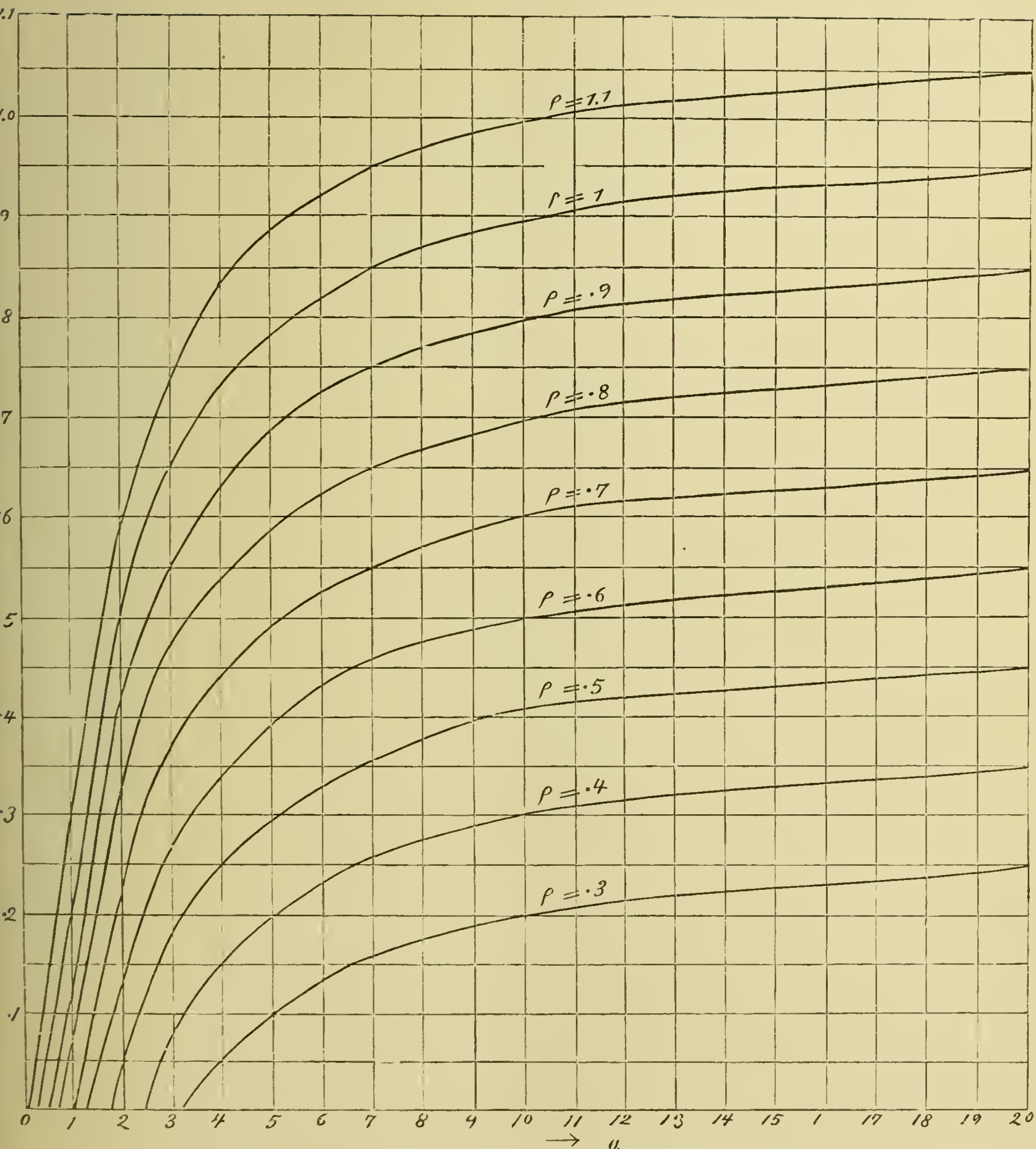


Fig. 14.

$$m' = \frac{\alpha}{\sqrt{1+\alpha^2}}\sqrt{1+\rho^2-2\rho\frac{1}{\sqrt{1+\alpha^2}}}$$

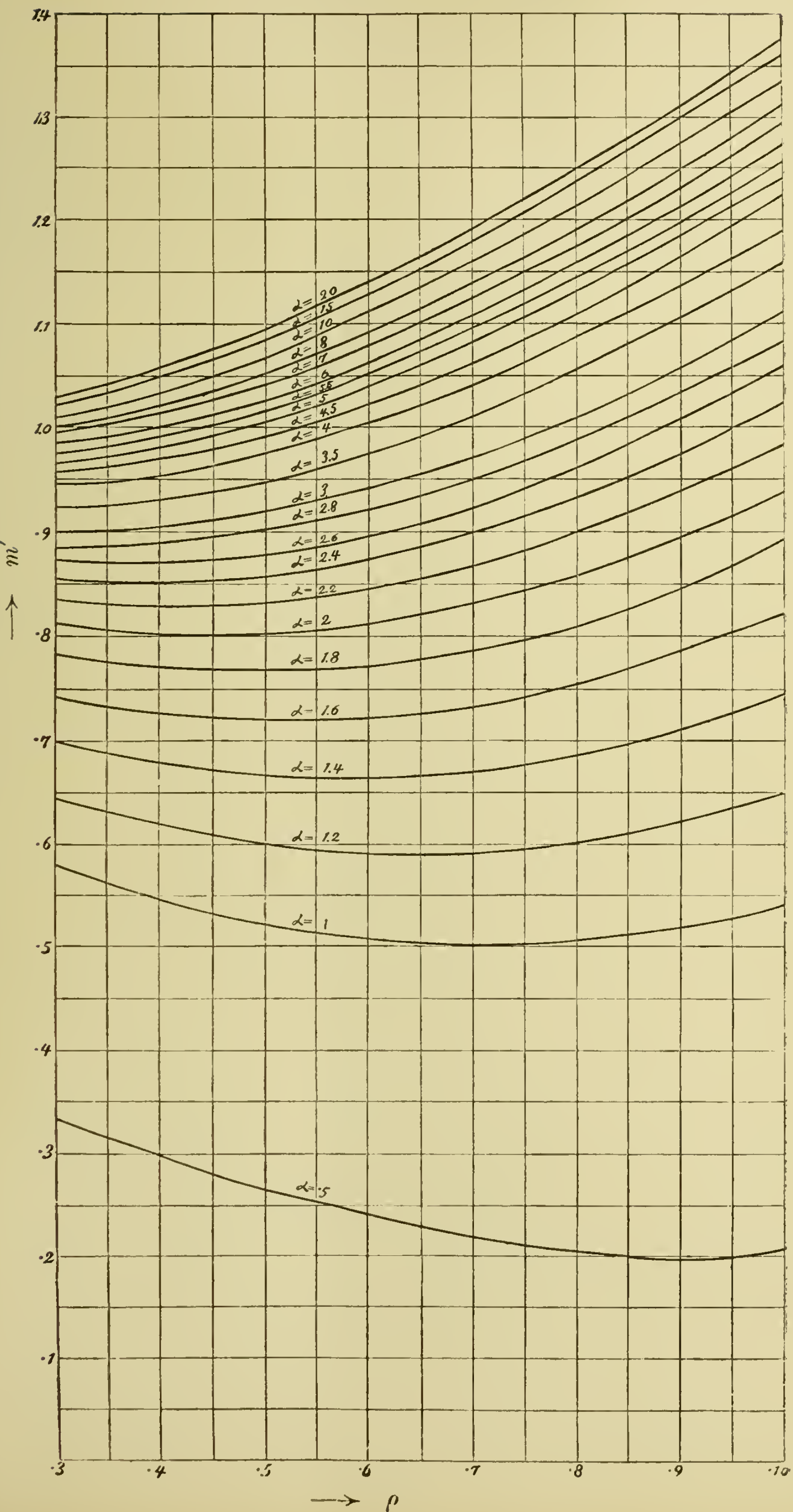
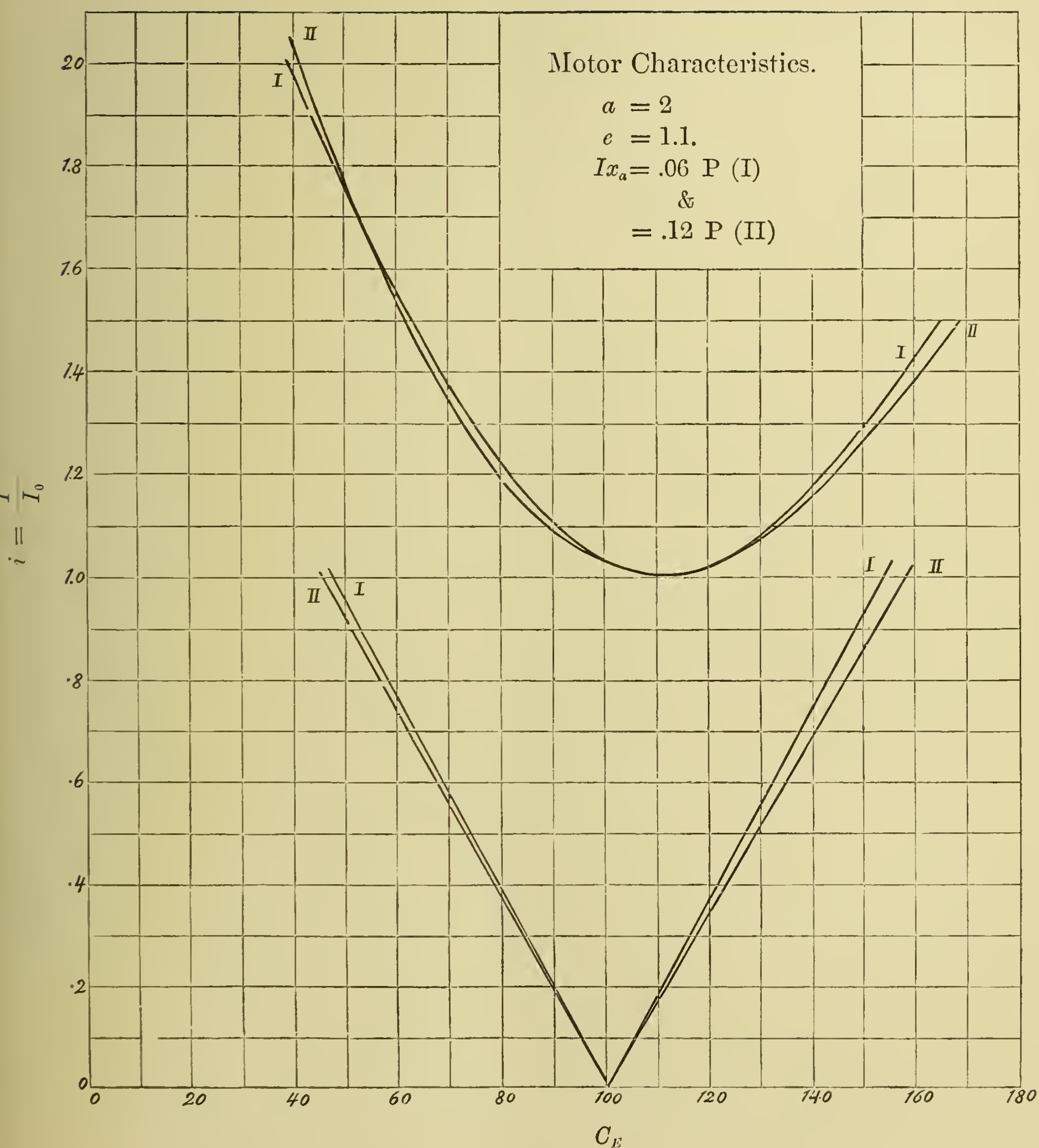
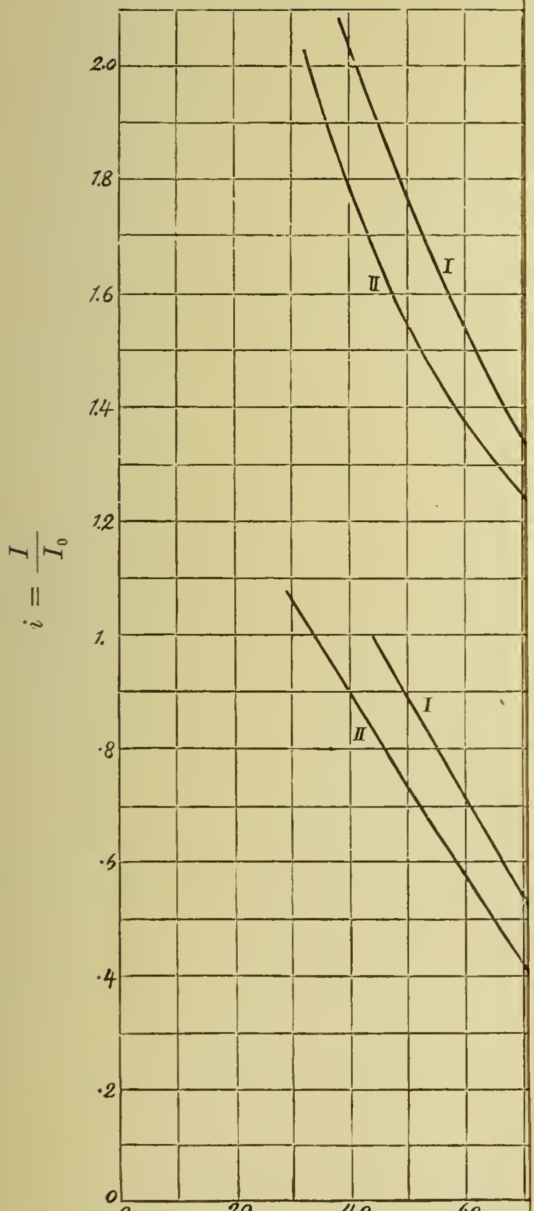


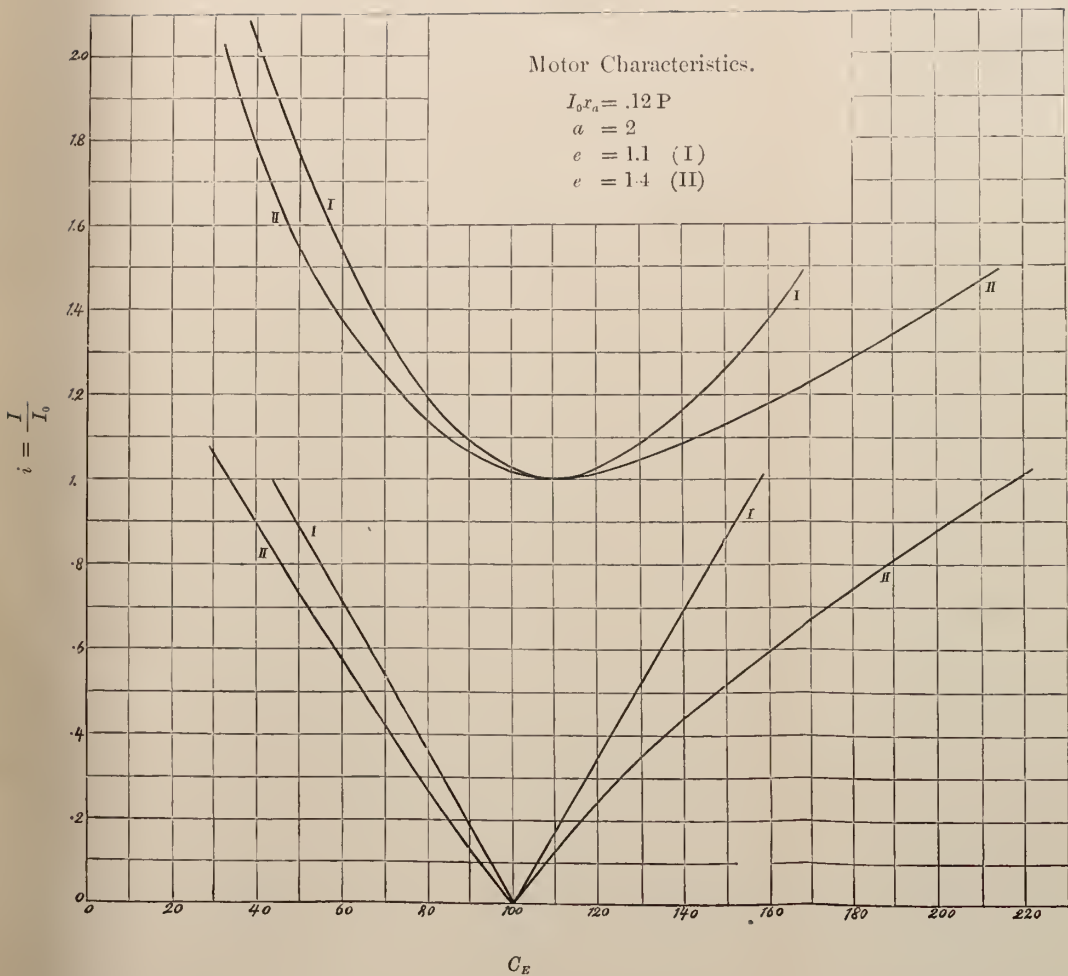
Fig. 16.

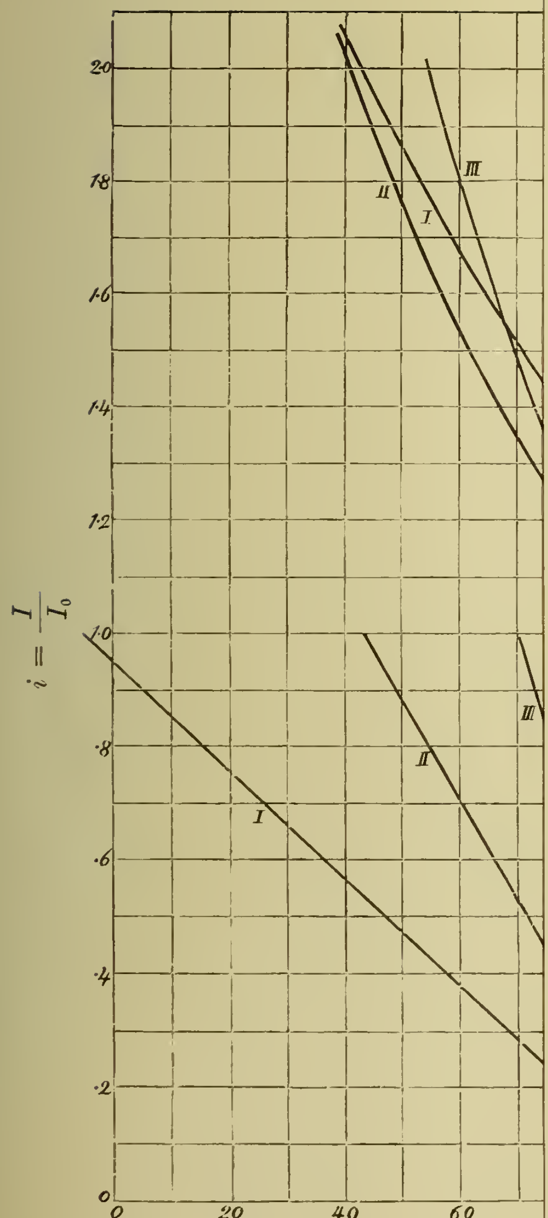

Motor Characteristics.
$a = 2$
$e = 1.1.$
$Ix_a = .06$ P (I)
&
$= .12$ P (II)
I
II
$i = \frac{I}{I_0}$
2.0
1.8
1.6
1.4
1.2
1.0
·8
·6
·4
·2
0
0
20
40
60
80
100
120
140
160
180
C_E



$i = \frac{I}{I_0}$
2.0
1.8
1.6
1.4
1.2
1.
·8
·6
·4
·2
0
0
20
40
60
I
II
I
II

Fig. 17.


Motor Characteristics.
$I_0 x_a = .12\,P$
$a = 2$
$e = 1.1$ (I)
$e = 1.4$ (II)
I
II
I
II
I
II
I
II
2.0
1.8
1.6
1.4
1.2
1.
·8
·6
·4
·2
0
0
20
40
60
80
100
120
140
160
180
200
220
$i = \frac{I}{I_0}$
C_E



$i = \frac{I}{I_0}$
2·0
1·8
1·6
1·4
1·2
1·0
·8
·6
·4
·2
0
0
20
40
60
I
II
III
I
II
III

Fig. 18.

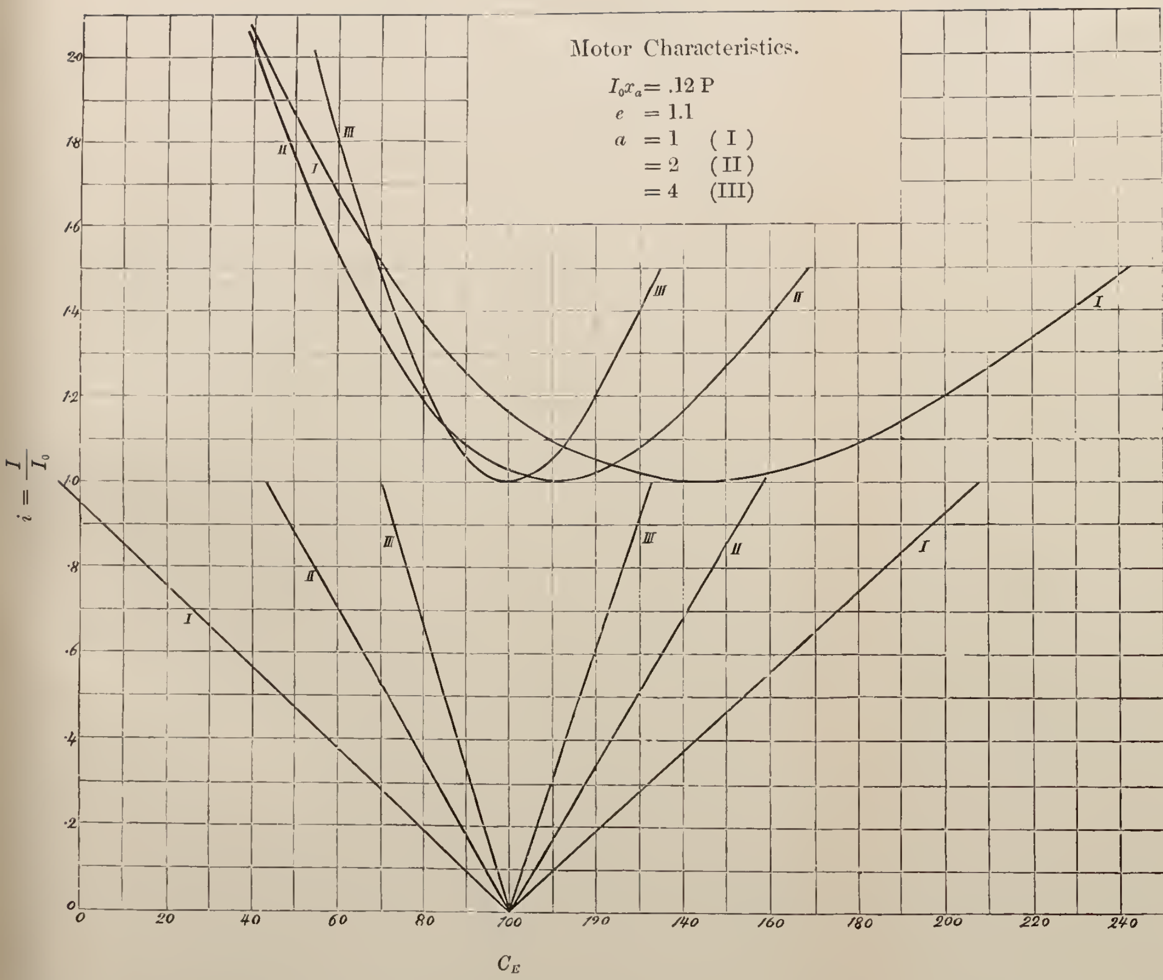
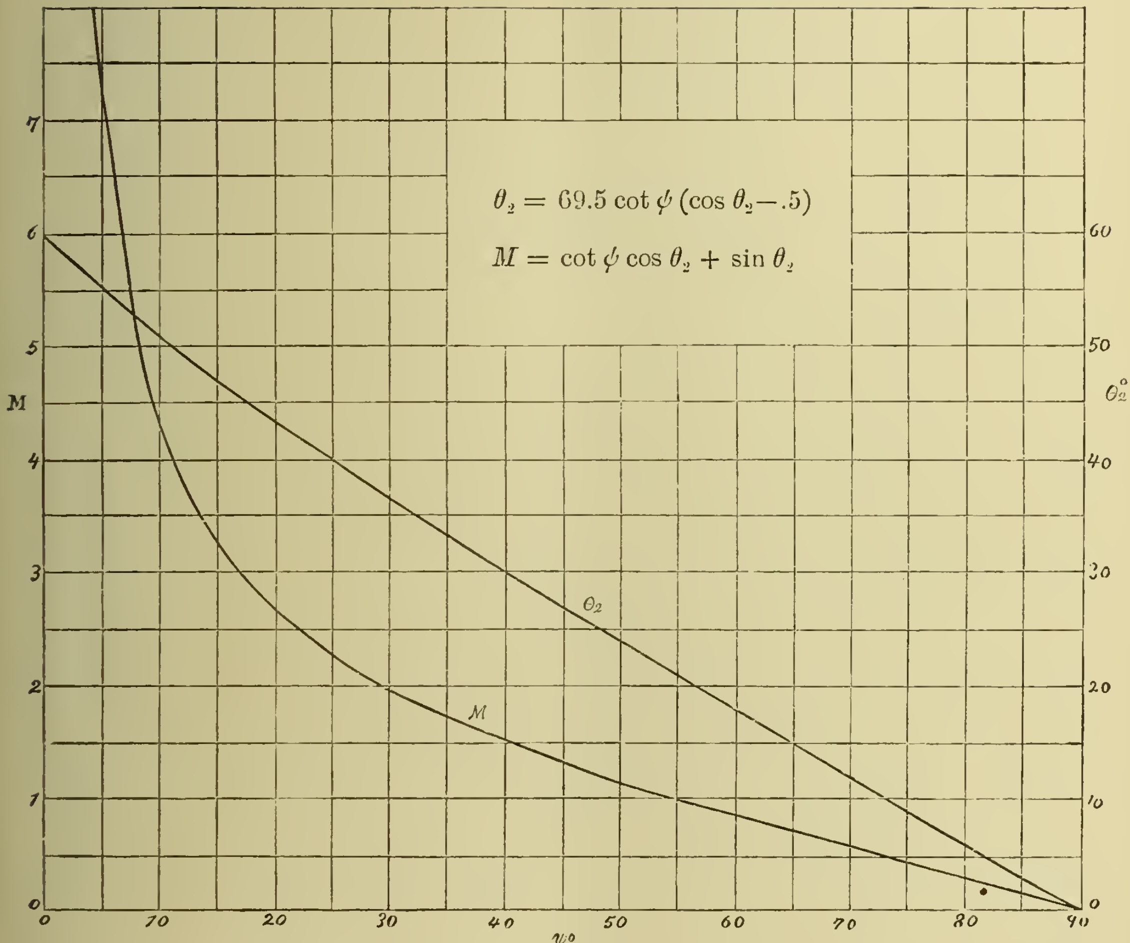

Motor Characteristics.
$I_0 x_a = .12$ P
$e = 1.1$
$a = 1$ (I)
$= 2$ (II)
$= 4$ (III)
$i = \frac{I}{I_0}$
C_E
0
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8
2·0
0
20
40
60
80
100
120
140
160
180
200
220
240
I
II
III

Fig. 19.



$\theta_2 = 69.5 \cot\psi (\cos\theta_2 - .5)$
$M = \cot\psi \cos\theta_2 + \sin\theta_2$
M
θ_2°
ψ°
θ_2
M
0
1
2
3
4
5
6
7
0
10
20
30
40
50
60
70
80
90
0
10
20
30
40
50
60

Fig. 20.

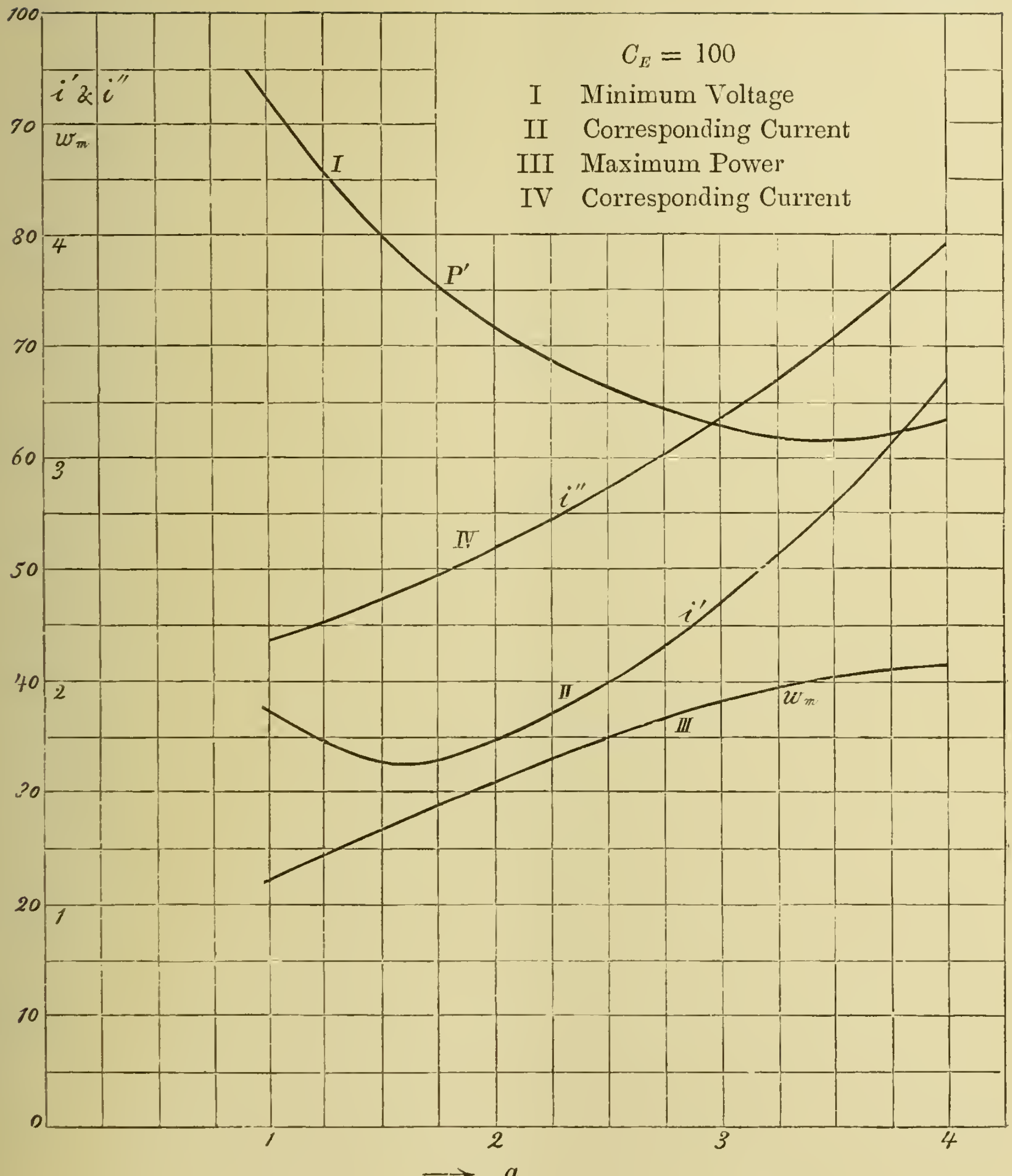
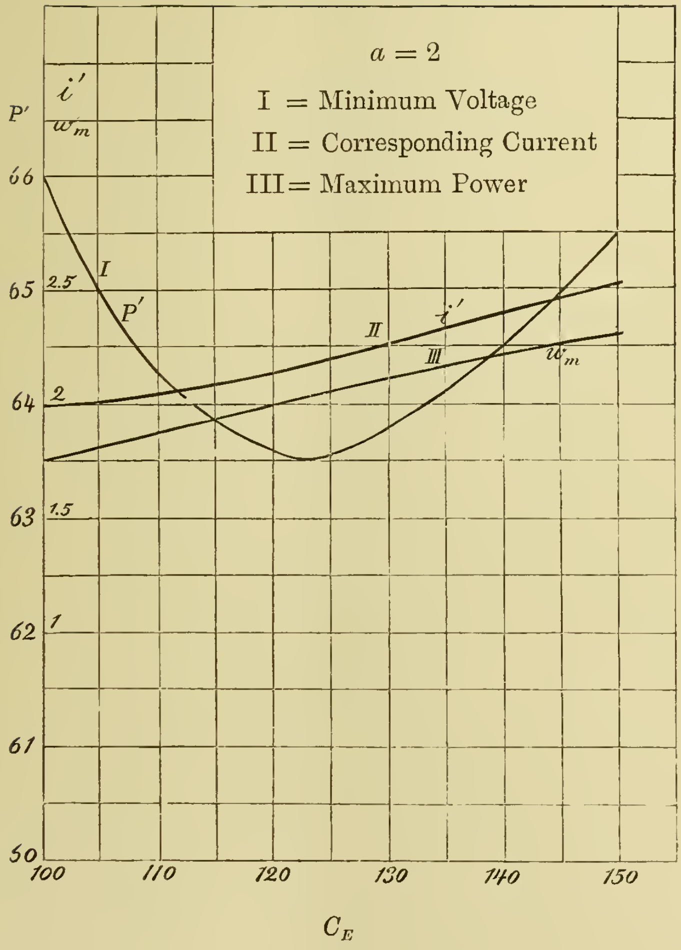
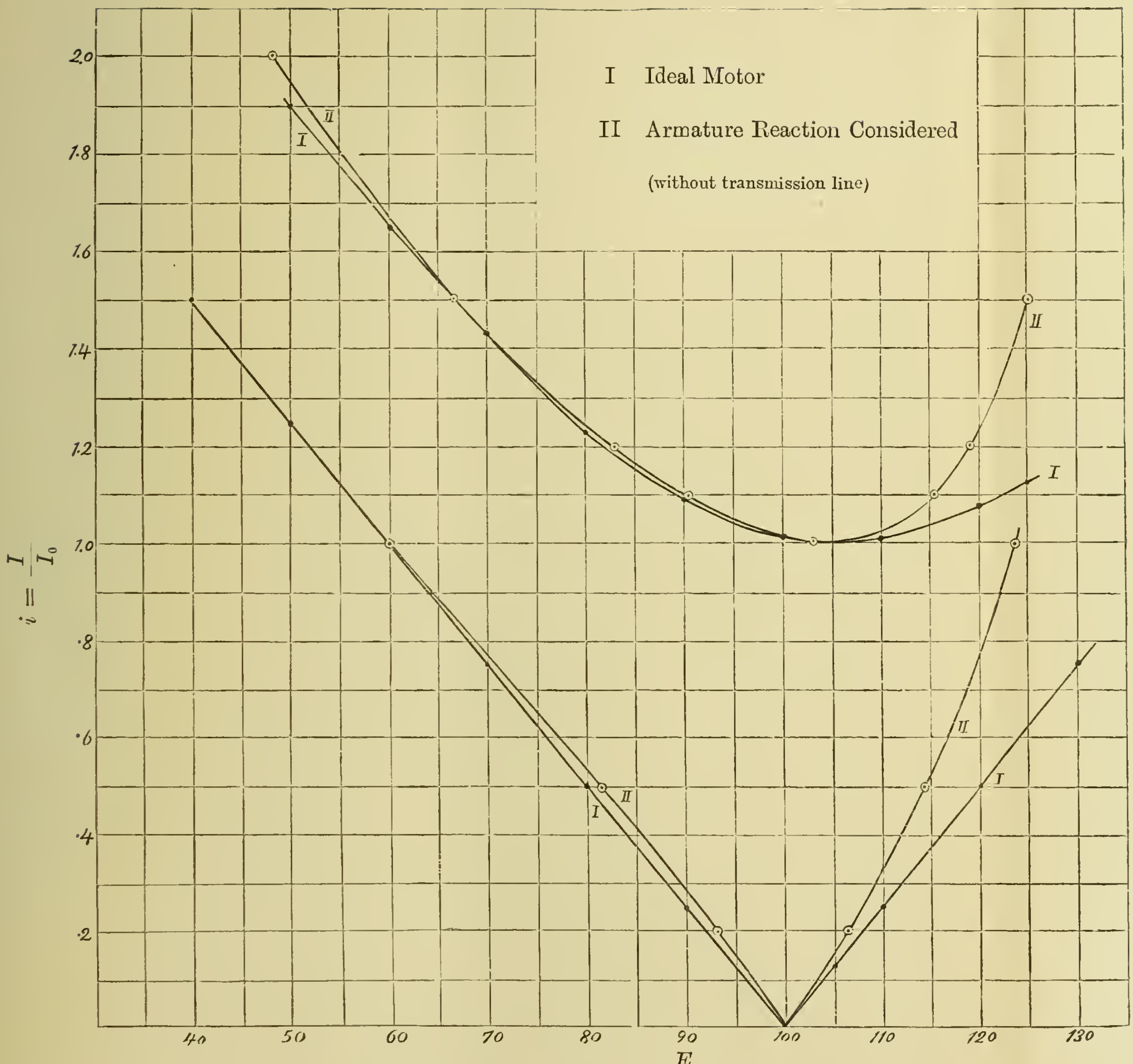

$C_E = 100$
I Minimum Voltage
II Corresponding Current
III Maximum Power
IV Corresponding Current
i' & i''
w_m
P'
a

Fig. 21.



$a = 2$
I = Minimum Voltage
II = Corresponding Current
III = Maximum Power
P'
i'
w_m
66
65
64
63
62
61
60
2.5
2
1.5
1
I
P'
II
III
i'
w_m
100
110
120
130
140
150
C_E

Fig. 22.



I Ideal Motor
II Armature Reaction Considered
(without transmission line)
2.0
1.8
1.6
1.4
1.2
1.0
·8
·6
·4
·2
$i = \frac{I}{I_0}$
40
50
60
70
80
90
100
110
120
130
E
I
II

Fig. 23.

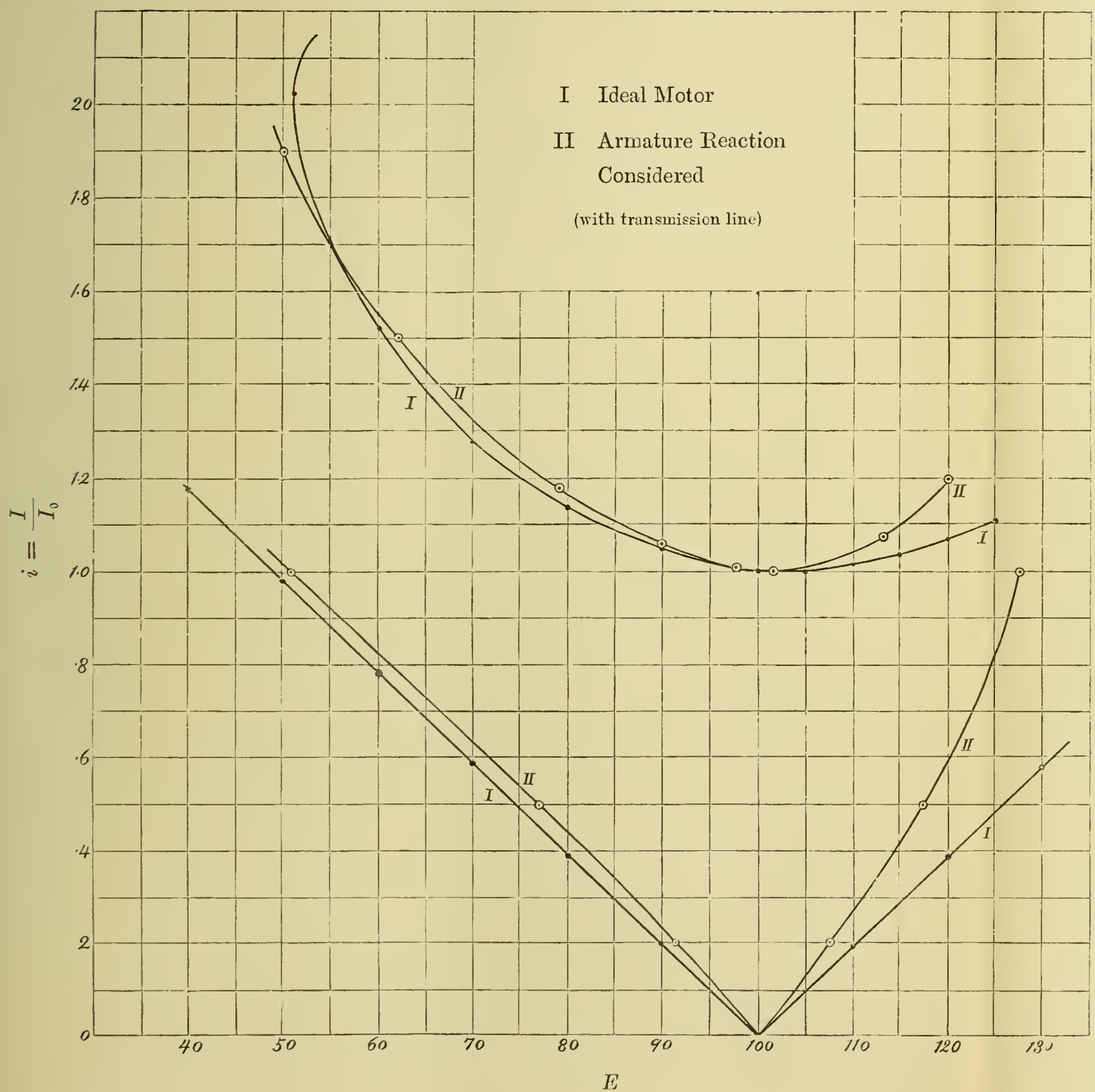

I Ideal Motor
II Armature Reaction Considered
(with transmission line)
I
II
I
II
I
II
I
II
$i = \frac{I}{I_0}$
2.0
1.8
1.6
1.4
1.2
1.0
·8
·6
·4
2
0
40
50
60
70
80
90
100
110
120
130
E

Fig. 24.

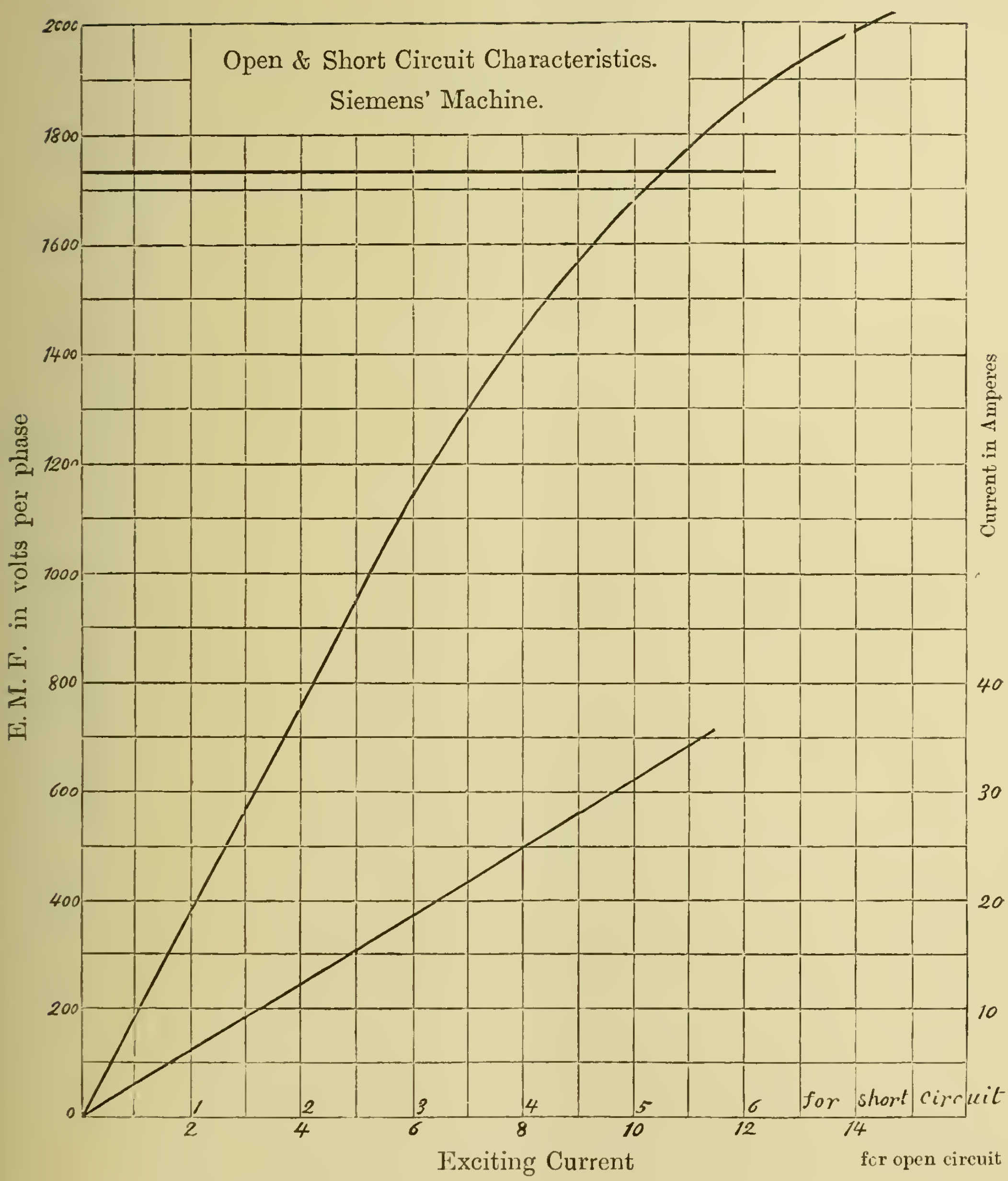

Open & Short Circuit Characteristics.
Siemens' Machine.
E. M. F. in volts per phase
2000
1800
1600
1400
1200
1000
800
600
400
200
0
Current in Amperes
40
30
20
10
1
2
3
4
5
6
for short circuit
2
4
6
8
10
12
14
Exciting Current
for open circuit

Fig. 25.

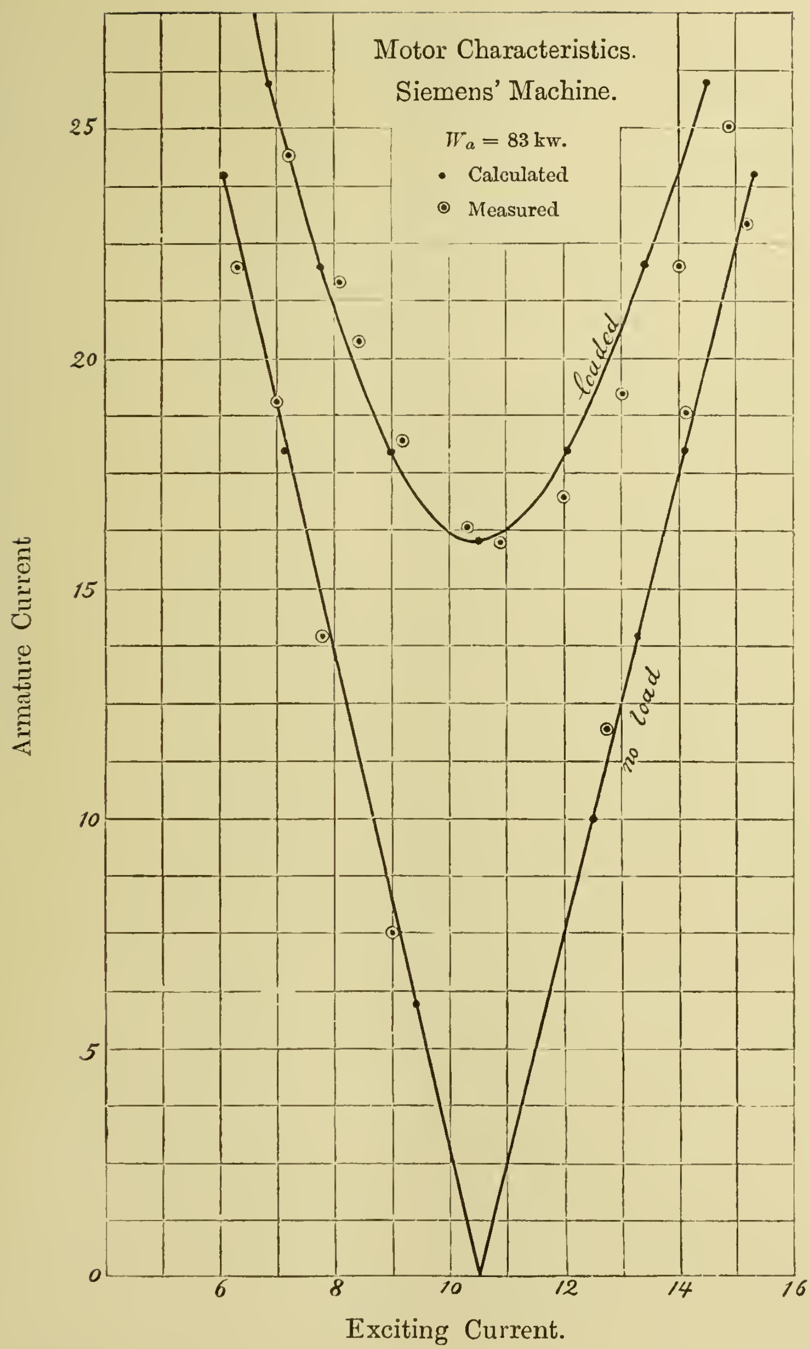

Motor Characteristics.
Siemens' Machine.
W_a = 83 kw.
Calculated
Measured
loaded
no load
Armature Current
Exciting Current.
0
5
10
15
20
25
6
8
10
12
14
16

Fig. 26.

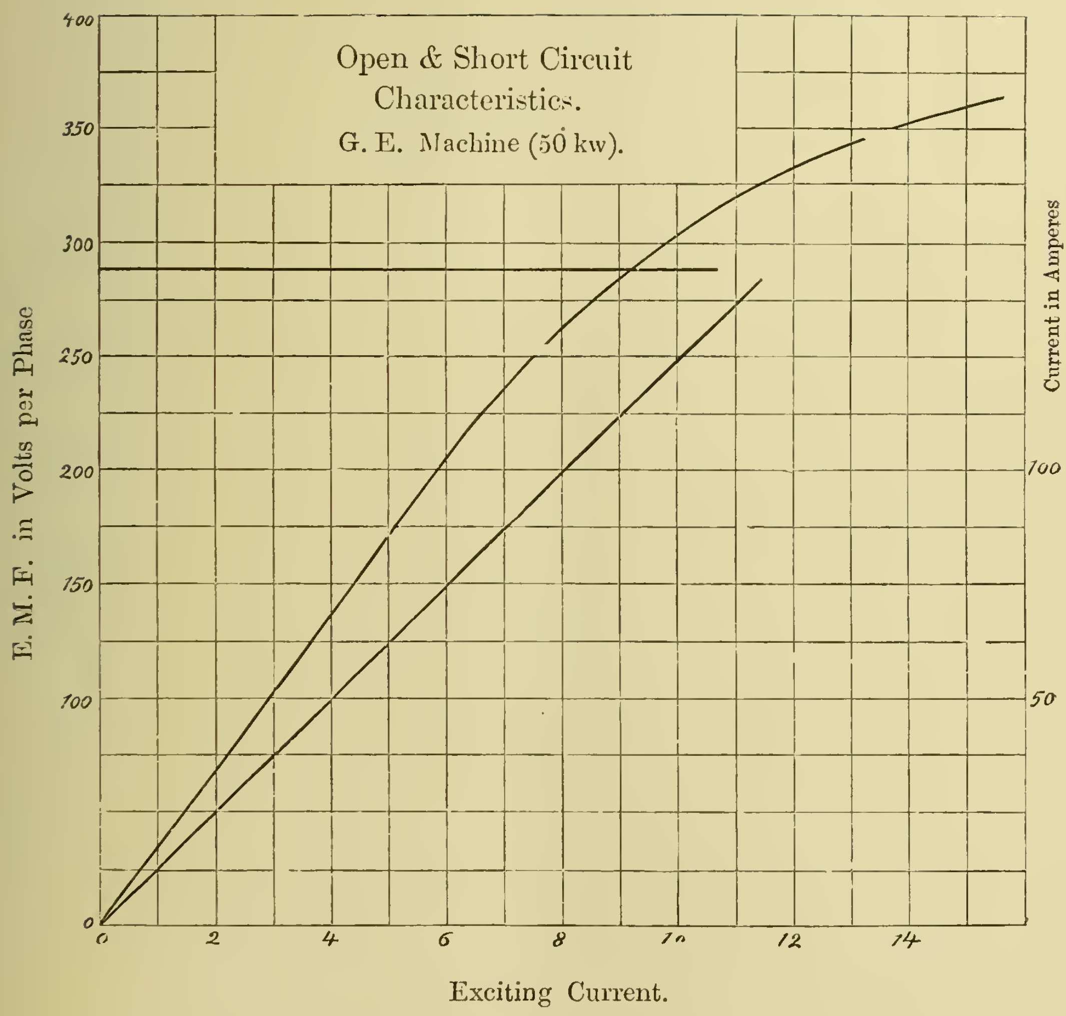
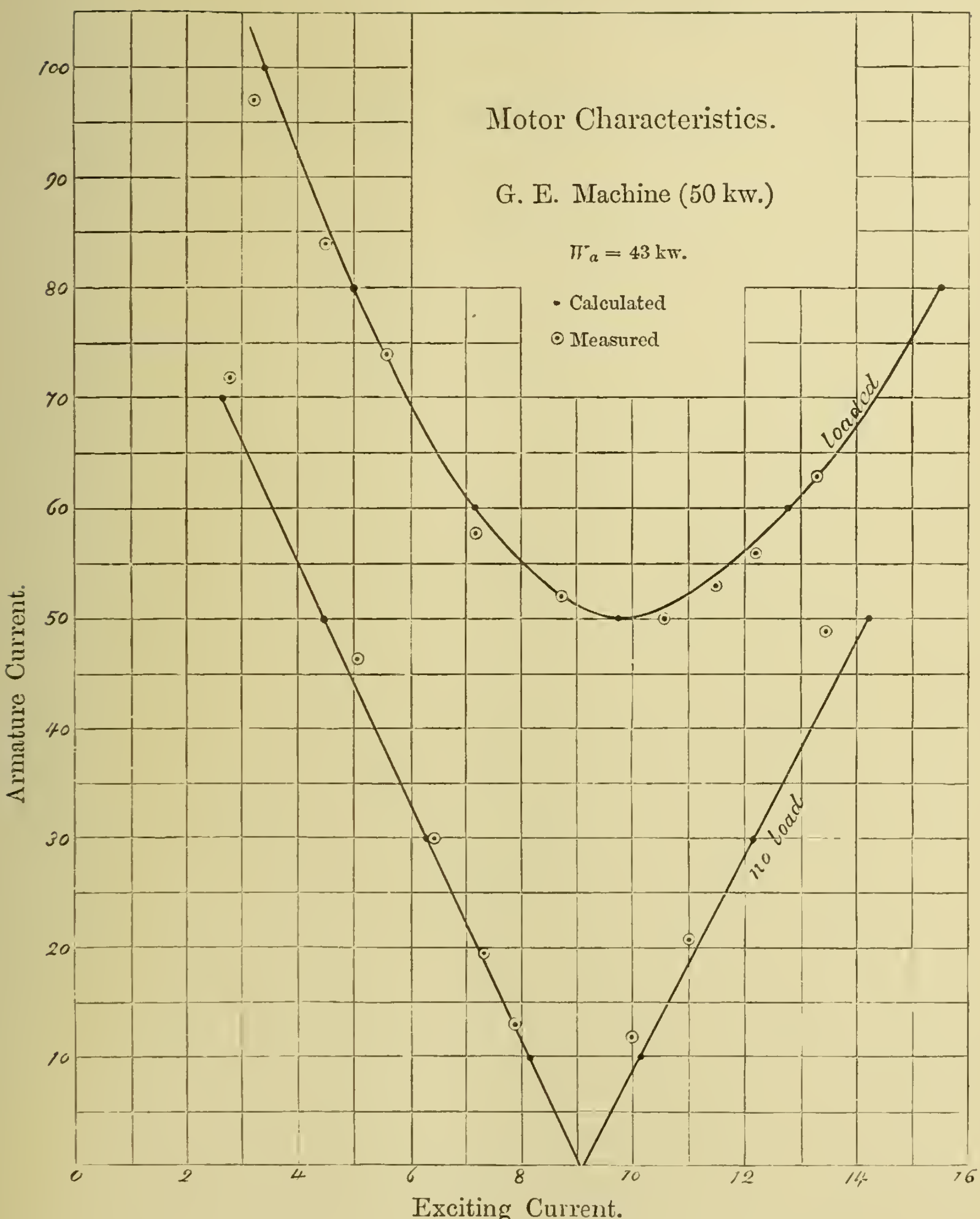

Open & Short Circuit
Characteristics.
G. E. Machine (50 kw).
E. M. F. in Volts per Phase
Current in Amperes
400
350
300
250
200
150
100
0
100
50
0
2
4
6
8
12
14
Exciting Current.

Fig. 27.


Motor Characteristics.
G. E. Machine (50 kw.)
W_a = 43 kw.
• Calculated
⊙ Measured
loaded
no load
Armature Current.
Exciting Current.
100
90
80
70
60
50
40
30
20
10
0
2
4
6
8
10
12
14
16

THE JOURNAL OF THE COLLEGE OF ENGINEERING.
IMPERIAL UNIVERSITY, TOKYO, JAPAN.
Vol. V, No. 3.

Published March, 1912.

The Protective Action of the Ground Wire.

By

H. Ho, *Professor of Electrical Engineering.*

Introduction.

The foremost requirement of a long distance electric transmission system is the reliability of its service. It is not always possible to provide reserve power stations or battery installations which can take up the entire load on occasion of a shut-down even for a short time, and the question of the protection of the transmission line, which is the weakest point in a system, becomes more and more important as the distance and the volume of power transmitted increase.

No doubt, the most annoying cause of trouble in the transmission line and the apparatus in immediate connection with it, is cloud lightning, that is, the disturbance caused by the discharge of atmospheric electricity. Other causes such as wind and sleet, destructive as they are in their effects, are simple in action and easier to provide against. Line protection against lightning has therefore been one of the most favorite topics of discussion with electrical engineers for the past five or six years, and many are the inventions of new lightning arresters and the elaborate papers on the nature of lightning disturbances that have appeared, the consequence being that we are now much more familiar with the power and behavior of lightning, and the line and apparatus are now protected with better devices and better knowledge than was the case a few years ago.

The opinions of the recognized authorities on this subject, and the reports of the committees of learned societies agree in one point at least, that the protective effect of the ground wires, that is earthed wires on the transmission line above the conductors, is certain beyond doubt, the confirmation by the actual diminution of lightning broubles by this simple device in many and various transmission systems being quite indisputable.

This idea of using the ground wire for line protection is quite old; but originally inadequately thin wires, indeed very often the barbed galvanized wires used for cattle fences, were employed; and very naturally they caused more trouble than benefit by breaking and falling upon the line conductors. In consequence the ground wire lost favor with engineers for some time. However, its real merits were gradually recognized till its adoption was revived very generally, this time with better method and better material, so that it is now considered decidedly bad practice to neglect to specify the ground wire for a transmission line of any importance exposed in any degree to atmospheric disturbances.

Very strange to say, the principles of the protective action of the ground wire are very little understood by engineers in general, their notions about it being not much advanced beyond the barbed wire stage. Even in the writings of eminent engineers of the day, the ground wire is usually treated simply as a particular form of lightning conductor, recognizing only its protective action against the direct stroke of lightning.

In our opinion, its action in diminishing the havoc done to the transmission line and apparatus by the disturbances caused by cloud lightning can be analyzed into four kinds.

(1) Protection from direct strokes of lightning.

(2) The shielding effect, by which the charge induced

on the transmission line by the thunder cloud is diminished.

(3) The increase of the capacity of the transmission line, in virtue of which the rise of the line potential due to the above said induced charge, upon the discharge of the inducing cloud to other clouds or to the earth, is decreased.

(4) The formation of a short-circuited secondary to the transmission line, by which an increase of the effective resistance and a decrease of the effective inductance of the line are obtained, which, in conjunction with the increase of the capacity stated above, causes an increase of the attenuation constant, which means an increased rapidity with which the travelling wave of electric potential is diminished in its wave height.

The first of these is of course well known, and was in truth the sole object for which the barbed wire was originally strung on the poles. The second is understood pretty well now-a-days; but as to the third, it is passed unheeded by most writers, and with some of them it is difficult to tell whether they recognize this action or not. The only instances of its statement that I have ever come across are in W. L. Water's short discussion of R. D. Mershon's paper (High Tension Power Transmission, compiled by A. I. E. E., p 111), and a few lines in R. P. Jackson's paper in the Proceedings of A. I. E. E., 1907.

With the fourth, the case is still worse. The only reference to this action, to my knowledge, is to be found in Rushmore and Dubois' paper in the same volume of the said Proceedings and the authors dismiss the subject with the following three lines.

"The ground wire in connection with the earth forms a short-circuited secondary which helps to dampen out disturbing oscillations."

This curious lack of complete explanation of the real action of the ground wire has led to the preparation of the following short essay.

Chapter I.

Lightning and its Effects upon the Transmission Line.

Opinions still differ concerning the origin and source of the atmospheric electricity which produces the lightning discharge; but according to the most probable of the theories, the minute water particles in the air are endowed by the action of the sun with an electric charge, which is always positive. According to Prof. E. Thomson, the years in which sun-spots, are abundant coincide with the years of great activity of lightning discharges on the earth. As these water particles unite to form larger drops, the charge on each spheroid increases in direct proportion to its volume while the surface does not increase at the same rate, consequently the surface density of the electric distribution and therefore the electric intensity outside it increases gradually until the intensity becomes so great that the air is torn disruptively at a certain spot and the adjoining parts follow in succession, producing the irregular path of the spark in the cloud which is determined by the line of least resistance at every stage of its progress, just like the cracking of a solid substance.

Thus the charge of a thunder cloud is not distributed on the outer surface as is the case with a charged conductor, but is disseminated throughout the cloud. But in explaining the effect

of the lightning discharge upon the transmission line, there is of course no harm in treating the thunder clouds for the sake of simplicity as insulated charged conductors.

The ways in which the lightning discharge affects the transmission line and causes disturbance in it may be classified as follows;—

(a) Direct stroke.

(b) Induction.

By direct stroke is meant the discharge of the thunder cloud directly on the line, the consequence being either the puncturing of the insulators or the spill-over on their surface, with the consequent cracking by the heat, and very often the splitting and burning of the wooden arms and poles. The region thus affected is generally very much restricted in extent, rarely exceeding a few pole spans. This can be explained by the exceedingly high frequency of the discharge, which forces the spark to take the shortest possible path to the earth, on account of the predominant effect of the inductance. If the stroke be so weak as to be unable to punture the insulators or cause a spill-over, it produces simply traveling waves of electric charge and therefore of potential, which propagate in both directions, as in the case of the induced charge to be treated below.

To take the blunt of such a direct stroke and conduct the discharge to the earth, is the first of the actions of the ground wire which we have enumerated above, and it is exactly similar to that of an ordinary lightning conductor in principle and requirements.

Next, to consider the case of induction: suppose a thunder cloud, which is almost always positively charged, approaches a transmission line. Then the corresponding distribution of

negative charge on the earth's surface approaches also, and a part of it is produced on the line conductors; that is, although most of the Faraday tubes originating in the charged cloud end on the ground, a small part terminate on the line.

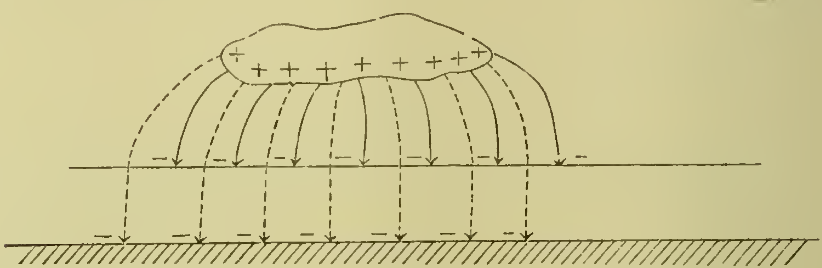


Fig. 1.

If the transmission line be grounded at its neutral point at either one or both of the ends, the state of the induction would be as shown in Fig. 1, negative distribution only being produced in that part of the line opposite the inducing cloud (the dotted lines show the Faraday tubes which go directly to the ground).

Now suppose the charge of the cloud suddenly disappears being discharged to the ground or to other clouds,—as is clear from the nature of the charge on a thunder cloud, as explained above, it is impossible that the whole charge should disappear with one discharge, but as we are considering the phenomena accompanying a single discharge, it is sufficient to contemplate only that portion of the Faraday tubes and electric intensity which disappear at the single discharge,—then we have momentarily the distribution of the negative charge left on the line, and the potential of that part is no more zero as it was before the discharge, but we have a distribution of negative potential, which is obtained by dividing the charge per unit length by the capacity

per unit length; and the Faraday tubes bridge the space between the line and the ground as indicated in Fig. 2.

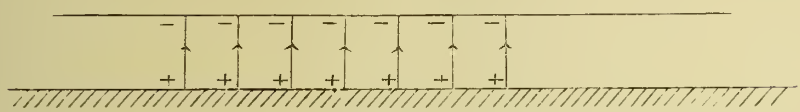


Fig. 2.

The distribution of charge and therefore that of potential now takes the form of traveling waves which are propagated in both directions.

If the potential be sufficiently high either to puncture the insulators or to cause a spill-over, the maximum potential of the traveling waves would naturally be decreased; according to Creighton it may sometimes reach as high a value as 500,000 volts.

The most comprehensive analogue, in our opinion, of distributed capacity and inductance like those of a transmission line, is a spiral spring as shown in Fig. 3; the mass per unit length of it in the unstrained state represents the inductance per unit length of the line; the fractional contraction—that is the ratio of the decrement of the length of an element to its original length—and the fractional elongation represent the positive and the negative charge per unit length respectively. The tension and the pressure correspond to the negative and the positive potential; the extensibility of the spring, that is the ratio of the fractional elongation to the tension, corresponds to the capacity per unit length of the line; the velocity of a point represents the current, and the displacement of a point from its original position shows the total quantity of electricity that has passed through the corresponding point of the transmission line. It is to be

remembered that the number of convolutions, that is to say the original length of the spiral in the unstrained state, must be taken as the measure of the length of the transmission line.

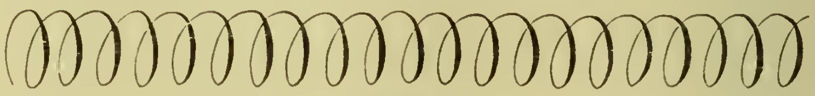


Fig. 3.

By means of this analogue, all kinds of free and forced oscillations and all other transient phenomena of a line can be explained; the reflections at the ends, and the partial reflections at the transition points between two parts of a line with different constants represented by the junctions of different kinds of spirals, can be beautifully demonstrated.

Thus, the problems of the line phenomena are exactly similar to those usually found in the text books of acoustics, the explanations and the conclusions for the mechanical cases being at once applicable to cases electrical. In proposing the above analogue, we have not mentioned the resistance and the leakage conductance of the line, but they are easily taken into account by supposing that there is some amount of friction between the spiral and the solid smface on which it rests or the fluid in which it is immersed, which is proportional to the velocity; and by supposing that the elements of the spiral are connected by friction junctions which permit a slipping between neighboring elements, the velocity of the slipping being proportional to the force at that point.

Now to return to our case of induction by a thunder cloud, the state of things at the instant of the disappearance of the inducing charge on the cloud can be represented by means of the above analogue, by supposing a postion of the spiral spring,

corresponding to that portion of the transmission line which has the induced charge, is stretched as shown in Fig. 4, and then suddenly set free. The traveling of the stretch towards right and left, gives us a vivid mental picture of the starting of the traveling waves of the charge and therefore of the potential in both directions, the height of these two waves being one half of the original distribution, as shown in Fig. 5.

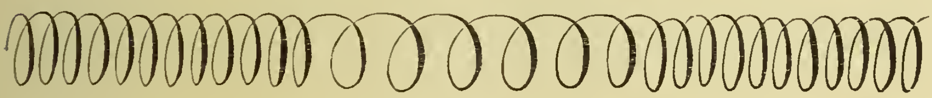


Fig. 4.

Fig.5.

These traveling waves arrive at both the terminals of the line, whence they are reflected back according to the conditions of the terminals,—partial reflections more or less complicated occur generally on account of the apparatus and the circuits in direct or indirect connection with the transmission line,—and the waves pass right and left repeatedly, gradually decreasing in their height through loss of energy till they totally subside. This explains why the generating and the receiving stations are affected by lightning discharges that take place at a considerable distance from them.

Let us next consider the induction on a transmission line completely insulated from the ground. In this case, equal and opposite charges must be produced on the line, of which the negative distribution is of course in the part immediately facing the inducing cloud, while only a small fraction of the positive

charge resides in the same part, the main portion of it being distributed in the remaining part of the line, as shown diagramatically with Faraday tubes in Fig. 6.

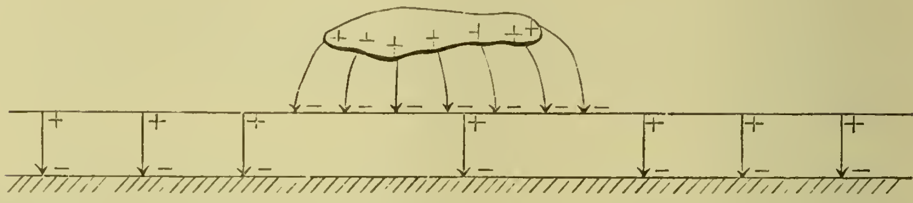


Fig. 6.

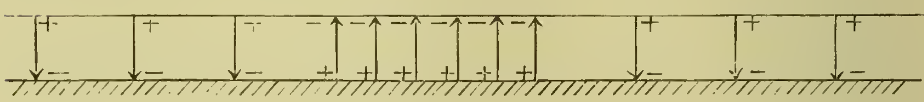


Fig. 7.

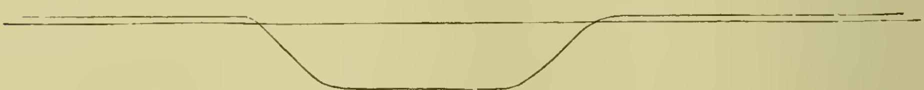


Fig. 8.

The sudden disappearance of the inducing charge leaves on the line such distributions of the negative and positive charges as appear in Fig. 7, the negative being concentrated in a short length and the positive being distributed with considerably less density on the remaining part of the line and the apparatus connected with it. As in the former case, this distribution of the charge and therefore of the potential, as shown in Fig. 8, instantly starts two traveling waves, one in each direction, the height of the waves being one half of the original distribution.

The most remarkable thing in connection with induction by a thunder cloud is the concentration of the induced negative

charge and the consequent negative potential, the scene of the considerable rise of the latter being usually 1000 to 5000 feet in extent at most.

It is not the object of the present paper to discuss the protective action of the ground wire against direct strokes of lightning but to explain what part it plays in diminishing the potential rise caused by the induction of the thunder cloud and the height of the traveling waves thereby produced.

In truth, it is this inductive action and not the direct stroke that a transmission line most frequently experiences.

Chapter. II.

The Shielding Action of the Ground Wire against Induction by Thunder Clouds.

By the use of the ground wire the amount of the induced negative charge on a transmission line is diminished by the following reason. For the sake of the simplicity of the formulae and the calculations, let us consider a simple line such as is shown in Fig. 9, consisting of two parallel cylindrical conductors at the same height above the ground. In practice such a simple line would be rare, except with Thury's system of direct current transmission, but it is hoped the results of the calculations for this simple case may suffice to give a general idea of the magnitudes of the various effects produced by the presence of the ground wire.

Let the diameter of the conductors be d cm, their height above the ground h cm, the axial distance between them D cm, and let the line have grounded neutral.

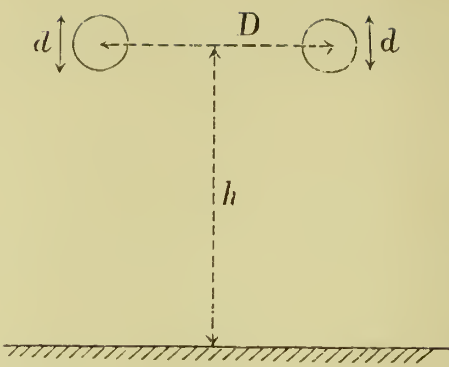


Fig. 9.

First let us consider the induction when we have no ground wire.

If there were no conductors whatever above the ground, the Faraday tubes which start in the thunder cloud and end on the earth would produce a distribution of negative charge on the earth's surface, and if we take the earth as conducting, the Faraday tubes may be taken to be everywhere normal to the earth's surface at all points not very distant from the ground. Consequently the potential of a point at the height of h cm above the ground may be expressed by,

$$V = kh,$$

where k is a constant of the locality, and in truth it is no other than the electric intensity itself; V and k are in electrostatic units.

Now suppose such a simple grounded transmission line as is shown in Fig. 9 is brought to this place, and let us consider the change of the electric distribution thereby produced. Let q be the induced charge per unit length of each of the conductors. Since the two conductors my be taken as similarly situated with respect to the cloud and the earth, they must be equally charged by induction. On account of the great distance of the inducing cloud from the conductors and the ground, we may take the distribution in the cloud as not at all affected by the introduction of the transmission line, hence the distribution on the ground must be the superposition of the distribu-

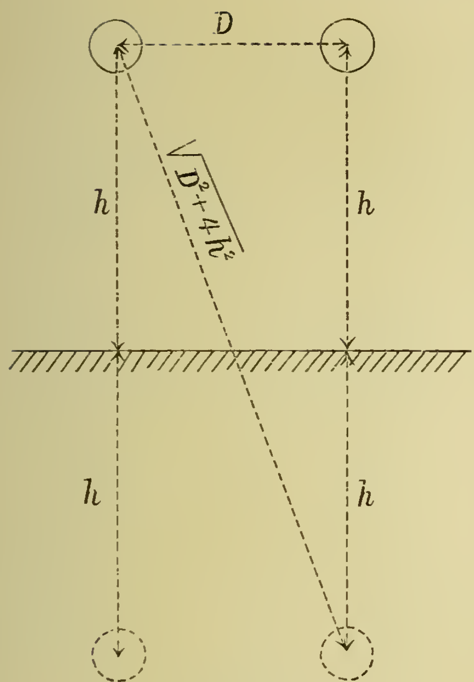


Fig. 10.

tion when there were no conductors, (which we shall call 'distribution No. 1'), and the distribution when there is no thunder cloud but only the said transmission line with q per unit length of each conductor (which we shall call 'distribution No. 2').

The potential at the conductors due to distribution No. 1 and the thunder cloud is,

$$V = kh,$$

as already explained, and that due to distribution No. 2 and q is easily obtained by the well known method of electrical image.

Thus the potential of the line is expressed by,

$$V = kh + 2q \log \frac{4h}{d} + 2q \log \frac{\sqrt{D^2+4h^2}}{D} = 0.$$

It is zero because the line is supposed to have grounded neutral. From this we have,

$$q = -\frac{kh}{2} \frac{1}{\log \frac{4h\sqrt{D^2+4h^2}}{dD}}, \quad \text{......................} \quad (1)$$

and the charge per unit length of the transmission line—*i. e.* the two conductors taken together—is

$$2q = -\frac{kh}{\log \frac{4h\sqrt{D^2+4h^2}}{dD}}. \quad \text{.............................} \quad (2)$$

Next, let a ground wire, d' cm in diameter, be added above the middle of the line conductors as indicated in Fig. 11, and consider the induced charge on the line for this case. Let the induced charge per unit length of the ground wire be denoted by q', then for the same reason as in the preceding case, the distribution on the ground is the superposition of the aforesaid distribution No. 1, and the distribution when there is no thunder cloud but only these three conductors, with q per unit length of the line conductors and q' per unit length of the ground wire.

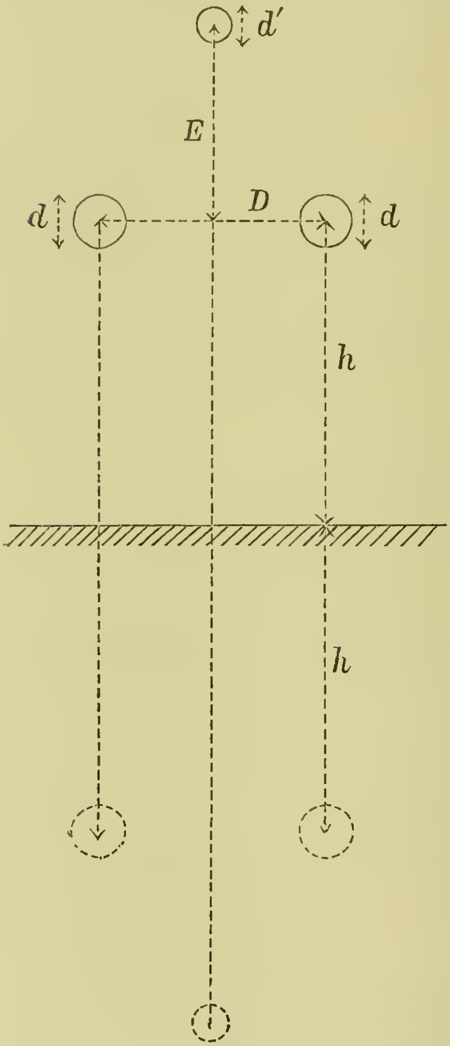


Fig. 11.

Therefore, again adopting the method of electrial image, we can express V and V', the potential of the line and the ground wire respectively, as follows:—

$$V = kh + 2q\log\frac{4h}{d} + 2q\log\frac{\sqrt{D^2+4h^2}}{D}$$

$$+2q'\log\frac{\sqrt{\frac{D^2}{4}+(2h+E)^2}}{\sqrt{\frac{D^2}{4}+E^2}} = 0$$

$$V' = k(h+E) + 2q'\log\frac{4(h+E)}{d'}$$

$$+4q\log\frac{\sqrt{\frac{D^2}{4}+(2h+E)^2}}{\sqrt{\frac{D^2}{4}+E^2}} = 0$$

From these two we have the charge per unit length of the line, that is the two conductors taken together,

$$2q = -k \frac{2h \log \frac{4(h+E)}{d'} - (h+E) \log \frac{\frac{D^2}{4} + (2h+E)^2}{\frac{D^2}{4} + E^2}}{2 \log \frac{4(h+E)}{d'} \log \frac{4h\sqrt{D^2+4h^2}}{dD} - \left(\log \frac{\frac{D^2}{4} + (2h+E)^2}{\frac{D^2}{4} + E^2} \right)^2} \qquad (3)$$

(in electrostatic unit.)

To compare this with (2) which is for the case without ground wire, and to give a definite idea of the effect of the presence of it, we take such a numerical example as the following:—

$$\begin{aligned} d &= 1.5 \text{ cm}, \\ d' &= 1 \text{ cm}, \\ h &= 1000 \text{ cm}, \\ D &= 200 \text{ cm}, \\ E &= 173 \text{ cm}, \end{aligned}$$

(three wires at the corners of an equilateral triangle).

By (2) we calculate,

$$2q = -k\,98.8,$$

and from (3) we get,

$$2q = -k\,75.6.$$

Thus we see that by adding the ground wire we can reduce the induced charge by 23 per cent.

Chapter III.

The Increase of the Electrostatic Capacity by the Ground Wire.

The potential of the transmission line at the moment immediately following the discharge of the inducing cloud is obtained by dividing the induced charge per unit length of the line by its electrostatic capacity per unit length; thus for the same

amount of the induced charge, a line with a larger capacity would acquire a lower potential. We shall now explain the increase of the line capacity caused by the presence of the ground wire.

Taking up again the simple transmission line of Fig. 9 consisting of two conductors at the same height above the ground, if we give the charge q per unit length to each of the line conductors, their potential will be, by the principle of electrical image,

$$V = 2q \log \frac{4h}{d} + 2q \log \frac{\sqrt{D^2+4h^2}}{D}.$$

The capacity per unit length of the line, that is the two conductors taken together, therefore is,

$$C = \frac{2q}{V} = \frac{1}{\log \dfrac{4h\sqrt{D^2+4h^2}}{dD}}. \quad \text{.................. (4)}$$

Next, add the ground wire in the position indicated in Fig. 11, and give the charge q per unit length to each of the line conductors; then their potentials will be, by the principle of electrical image again,

$$V = 2q \log \frac{4h}{d} + 2q \log \frac{\sqrt{D^2+4h^2}}{D} + 2q' \log \frac{\sqrt{\dfrac{D^2}{4}+(2h+E)^2}}{\sqrt{\dfrac{D^2}{4}+E^2}},$$

and that of the ground wire,

$$V' = 2q' \log \frac{4(h+E)}{d'} + 4q \log \frac{\sqrt{\dfrac{D^2}{4}+(2h+E)^2}}{\sqrt{\dfrac{D^2}{4}+E^2}} = 0$$

Eliminating q' between these two equations, we have,

$$C' = \frac{2q}{V} = \frac{1}{\log \dfrac{4h\sqrt{D^2+4h^2}}{dD} - \dfrac{1}{2}\left(\log \dfrac{\dfrac{D^2}{4}+(2h+E)^2}{\dfrac{D^2}{4}+E^2}\right)^2 \dfrac{1}{\log \dfrac{4(h+E)}{d'}}}. \quad (5)$$

This is the capacity per unit length of the line (that is the two conductors taken together), as increased by the presence of the ground wire, as may be seen by comparing it with (4).

Substituting again the numerical values as adopted before, we obtain from (4) and (5),

$$C = 0.099,$$
$$C' = 0.114.$$

The ratio is,

$$\frac{0.114}{0.099} = 1.16,$$

that is, for the same amount of charge per unit length, the potential of the line conductors will be only

$$\frac{1}{1.16} = .86$$

times its value without the ground wire.

Now, as explained in the preceding chapter, the induced charge is decreased to 77 percent in virtue of the ground wire; thus, the result of these two actions, the increasing of the capacity and the screening, is that the potential of the line at the instant immediately after the discharge of the inducing cloud is only,

$$0.86 \times 0.77 = 0.66$$

times the value without the ground wire.

We wish to emphasize this fact, viz., that the part played by the increase of capacity in diminishing the height of the distribution of the potential is not much less important than that of the screening.

Chapter IV.

The Ground Wire as a Short-Circuited Secondary.

The distribution of electric potential on the transmission line as diminished in its height by these two actions of the ground wire as described above, will now propagate in both directions as

traveling waves, and then begins the damping action of the ground wire as a short-circuited secondary to the line.

To make the ground wire most effective in this action as well as for protection against direct strokes, it is essential that it be effectively grounded at as many poles as possible. If tower construction is adopted, grounding at every tower is recommended.

To investigate the effect of the presence of the short-circuited secondary upon the effective inductance and the effective resistance of the line, let us consider two circuits with the same number of turns inductively related to each other and r_1 and r_2 in resistance respectively.

If L_1 and L_2 be the total self-inductance of these two circuits, we may divide L_1 into $L_1-M=l_1$ and M, and L_2 into $L_2-M=l_2$ and M, where M is the mutual inductance, and take instead of the given circuits such an equivalent system as that shown in Fig. 12, in which the coils MM are supposed to have an equal number of turns and are upon a magnetic circuit without leakage, with the mutual inductance M between them.

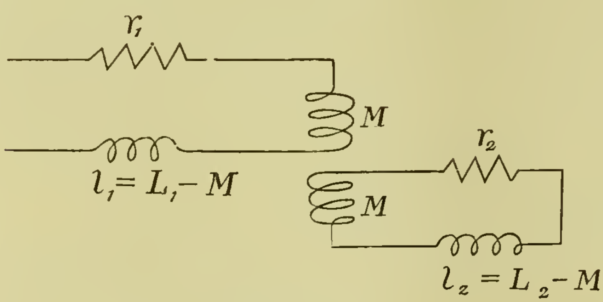


Fig. 12.

As is well known, this may be replaced with a simpler equivalent in Fig. 13.

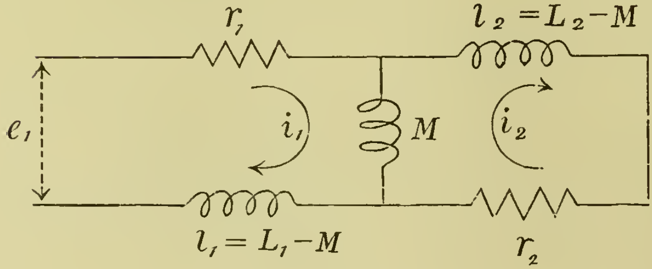


Fig. 13.

If e_1 be the impressed electromotive force at the terminals of the first circuit, we have

$$r_1 i_1 + l_1 \frac{di_1}{dt} + M \frac{d(i_1 - i_2)}{dt} = e_1 \quad \text{......................} \quad (6)$$

$$r_2 i_2 + l_2 \frac{di_2}{dt} + M \frac{d(i_2 - i_1)}{dt} = 0, \quad \text{.........................} \quad (7)$$

and from this,

$$[M(l_1+l_2)+l_1 l_2] \frac{d^2 i_1}{dt^2} + [M(r_1+r_2)+r_1 l_2 + r_2 l_1] \frac{di_1}{dt} + r_1 r_2 i_1 = (M+l_2) \frac{de_1}{dt} + r_2 e_1.$$

Looking at this equation, it is quite evident that such a system can not be represented by an equivalent consisting of an effective resistance and an effective inductance, which is correct for any kind of impressed electromotive force. But if we put $r_2 = 0$, the equation reduces to,

$$r_1 i_1 + \left(l_1 + \frac{M l_2}{M + l_2}\right) \frac{di_1}{dt} = e_1.$$

Therefore, in such a case we can replace the two given circuits with a resistance r_1 and an inductance

$$l_1 + \frac{M l_2}{M + l_2}$$

in series; that is, the effect of the presence of the second circuit can be represented by the decrease of the inductance of the first circuit by the amount

$$(M + l_1) - \left(l_1 + \frac{M l_2}{M + l_2}\right). \quad \text{......................} \quad (8)$$

And the current in the second circuit is, from (7)

$$i_2 = \frac{M}{M + l_2} i_1. \quad \text{..............................} \quad (9)$$

If we denote the inductance and the resistance per unit length of the transmission line (all wires taken together) by L_1 and r_1, and those of the ground wire by L_2 and r_2 respectively,

the case is no other than that of Fig. 12, and therefore of Fig. 13; the effect of the presence of the ground wire cannot be represented simply by the change of the effective resistance and inductance, as explained above. But if r_2 be very small compared with L_2, we may represent the effect approximately by a decrease of the effective inductance and an increase of the effective resistance as follows.

Firstly, since r_2 is small compased with l_2, we neglect it and take our case to be that of (8), that is, the inductance of the line is considered as diminished from $L_1 = l_1 + M$ to

$$L_1' = l_1 + \frac{M l_2}{M + l_2} \quad \text{.....................} \quad (10)$$

The current through the secondary circuit is taken to be approximately as shown by (9), and then the change of the effective resistance is calculated from the ohmic loss in this way;— since the loss in r_2 must be,

$$i_2^2 r_2 = \left(\frac{M}{M+l_2} i_1\right)^2 r_2 ,$$

the increase of the effective resistance to i_1 caused by this must be (see Fig. 13),

$$r_1' = \frac{i_2^2 r_2}{i_1^2} = \left(\frac{M}{M+l_2}\right)^2 r_2 \quad \text{.......................} \quad (11)$$

We consider such an increment of the effective resistance is given to r_1.

To represent the effect of the presence of the ground wire simply by the decrease of L_1 to L_1' and the increase of r_1 to $r_1 + r_1'$ is only an approximation, as was said above; but we hope it may serve to give an idea of the magnitude of the damping action.

We assume that the grounding of the ground wire is effected so frequently that the resistance and inductance of the ground

connections can be neglected in the following calculations.

According to Heaviside's formula, as is given on *p.* 101, Vol. I of the collection of his papers, the inductance per unit length of a single overhead wire with earth return is,

$$L = \frac{\mu}{2} + 2 \log \frac{4h}{d}, \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (12)$$

and the mutual inductance between two parallel conductors, with earth return each h_1 and h_2 in height above the ground and D_h in the horizontal distance, is

$$M = \log \frac{D_h^2 + (h_1 + h_2)^2}{D_h^2 + (h_1 - h_2)^2}. \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (13)$$

In a case where the current varies so rapidly that it can be taken as concentrated on the surface of the conductors, we have to change (12) into

$$L = 2 \log \frac{4h}{d}. \quad \dots\dots \quad \dots\dots\dots\dots\dots\dots \quad (14)$$

Since in our case of the traveling wave the charge equal and opposite in sign to that on the conductors progresses along the earth's surface, we are justified to adopt these formulae for the calculation of the self-and the mutual inductance. The inductance of our simple transmission line consisting of two parallel conductors d cm in diameter and D cm in axial distance and at the height of h cm above the ground, is easily calculated by these formulae. It is, per unit length of the transmission line, the two conductors taken together,

$$L_1 = \frac{1}{2}\left[2 \log \frac{4h}{d} + 2 \log \frac{\sqrt{D^2+4h^2}}{D}\right] = \log \frac{4h\sqrt{D^2+4h^2}}{dD} \quad (15),$$

and the inductance per unit length of the ground wire is,

$$L_2 = 2 \log \frac{4(h+E)}{d'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The mutual inductance per unit length between the ground wire and one of the line conductors is,

$$M = \log \frac{\left(\frac{D}{2}\right)^2 + (2h+E)^2}{\left(\frac{D}{2}\right)^2 + E^2}. \qquad (17)$$

Since both the line conductors are similarly situated with respect to the ground and the ground wire, (17) may be taken as the mutual inductance between the ground wire and the two conductors taken together.

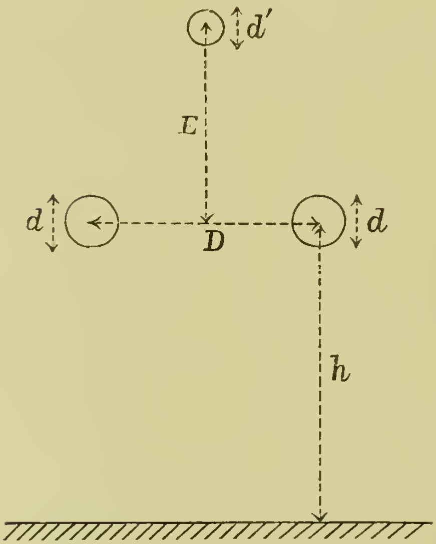


Fig. 14.

Taking up again the numerical example as stated above, we get the following as the result of calculation by the above formulae.

$$L_1 = 10.17,$$
$$L_2 = 16.88,$$
$$M = 4.76,$$

consequently

$$l_1 = L_1 - M = 5.41,$$
$$l_2 = L_2 - M = 12.12,$$

both in electromagnetic unit.

Thus (10) becomes

$$L_1' = l_1 + \frac{Ml_2}{M+l_2} = 8.83,$$

that is, the inductance is decreased to

$$\frac{8.83}{10.17} = 0.87$$

times its former value.

Next, we have to compute the resistance; to do this, let us suppose that the traveling wave of electric potential is sinusoidal and 2 kilometres in extent, as in Fig. 15. Of we had a succession of such half waves, since the velocity of propagation is approximately that of light, the frequency would be,

$$f = \frac{3 \times 10^{10}}{2 \times 2 \times 10^{5}} = 75000 \text{ per sec.}$$

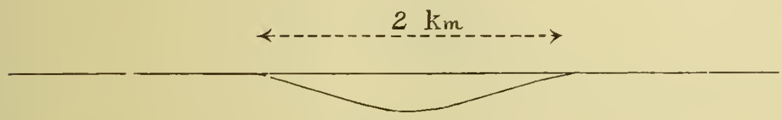


Fig. 15.

Let us assume that we may use without much errorthe value of the resistance calculated for an alternating current of this frequency, in the present case of the traveling wave. Calculated as a solid conductor, the penetration of current of our line conductor, which is 1.5 cm in diameter, is 0.23 cm, corresponding to the above frequency (see Steinmetz's Transient Electric Phenomena, p 377); the resistance of the two conductors taken together is therefore,

$$r_1 = 0.0000075 \text{ ohms.} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

The ground wire is supposed to be a double galvanized stranded irox wire, but let us for the sake of simplicity regard it as a solid cylinder in calculating the resistance, and let us take the thickness of the zinc layer to be about .005 cm. As the penetration of current of a zinc cylinder corresponding to the above frequency is 0.081 cm, we take the whole thickness of the zinc as effective for carrying current, and neglect the iron part entirely, since the high permeability of the latter makes the penetration extremely

small. In this way we get the approximate value,

$$r_2 = 0.0003 \text{ ohms.} \qquad \text{(19)}$$

Hence (11) becomes

$$r_1' = \left(\frac{M}{M+L_2}\right)^2 r_2 = 0.0000238 \text{ ohms.}$$

That is the effective resistance of the transmission line is increased from r_1 to

$$r_1 + r_1' = 0.0000075 + 0.0000238 = 0.0000313,$$

which is

$$\frac{0.0000313}{0.0000075} = 4.17$$

times the former value of r_1.

The traveling waves, which start in both directions after the discharge of the thunder cloud, are attenuated gradually in their height by the loss of energy, the value of which after traveling through the distance of x cm being $\varepsilon^{-\alpha x}$ times its original value; α is the so-called attenuation constant, and is

$$\alpha = \frac{1}{2}\left(r\sqrt{\frac{C}{L}} + g\sqrt{\frac{L}{C}}\right),$$

where r C L g are the resistance, capacity, inductance, and leakage conductance per unit length of the line respectively (see Steinmetz's Transient Elec. Phenomena p. 462).

Since g is very small, let us simplify this to

$$\alpha = \frac{1}{2} r\sqrt{\frac{C}{L}} \qquad \text{(20)}$$

Now for our transmission line, the line constants, when there is no ground wire, are as already given,

$$r_1 = 0.0000075 \text{ ohms,}$$

$$C = 0.099 \times \frac{1}{9 \times 10^{20}} \times 10^9 \text{ farads,}$$

$$L_1 = 10.17 \times 10^{-9} \text{ henrys.}$$

Substituting these in (20), we have

$$\alpha = 1.23 \times 10^{-8}. \qquad (21)$$

Of we take 100 kilometres as the total length of our transmission line, and assume that the lightning discharge takes place at about the middle of the line, the traveling wave has to pass through 50 km before it arrives at either the receiving or the sending end. Thus, taking q as 50×10^5 cm, the height of the wave at its first arrival at the end must be,

$$\varepsilon^{-\alpha x} = \varepsilon^{-1.23 \times 10^{-8} \times 5 \times 10^{6}} = \varepsilon^{-6.15 \times 10^{-2}} = .94 \qquad (22)$$

times its original value, that is, it is reduced by 6 per cent.

Now let us suppose that the ground wire is added, and let us regard the influence of its presence as a closed secondary circuit approximately represented by the increase of the effective resistance and the decrease of the effective inductance, as explained above. We have already shown that the effective inductance and resistance become 0.87 and 4.17 times their former values without the ground wire, while, as explained in Chapter III, the capacity is multiplied 1.16 times. The consequence is, the attenuation constant is multiplied

$$4.17\sqrt{\frac{1.16}{0.87}} = 4.83 \qquad (23)$$

times, that is

$$\varepsilon^{-\alpha' x} = \varepsilon^{-6.15 \times 4.83 \times 10^{-2}} = 0.74. \qquad (24)$$

Comparing this with (22), the ratio is

$$\frac{\varepsilon^{-\alpha' x}}{\varepsilon^{-\alpha x}} = \frac{0.74}{0.94} = 0.79. \qquad (25)$$

This means that the height of the wave after it has traveled through 50 km is less than that without the ground wire, by as much as 21 per cent.

It is needless to say, that the increase of the attenuation constant is effective also for the traveling wave caused by the direct strope of lightning.

Chapter V.

Summary,

Summing up all the above, we may enumerate the effect of the presence of the ground, wire, as follows;—

(a) By the screening action (Chapter II) the induced charge becomes only 77 per cent of its value without the ground wire.

(b) By the increase of capacity (Chapter III) the potential after a lightning discharge ought to be decreased to 86 per cent for the same amount of induced charge.

(c) If a wave traveled through 50 km, the height would be decreased to 79 per cent of its value without the ground wire, for the same original value.

Since (a) and (b) occur at the same time, the potential immediately after the lightning discharge is diminished to

$$0.77 \times 0.86 = 0.66$$

times the value without the ground wire.

And if we consider the height of the traveling wave after it has traveled through 50 km, the decrease is to

$$0.77 \times 0.86 \times 0.79 = 0.52 \quad \text{......................} \quad (26)$$

times the value when no ground wire is used that is, we obtain the remarkable decrese of 48 per cent.

This numerical example shows clearly the effectiveness of the ground wire; above all, we wish to point out that the effects of the increases of capacity and attenuation constant are by no means inconsiderable.

Chapter VI.

Insulated Transmission Line.

In Chapters II and V we have considered a transmission line with grounded neutral; we now proceed to investigate one without grounding.

In treating the inducing action of thunder clouds on an insulated line, even some of the most eminent engineers consider the leakage over the insulators as a necessary condition; but the following will clear up such an erroneous conception and explain that such leakage is by no means necessary for the appearance of the induced charge.

We take up again our simple transmission line with a ground wire as in Fig. 16, and suppose the line is nowhere grounded; then the potential of the line and the ground wire may be expressed as,

$$V = kh + 2q\log\frac{4h}{d} + 2q\log\frac{\sqrt{D^2+4h^2}}{D} + 2q'\log\frac{\sqrt{\frac{D^2}{4}+(2h+E)^2}}{\sqrt{\frac{D^2}{4}+E^2}}, \quad (27)$$

$$V' = k(h+E) + 2q'\log\frac{4(h+E)}{d'} + 4q\log\frac{\sqrt{\frac{D^2}{4}+(2h+E)^2}}{\sqrt{\frac{D^2}{4}+E^2}} = 0, \quad (28)$$

where q and q' are the charges per unit length of the line conductors and the ground wire respectively.

Eliminating q' between these,

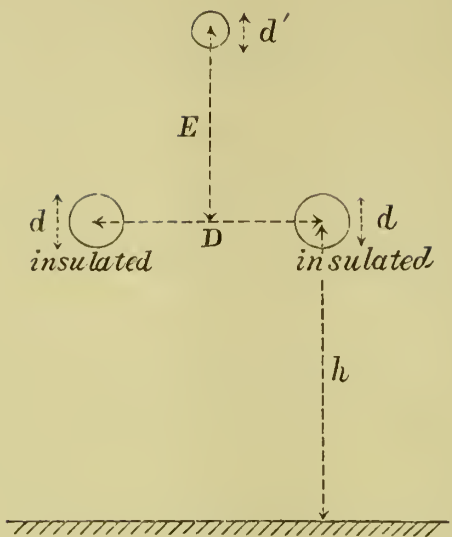


Fig. 16.

$$2q = \frac{V-k\left[h-(h+E)\log\dfrac{\dfrac{D^2}{4}+(2h+E)^2}{\dfrac{D^2}{4}+E^2}\,\dfrac{1}{2\log\dfrac{4(h+E)}{d'}}\right]}{\log\dfrac{4h\sqrt{D^2+4h^2}}{dD}-\left(\log\dfrac{\dfrac{D^2}{4}+(2h+E)^2}{\dfrac{D^2}{4}+E^2}\right)^2\dfrac{1}{2\log\dfrac{4(h+E)}{d'}}} \,. \qquad (29)$$

Let us, for brevity, write this as,

$$2q = \frac{V-kA}{B}\,, \qquad (29')$$

where $2\,q$ is the charge per unit length of the transmission line (two conductors taken together).

Suppose, for the sake of simplicity, the distribution of k produced by the thunder cloud is a sinusoidal half wave $2Z$ cm in extent as indicated in Fig. 17, and let its maximum value at the middle be K, then

$$k = K\cos\omega z, \qquad (30)$$

where z is the distance from the middle point, and $\omega=\frac{\pi}{2Z}$. Substituting this in (29) and integrating, we get,

$$Q=2\int_0^{\frac{\pi}{2\omega}}2q\,dz=2\int_0^{\frac{\pi}{2\omega}}\frac{1}{B}(V-AK\cos\omega z)dz=\frac{2}{B}\left(VZ-\frac{AK}{\omega}\right),\ldots(31)$$

which is the total electric charge induced in the whole region $2Z$.

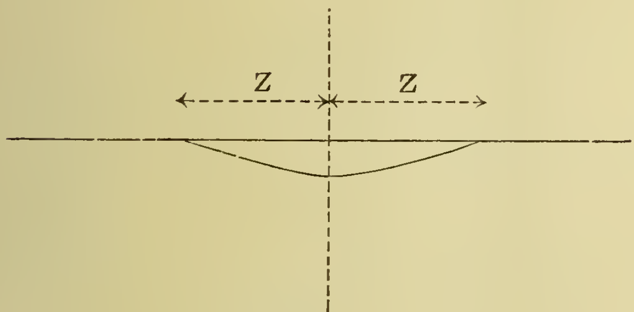


Fig 17.

If we consider the remaining part of the line outside of $2Z$ as wholly out of the influence of the cloud, the induced charge per unit length of that part must be the product of this potential V, which is, of course, canstant everywhere on the line, and the capacity given by the equation (5).

Therefore the total charge outside of $2Z$ must be,

$$Q'=V\frac{1}{\log\frac{4h\sqrt{D^2+4h^2}}{dD}-\frac{1}{2}\left(\log\frac{\frac{D^2}{4}+(2h+E)^2}{\frac{D^2}{4}+E^2}\right)^2\frac{1}{\log\frac{4(h+E)}{d'}}}$$

$$\times(S-2Z)=\frac{V(S-2Z)}{B}, \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (32)$$

and since the line is insulated from the ground, we must have,

$$Q+Q'=0.$$

Thus, from (30) and (32),

$$\frac{2}{B}\left(VZ-\frac{AK}{\omega}\right)+\frac{V}{B}(S-2Z)=0,$$

that is,

$$V=\frac{2}{\pi}AK\frac{2Z}{S}\text{ .... } (33)$$

By putting $k=K$, and using V of (33), in the equation (29), we obtain the charge at the middle of $2Z$, where the charge and therefore the potential after the discharge of the cloud are maxima; it is

$$2q_{max}=-\frac{KA}{B}\left(1-\frac{2}{\pi}\frac{2Z}{S}\right)$$

$$=-K\frac{h-(h+E)\log\dfrac{\frac{D^2}{4}+(2h+E)^2}{\frac{D^2}{4}+E^2}\,\dfrac{1}{2\log\dfrac{4(h+E)}{d'}}}{\log\dfrac{4h\sqrt{D^2+4h^2}}{dD}-\left(\log\dfrac{\frac{D^2}{4}+(2h+E)^2}{\frac{D^2}{4}+E^2}\right)^2\dfrac{1}{2\log\dfrac{4(h+E)}{d'}}}\times\left(1-\frac{2}{\pi}\frac{2Z}{S}\right)$$

$$=-K\frac{2h\log\dfrac{4(h+E)}{d'}-(h+E)\log\dfrac{\frac{D^2}{4}+(2h+E)^2}{\frac{D^2}{4}+E^2}}{2\log\dfrac{4(h+E)}{d'}\log\dfrac{4h\sqrt{D^2+4h^2}}{dD}-\left(\log\dfrac{\frac{D^2}{4}+(2h+E)^2}{\frac{D^2}{4}+E^2}\right)^2}\times\left(1-\frac{2}{\pi}\frac{2Z}{S}\right).$$

Comparing this with (3) of Chap. II, we see that it is

$$\left(1-\frac{2}{\pi}\frac{2Z}{S}\right)$$

times that for the case of a grounded transmission line.

Next, let there be no ground wire. Starting with the two equations

$$V=kh+2q\log\frac{4h}{d}+2q\log\frac{\sqrt{D^2+4h^2}}{D},\text{ .... } (35)$$

$$V' = k(h+E)+4q\log\frac{\sqrt{\frac{D^2}{4}+(2h+E)^2}}{\sqrt{\frac{D^2}{4}+E^2}} = 0, \qquad (36)$$

and proceeding as we have done above, using the capacity given by (4), we get the following result,

$$2q_{max} = -K\frac{h}{\log\frac{4h\sqrt{D^2+4h^2}}{dD}}\left(1-\frac{2}{\pi}\frac{2Z}{S}\right). \qquad (37)$$

Comparing this with (3) of Chapter II, we see that it is

$$\left(1-\frac{2}{\pi}\frac{2Z}{S}\right)$$

times that of the case of the grounded line.

These equations (34) and (37) inform us that, with respect to the ratio showing the screening action of the ground wire against the induction by the thunder cloud, there is no difference between the grounded and the insulated transmission line. As to the ratio which shows the decrease of potential due to the increase of the capacity by the presence of the ground wire, and the ratio which shows the decrease of the wave height on its arrival at the end due to the increased attenuation constant, they are evidently the same for both cases.

Thus, the whole of Chapter V holds good for both the grounded and the insulated line.

The difference lies in this, that the induced charge, and therefore the potential after the lightning discharge, for the insulated line, is

$$\left(1-\frac{2}{\pi}\frac{2Z}{S}\right)$$

times that of the grounded.

But a great number of experiences shows us that $2Z$, the extent of the distribution of the induced charge, is remarkably limited, being 1000 to 5000 ft generally; if we take $2Z = 2$ km and S, the total distance of transmission, as 100 km,

$$1-\frac{2}{\pi}\frac{2Z}{S} = 0.987,$$

or very nearly unity. Besides, if we take account of the capacities of the transformers, etc. at the ends of the line, which we have neglected in the above, this $\frac{2Z}{S}$ has to be considerably diminished from the actual ratio of the lengths, and consequently the above number ought to be much nearer to unity.

Hence we may conclude as follows; —Comparing the grounded and the insulated transmission line, the protection which is afforded by the presence of a ground wire is nearly the same for both cases.

Oct. 1910.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
VOL. V., No. 4.

Published September, 1912.

The Preparation of "Lipase Powder" acting in Neutral Medium and its Technical Application.

By

Yoshio Tanaka, *Kōgakuhakushi.*

The ferment process of hydrolysing fats on an industrial scale was suggested by Connstein, Hoyer and Wartenburg[1], whose method consisted in triturating a quantity of the crushed castor seed with the oil or fat, together with acidified water. This method however did not meet with extended application, on account of the inferior quality of the product and also of the loss of considerable quantities of fatty matter by the formation of a mixed layer consisting of castor seed, fatty matter and glycerine. As the impurities in the products obtained by this method are undoubtedly derived from the castor seed used, it becomes necessary to isolate the lipase from the castor seeds. It has however been shown by several investigators that no solution of the lipase can be obtained by extraction with solvents, the enzyme being insoluble in the latter. Practically all the attempts hitherto made to isolate a pure lipase have proved unsuccessful.

Much attention has been devoted to obtaining the active preparation in a purer and more concentrated form than the seeds themselves, and several methods have been devised for the purpose. Nicloux[2] has patented a preparation of an "extract"

---

[1] Connstein, Hoyer and Wartenburg; Ber. Deutsch. chem. Ges., 1902, 3988—4006.
[2] Nicloux; Eng. Pat. 8304, 1904.

made by triturating crushed castor seed with castor or cotton seed oil, filtering the mass through a fine silk gauze and subjecting the resulting oily extract to centrifugal force. Hoyer[1] has obtained an active product in the form of an emulsion which he terms "ferment oil," by extracting the crushed castor seed with a solvent for oils. This turbid product, consisting of a mixture of castor oil and protoplasma, is very active in the presence of dilute acid. He[2] has also prepared an active creamy substance consisting of about 38 per cent. of ricinoleic acid, 4 per cent. of proteids and 58 per cent. of water, by treating the crushed castor seeds with water, and leaving it to ferment. This lipolytic agent is termed "ferment" by him, and it is at present being manufactured in marketable quantities. The active product, however, rapidly loses its lipolytic power on keeping, and, moreover, the glycerine resulting from its use contains impurities derived from the "ferment."

The writer[3] showed decidedly in his previous communication that the acid added to the resting seed of the castor oil plant in its optimum quantity is useful for developing lipase from its zymogen, but not for acidifying the lipolytic medium, and that the actual lipase is most active in a neutral medium, and less active in the presence of free acid, especially mineral acid. The author also found that an insoluble residue obtained by treating pressed castor seed with a proper amount of acid, and then washing with water until the residue was free from soluble matter, had in high degree the power of hydrolysing fats without the use of any acid.

From these results the author undertook to prepare an active product in a convenient form from castor seeds. At first he found

---

1 Hoyer ; Seifenfabrikant, 1907, 253.

2 Hoyer ; ibid., 1907, 304.

3 Y. Tanaka ; this Jour., 1910, Vol. V., 25.

it necessary to determine the condition, under which the zymogen of lipase is most favourably activated.

Experimental.

Conditions most favourable to the Conversion of Zymogen into Active Lipase.

To determine what conditions were most favourable to the conversion of zymogen into active lipase, the author undertook to investigate the influence of the quantity of acid, necessary to digest the pressed castor seed and the influence of the time and temperature of the digestion, on the activity of the resulting lipase.

(a) Influence of the Quantity of the Digesting Acid.

To determine the optimum amount of acid, by which the zymogen of lipase may be most favourably developed, the author prepared the lipolytic substance according to the following method and compared the activities.

Experiment 1.

Two grams of pressed castor seed were triturated with different amounts of sulphuric acid for twenty minutes at ordinary temperature and then filtered and thoroughly washed:

No. of lipolytic substance.	Grams of pressed castor seeds.	C.c. of N/10 sulphuric acid.	C.c. of water added.
(1)	2	6	8
(2)	2	8	6
(3)	2	10	4
(4)	2	10	10
(5)	2	12	0
(6)	2	14	0

The washed residue obtained from the above was rubbed up with thirty grams of soja bean oil, together with about 5 c.c. of water and allowed to hydrolyse at 35°C. The amounts of decomposition effected in one and three hours were determined:

No. of lipolytic substance.	Per cent. of oil decomposed in	
	one hour.	three hours.
(1)	22.0	42.5
(2)	26.7	55.7
(3)	36.6	61.5
(4)	36.4	61.9
(5)	32.5	54.8
(6)	21.5	41.3

These figures show that the activity of the liberated lipase depends upon the absolute quantity of the acid used, but not on its concentration, a fact which was observed in the hydrolysis of oil with the original pressed castor seeds. The optimum quantity of the digesting acid which activates the zymogen most favourably into lipase is 5 c.c of N/10 strength for each gram of the pressed castor seeds.

This optimum of the acid is larger than that required in the hydrolysis of oil by the original pressed castor seeds, as has been shown in the previous communication. It seems probable that as the excess of acid is however subsequently removed by washing out with water, its inhibitory action does not show itself. Similar experiments with acetic acid yielded analogous results. The optimum quantity of acetic acid most favourable for developing lipase from its zymogen was found to be 6—7 c.c. of N/10 acid for each gram of the pressed castor seed which was used in the present investigation.

(b) Influence of Time of Digestion.

This experiment is to ascertain the influence of the time of digestion on the activity of the resulting lipolytic substance.

Experiment 2.

Two grams of pressed castor seed were digested with 10 c.c. of N/10 sulphuric acid for different lengths of time at 15°C., after which the mixture was completely washed out with water and a pasty mass obtained. Each of these preparations was triturated with 30 grams of soja bean oil, together with 5 c.c. of water, and allowed to hydrolyse for one hour at 38°C. The results were as follows:

Minutes of digestion in the preparation of lipolytic substance	Per cent. of oil decomposed in one hour.
5	33.5
20	35.0
30	35.5
60	34.9
120	33.0

The fact that the time varied from 5 to 120 minutes does not appear to have had an appreciable influence on the result.

(c) Influence of Temperature of Digestion.

Experiment 3.

Two grams of pressed castor seed were digested in 10 c.c. of N/10 sulphuric acid at different temperatures and the mixtures were then washed free from soluble matter. The hydrolysis of oil by the washed residue was carried out, the procedure being the same as above:

Temperature of digestion.	Time of digestion in minutes.	Per cent. of oil decomposed in one hour.
10—15°C.	30	29.3
10—15°C.	120	28.8
20—25°C.	30	36.6
30—35°C.	30	39.5
30—35°C.	60	39.8
35—40°C.	30	34.2

From these results, it was found that the optimum temperature of digestion, whereby the most active preparation was obtained, was 30—35°C.

Preparation of "Lipase Powder."

Based upon the results of the above experiments, the author has prepared a pasty lipolytic substance by treating pressed castor seed with the optimum amount of acid under the most suitable conditions and washing thoroughly. It has however two disadvantages; it can not be kept long, and the formation of emulsion with oil is not so satisfactory as with pressed castor seed.

The author has found that a lipolytic powder which is active in the absence of any acid and can be kept for a long time without impairing its activity, can be obtained by drying the active pasty substance at low temperatures. This lipolytic product is termed "Lipase powder" by the author.

The preparation of the "lipase powder" is very simple and is as follows:

100 grams of pressed or extracted castor seed are triturated with 600—700 c.c. of N/10 acetic acid (or about 500 c.c. of N/10 sulphuric acid) for thirty minutes at 30—35°C. The milky mixture thus obtained is filtered, and the residue washed thoroughly with water and dried at temperatures not exceeding 40°C.

The "lipase powder" thus obtained is an odourless, tasteless, white powder and contains no soluble matter. Its chemical composition varies with the pressed castor seed used. One example had the following composition:

Water	5.3%
Crude fatty matter	37.2 ,,
Nitrogenous matter	46.3 ,,
Mineral matter	1.2 ,,
Non-nitrogenous organic matter	10.0 ,,

Hydrolysis of fats and fatty oils can be rapidly brought about by a small quantity of the "lipase powder" in the presence of water alone, as may be seen from the following experiments.

Experiment 4.

In typical experiments in which 25 grams of fats or fatty oils were mixed with one grams of the "lipase powder" and 8 c.c. of water and allowed to stand at 38°C., the following percentages of oils and fats were hydrolysed in one and in six hours:

Oil or fat.	Per cent. of oil or fat decomposed in	
	One hour.	Six hours.
Soja bean oil	44.4	81.7
Groundnut oil	49.8	84.7
Cocoanut oil	40.4	70.8
Lard	51.7	85.9
Tallow	50.6	84.4

These results show that the "lipase powder" has a stronger

power of decomposing oils and fats than the original pressed castor seed. It is somewhat remarkable that tallow can be rapidly decomposed by the "lipase powder" in a pasty condition which can be brought about at 38°—40°C.

From the possibility of the decomposition of fats and oils with good results by the "lipase powder," the author continued to make a number of further experiments in the hope that this process might be worked out technically.

Technical Application of "Lipase Powder."

The manner of applying "lipase powder" in practice is very simple. The "lipase powder" is mixed with oil and then a proper quantity of water is added to the mixture, which is allowed to hydrolyse at a temperature not exceeding 40°C. Three or four per cent. of the powder is enough to hydrolyse about 90 per cent. of oil in 7—10 hours. The amount of water depends upon the amount of lipolytic powder; a large excess of water affects the production of a satisfactory emulsion disadvantageously, whereas too little water causes a rapid accumulation of glycerine which has an inhibitory effect[1] on the action of the lipase, causing a retardation of the decomposition velocity of the oil. It was found that the proper amount of water was 6—10 times the weight of "lipase powder."

When tallow is to be hydrolysed it must be previously melted. Then the "lipase powder" and a proper quantity of warm water are added when the fat has cooled to 40°C. The mixture is stirred continuously, the working temperature being preferably 38—40°C.

---

[1] See next communication.

At the expiration of the working period the mixture is heated to a temperature which keeps it fluid, and allowed to settle. Two layers are formed; clear fatty acids separate forming an upper layer, while the lower one is an emulsion containing ferment, fatty matters and glycerine water. Heating the mixture to higher temperature or blowing steam through it causes the swelling and gelatinisation of the "lipase powder" and makes the separation of it difficult. The fatty acid can be easily separated. The glycerine may be extracted with water from the lower layer.

The practical yields of the fatty acids and glycerine to be obtained by the present method depend upon the kind of fats or fatty oils, the amount of the "lipase powder" and other conditions. The following examples show the yields of the fatty acids and glycerine on the experimental scale.

Experiment 5.

Forty grams of the "lipase powder" were added to 1000 grams of tallow, together with 360 c.c. of water and allowed to act for 7 hours at 38°C. At the expiration of that time, the mixture was warmed to 60°C. Whereupon the fatty acids were separated, forming an upper layer. To the mixture which remained in the lower layer and consisted of free fatty acid, ferment, and glycerine, 400 c.c. of water were added and the whole pressed. The first glycerine liquor was thus obtained. The pressed residue was once more treated with 400 c.c. of water and again pressed, and the second glycerine liquor was obtained. These liquors were subjected to evaporation. The results may be seen in the following table:

Yield from 100 parts of tallow.

Fatty acid (containing about 13% of neutral fat).	89.0 per cent.
Crude glycerine (80% glycerol).	10.2 ,,
Mixture (containing 61% fatty matter).	15.8 ,,

The yield of glycerine may be increased by allowing more time for a further decomposition of the tallow.

A similar experiment was made with soja bean oil.

Experiment 6.

Thirty grams of the "lipase powder" were allowed to act on 1000 grams of soja bean oil for 11 hours, the experimental conditions being the same as before. The approximate yield from 100 parts of the oil was as follows:

Yield from 100 parts of soja bean oil.

Fatty acid (containing about 8% of neutral oil). ...	91.6 per cent.
Crude glycerine (80% glycerol).	11.0 ,,
Mixture (containing 53% fatty matter).	10.0 ,,

The fatty acids resulting from this process are much lighter in colour than those obtained by other methods of hydrolysis and require no further purification. The great purity of the glycerine liquor makes any treatment, or expensive evaporator, or other additional equipment unnecessary. The crude glycerine yielded has a pure sweet taste, pale colour, and is free from any unpleasant smell. It contains only a small amount of organic impurity or trace of mineral matter, as is shown by the following analysis, the sample analysed having been obtained from lard:

Specific gravity	1.2130
Glycerol (by acetin method)	77.5 per cent.
Mineral matter	0.08 ,,
Organic impurities (albuminoids, etc.)	0.45 ,,

The crude glycerine gave no precipitate with silver nitrate, barium chloride or lead acetate.

The Keeping Quality of "Lipase Powder."

The keeping quality of the "lipase powder" has been tested during a period of six months. As can be seen from the data adduced, it has shown no appreciable change in its lipolytic activity for a moderately long time.

Experiment 7.

Twenty-five grams of soja bean oil were decomposed by one gram of the "lipase powder" and 8 c.c. of water at 38°C:

Time of keeping in days.	Per cent. of oil decomposed in one hour.
1	45.4
60	45.0
120	43.5
180	40.0

Summary.

1. An active lipolytic powder has been prepared by treating pressed castor seed with the optimum amount of acid under suitable conditions, completely washing out all the soluble matters with water and drying the pasty mass thus obtained. This lipolytic powder I call "Lipase powder."

2. The optimum temperature of the digestion in which the zymogen of lipase may be most favourably developed, is 30—35°C.

3. The activity of the liberated lipase does not depend upon the concentration of the acid used in its liberation from the zymogen, but upon the absolute amount of the acid.

4. The time of digesting the zymogen or castor seed with the proper amount of acid shows little effect on the activity of the liberated enzyme.

5. The "lipase powder" is an odourless, tasteless, white powder, containing no soluble matter. It hydrolyses fats and fatty oils very rapidly in the absence of any soluble acid.

6. Not only can the "lipase powder" be kept for a long time without showing an appreciable change in its activity, but the products of hydrolysis resulting from its use are exceedingly superior. It is, therefore, eminently suitable for industrial purposes.

January 15, 1912.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
VOL. V., No. 4.

Published September, 1912.

Influence of the Products of Change on the Action of Lipase.

By

Yoshio Tanaka, *Kōgakuhakushi.*

It is well known that the products of the enzymic reaction exert a retarding influence on the rate of change. However, the retarding effect seems to be practically confined to one of the products of change, the others having very little or no effect on the rate of progress. Victor Henri[1] showed that the hydrolysis by invertase is retarded by fructose but not by glucose. E. F. Armstrong[2] investigated the influence of the products upon the action of sucroclastic enzymes. He found that the hydrolysis of lactose by lactase is retarded by only one of its products of action, viz. galactose, but not by the other products, dextrose and laevulose. A. Wohl and E. Glimm[3] observed that in the hydrolysis of starch by amylase, the inhibiting effect of maltose was greater than that of dextrin. Hence, it seems probable that the retardation of hydrolysis observed in the action of lipase on oil, is also due to this cause.

The present investigation relates to the influence of products of change on the rate of hydrolysis of oil by lipase.

---

1 V. Henri; Zeit. f. physik, Chem., 1901, 194.
2 E. F. Armstrong; Jour. Chem. Soc., 1904, Abst. 957.
3 A. Wohl and E. Glimm; Biochem. Zeits., 1910, 349.

Experimental.

Influence of Fatty Acid.

In the present experiment, the influence of fatty acid, chiefly oleic acid, was investigated.

Experiment I.

Different amounts of soja bean oil were hydrolysed by the "lipase powder" in the presence of different amounts of fatty acid at 38°C. The results were as follows:

Grams of soja bean oil.	Grams of 98% fatty acid added.	Grams of "lipase powder"	C.c. of water added	Grams of oil hydrolysed in		Per cent. of decomposed oil to the original amount of neutral oil after	
				one hour.	two hours.	one hour.	two hours.
50	0	2	18	22.00	32.50	44.0	65.0
40	10	2	18	17.73	26.33	44.1	65.5
30	20	2	18	13.13	20.31	43.2	66.8
10	40	2	18	4.70	7.33	43.5	67.9

These data show that the amount of oil hydrolysed is directly proportional to the amount of oil present, or the rate of change follows the law of mass action, even in the presence of a large amount of free fatty acid. This result proves without doubt that the fatty acid has no retarding effect upon the enzyme action, because, if that were the case, it would lead to a slowing of the reaction greater than that due to the diminished concentration of the neutral oil.

Influence of Glycerine.

The author next investigated the influence of glycerine which is also a product of the hydrolysis of oil.

Experiment 2.

Equal amounts of soja bean oil were hydrolysed by the "lipase powder" in the presence of varied amounts of glycerine at 38°C. respectively, the results obtained being the following:

	Grams of soja bean oil.	Grams of "lipase powder"	C.c. of 50% glycerine water	C.c. of water added.	Per cent. of oil decomposed in one hour.
(a)	25	1	0	7	42.3
(b)	25	1	1.0	6	40.0
(c)	25	1	2.0	5	36.5
(d)	25	1	3.0	4	33.5
(e)	25	1	4.0	3	29.2

It is clear that glycerine has a inhibitory effect on lipase. In the case (e) the concentration of glycerine in aqueous medium is about 28.6 per cent. which corresponds to the concentration of glycerine liquor to be obtained when about 80 per cent. of oil was decomposed.

As stated in a previous report,[1] the amount of water in the lipolytic medium using a definite quantity of castor seed has no marked influence on the action of lipase in the initial stage, provided the absolute amount of added acid is constant. However there are later stages in which the rate of change diminishes owing to the effect of the accumulation of glycerine produced.

The synthetic power of lipase has been demonstrated by several authors and experiments upon the lipolytic synthesis of true fats have been carried out by Pottevin,[2] Welter,[3] Dunlap and

[1] Y. Tanaka; this journal, 1910, Vol. V., 29.
[2] Pottevin; Ann. de l'Inst. Pasteur, 1906, 901
[3] Welter; Zeitsch. angew. Chem., 1911, 385.

Gilbert.[1] The reversible action of lipase can be recognised as a cause of the retardation of hydrolysis. But the experiments carried out by the authors named showed that the remarkable synthetic effect was produced in very little water. Now, from experiment 2, wherein marked retardation resulted on adding glycerine to the medium (in which a sufficient amount of water was present) at the commencement of the hydrolysis, it seems probable that the retardation of lipolytic action is chiefly due to a simple inhibitory effect of glycerine on lipase, although the reverse action of the enzyme may also be responsible in a less degree.

Hence it is beneficial to reduce the concentration of glycerine by using the maximum amount of water, which however does not prevent the formation of fine emulsion as the following experiments show.

Experiment 3.

100 grams of soja bean oil were hydrolysed at 38°C. by 4 grams of the "lipase powder," together with 20 c.c. of water. After two hours it was found that 65.2 per cent. of fatty acid was produced. Then two 30 gram portions of the emulsified mixture were put into two flasks, to one of which 4 c.c. of water were added and allowed to hydrolyse further for one hour. The increase of fatty acid in both samples was as follows:

Grams of emulsified mixture.	C.c. of water added.	Per cent. of increase of fatty acid in one hour.
30.0	4	9.0
30.0	0	5.1

[1] Dunlap and Gilbert; Jour. Amer. Chem. Soc. 1911, 1787.

It is discovered also that the complete hydrolysis of oil is easily obtained by the double treatment with lipase. For example, 500 grams of soja bean oil were hydrolysed by 3 per cent. of the "lipase powder," and fatty acid containing 8.6 per cent. of neutral oil was obtained. This fatty acid, after the glycerine had been completely removed, was treated again with 4 per cent. of the "lipase powder" at 38°C. In four hours it was found that the neutral oil in the fatty acid had been completely hydrolysed.

Summary.

The activity of lipase is inhibited by glycerine, whilst fatty acid is almost without effect. Hence the retardation of the enzymic hydrolysis of oil is chiefly attributable to the inhibiting effect of glycerine produced, although the reversible action of lipase may also be regarded as a less efficient cause. These results may be of some practical importance. If the maximum hydrolysis of oil is desired, it will always be preferable to use the maximum amount of water, which does not prevent the production of a good emulsion, or to carry out the double treatment with lipase, as shown above.

January 15, 1912.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
VOL. V., No. 4.

Published September, 1912.

Influence of Some Neutral Salts, Nitrogenous Matters and Castor Seed Extract on Lipase.

By

Yoshio Tanaka, *Kōgakuhakushi.*

R. Green[1] has shown that a neutral salt such as sodium chloride impedes the action of lipase. Connstein, Hoyer, and Wartenburg[2], on the other hand, has held that ferrous sulphate, sodium chloride, sodium sulphate, magnesium sulphate and ammonium sulphate has no effect on its action. Further, K. Braun and E. C. Behrendt[3] has found that compounds of magnesium, alkali-metals and tungsten has no influence on the activity of lipase, but that even a small amounts of mercury, copper and iron salts checks the enzymic activity, while E. Hoyer[4] maintains that an addition of 0.15 to 0.2 per cent. (of the weight of the fat) of manganese sulphate helps to increase the activity of the enzyme.

These contradictory results appeared to the present author to be due to the disturbing influence of the salts and protein substances naturally present in the castor seed or castor-seed cake, and he has, therefore, undertaken the study of the influence of some neutral salts, nitrogenous matters and castor seed extract on "lipase powder" which contains no soluble matter and is active in the presence of water alone.

---

1 Green's 'The Soluble Ferments and Fermentation,' 1901, 244.

2 Connstein, etc.; Ber. Deutsch. chem. Ges., 1902, 4004.

3 Braun and Behrendt; Ber. Deutsch. chem. Ges., 1903, 1905.

4 Hoyer; Chem. Centr., 1905, 2, 582.

Experimental.

Influence of Some Neutral Salts.

At first the influence of alkali-metal salts on the lipolysis of soja bean oil was investigated.

Experiment 1.

Twenty-five grams of soja bean oil were rubbed up with one gram of the "lipase powder," together with 6 c.c. of water and, after adding varying amounts of alkali-metal salts allowed to stand for 90 minutes at 38°C. The following tables show the percentage of the oil decomposed:

(i) Sodium chloride.

Grams of sodium chloride.	Per cent. of sodium chloride in aqueous medium.	Per cent. of oil decomposed in 90 minutes.
0	0	59.8
0.2	3.3	65.3
0.3	5.0	65.9
0.4	6.7	64.3
0.5	8.3	62.7
0.6	10.0	60.9

(ii) Potassium chloride.

Grams of potassium chloride.	Per cent. of potassium chloride in aqueous medium.	Per cent. of oil decomposed in 90 minutes.
0	0	58.6
0.1	1.7	60.9
0.2	3.3	63.3
0.3	5.0	64.2
0.6	10.0	64.0
1.2	20.0	56.2

(iii) Potassium sulphate.

Grams of potassium sulphate.	Per cent. of potassium sulphate in aqueous medium.	Per cent. of oil decomposed in 90 minutes.
0	0	58.6
0.1	1.7	62.7
0.2	3.3	64.5
0.3	5.0	66.8
0.6	10.0	62.7

(iv) Lithium chloride.

Grams of lithium chloride.	Per cent. of lithium chloride in aqueous medium.	Per cent. of oil decomposed in 90 minutes.
0	0	59.2
0.2	3.3	65.1
0.3	5.0	65.9
0.5	8.3	59.2

Thus it is evident that the lipase greatly increases its activity by the addition of neutral salts of alkali-metals and, even with so high a concentration as 10 per cent. of the salts in the watery medium their influence is not harmful.

Experiments with ammonium chloride and ammonium sulphate gave similar results.

Experiment 2.

The influence of some neutral salts of other metals on the activity of lipase was also investigated, the procedure in the experiments being analogous to that given above. The results are shown in the following tables:

(i) Magnesium acetate.

Grams of magnesium acetate.	Per cent. of magnesium acetate in aqueous medium.	Per cent. of oil decomposed in 90 minutes
0	0	59.8
0.01	0.16	57.1
0.02	0.33	55.6
0.06	1.00	51.6
0.12	2.00	45.4

(ii) Calcium chloride.

Grams of calcium chloride.	Per cent. of calcium chloride in aqueous medium	Per cent. of oil decomposed in 90 minutes.
0	0	59.8
0.02	0.33	56.5
0.04	0.66	53.2
0.08	1.33	48.8

(iii) Manganese sulphate.

Grams of manganese sulphate.	Per cent. of manganese sulphate in aqueous medium.	Per cent. of oil decomposed in 90 minutes.
0	0	61.5
0.025	0.41	64.2
0.05	0.83	65.1
0.10	1.66	63.9
0.25	4.16	62.7
0.30	5.00	60.1

(iv) Copper sulphate.

Grams of copper sulphate.	Per cent. of copper sulphate in aqueous medium.	Per cent. of oil decomposed in 90 minutes.
0	0	59.8
0.003	0.05	11.7
0.02	0.33	8.0

As can be seen from the table, the salts of magnesium, calcium and copper have an inhibitory influence on the activity of lipase, the retarding effect of copper sulphate being especially remarkable. A similar result was obtained with the salts of strontium, barium, iron and other heavy metals. That manganese sulphate produces a favourable action, as was found earlier by Hoyer, is worthy of remark, while a concentration of five per cent. in aqueous medium causes a slackening.

These influences of the several electrolytes mentioned do not stand in relationship to their absolute quantities nor to the amount of oil, but to the degree of their concentration in aqueous medium, as is shown by the following experiment.

Experiment 3.

In this experiment the influence of concentration of potassium chloride and magnesium acetate was investigated, the procedure being as before. The results were as follows:

(i) Influence of concentration of potassium chloride.

Grams of soja bean oil.	Grams of "lipase powder."	C.c. of water added.	Grams of potassium chloride added.	Concentration of pot. chloride in aqueous medium.	Grams of fatty acid produced in 90 minutes.
25	1	6.0	1.2	20.0	13.9
25	1	10.0	1.2	12.0	16.0
30	1	10.0	1.2	12.0	15.8

(ii) Influence of concentration of magnesium acetate.

Grams of soja bean oil.	Grams of "lipase powder."	C.c. of water added.	Grams of magnesium acetate added.	Concentration of mag. actate in aqueous medium.	Grams of fatty acid produced in 90 minutes.
25	1	6.0	0.12	2.0	11.3
25	1	10.0	0.12	1.2	12.6
30	1	10.0	0.12	1.2	12.5

The author also made the following experiment to determine whether or not the presence of accelerating salts in the lipolytic medium would show favourable influence when sufficient time for hydrolysis was allowed.

Experiment 4.

Twenty five grams of soja bean oil were hydrolysed with one gram of "lipase powder" and 6 c.c. of water with or without sodium chloride and manganese sulphate for six hours at 38°C., the comparative results obtained being the following:

Grams of salt added.	Per cent. of oil decomposed in six hours.
No salt added	79.5
Sodium chloride 0.3	80.4
Manganese sulphate 0.05	80.7

This table shows that there is practically no difference in the results. Hence the favourable influence exerted by the neutral salts of alkali-metals or manganese salts is noticeable only in the earlier stages of hydrolysis; as hydrolysis advances, the influence of the accelerating substances ceases to show itself. Consequently the utility of manganese salts in accelerating the hydrolysis of oil does not appear to have such a practical importance as at first seemed probable.

Influence of Castor Seed Extract and Some Nitrogenous Matters on Lipase.

That castor seed extract has also a favourable effect on the activity of lipase may be seen from the following experiment.

Experiment 5.

Two samples of 25 grams each of soja bean oil were hydrolysed for one hour at 38°C. with one gram of the "lipase powder," and adding 7 c.c. of water to one of them and 7 c.c. of a castor seed extract to the other. The extract was made by triturating 100 grams of the pressed castor seeds with 700 c.c. of water for one hour. The results were as follows:

	Per cent. of oil decomposed in one hour.
Without extract	40.5
With extract	46.0

In another experiment, it was shown that the castor seed extract itself had no lipolytic action.

This beneficial effect of the castor seed extract seems to be due to the simultaneous effect of its mineral and protein matters.

The author investigated the protein of castor seed extract and isolated a crystallisable globulin, a small proportion of coagulable albumin, and proteoses. The influence of the mineral constituents and protein matters isolated from castor seed extract has next been separately studied.

Experiment 6.

Two 25 gram samples of soja bean oil were hydrolysed for one hour at 38°C. by one gram of the "lipase powder," adding to one of them 6 c.c. of a two per cent. solution of the globulin dissolved

in a 3 per cent. sodium chloride solution, and to the other 6 c.c of the sodium chloride solution alone. The results were as follows:

	Per cent. of oil decomposed in one hour.
Without globulin	45.5
With globulin	45.8

As can be seen from these results, the globulin dissolved in dilute saline salution has little or no influence on the activity of the lipase.

In the following experiment the influence of coagulable albumin, proteose and the evaporated dialysate which contains chiefly phosphates, chlorides and sulphates of alkali-metals, was investigated.

Experiment 7.

Twenty-five grams of soja bean oil were hydrolysed by one gram of the "lipase powder" with the addition of dialysed extract[1] freed from globulin, and evaporated dialysate[2]:

	Per cent. of oil decomposed in one hour.
With 7 c.c. of water alone	40.5
With 7 c.c. of dialysed, filtered extracts (albumin & proteose solution)	43.9
With 7 c.c. of dialysed, filtered & boiled extract (proteose solution)	43.7
With 7 c.c. of evaporated dialysate...	42.7

[1] 100 grams of the pressed castor seed were triturated with 700 c.c. of water and filtered through paper. 300 c.c. of the filtrate were dialysed in running water for 72 hours and filtered.

[2] Another 300 c.c. of the filtrate obtained in the above were dialysed in 300 c.c. of water for 48 hours. The dialysate was evaporated up to 100 c.c. *in vacuo* and used in the experiment.

It may be seen from the figures that coagulable albumin exerts no influence on lipase, while proteose and dialysate have a beneficial action.

Thus, the favourable action of castor seed extract is due to the presence of proteose and neutral salts of alkali-metals and not to globulin or other coagulable albumins.

An analogous fact was observed in the influence of some protein substances.

Experiment 8.

In the following series of experiments fifty grams of soja bean oil were mixed with two grams of the "lipase powder" with the addition of some nitrogenous matters named, dissolved in 12 c.c. of water:

Grams of substance added.	Per cent. of oil decomposed in one hour.
0	41.5
0.12 gram of leucine	44.2
0.12 gram of asparagine	45.4
0.06 gram of gelatin...	41.2

From these results, it may be seen that the decomposition products of protein, such as lencine and asparagine, have also an accelerating effect on lipase, while gelatin does not appear to promote the lipase action.

Neutralised egg albumin behaves like gelatin in this respect.

Summary.

The results of the present investigation with regard to the influence of neutral salts, some nitrogenous matters and castor

seed extract on lipolysis, using the new "lipase powder" which contains no soluble matter and acts in the presence of water alone, may be summarised as follows:

1. The activity of lipase is shown to be greatly increased by an appreciable addition of the neutral salts of alkali-metals, and it is not retarded even in a very concentrated salt solution, as for example, 10 per cent. strength in the aqueous medium.
2. The salts of magnesium, calcium, and, especially, copper exert a retarding influence on lipase even when they are added in small quantities. That manganese salts should greatly accelerate the activity of the enzyme, a fact already observed by Hoyer, and confirmed by the present author, is clearly exceptional.
3. The favouring action of neutral salts of alkali-metals and manganese is however manifested only in the first phase of hydrolysis; when the hydrolysis has progressed it acts no longer.
4. The influence of the added salts on lipolysis has no relationship to their absolute amount nor to the amount of the oil, but rather to their concentration in the aqueous medium.
5. The hydrolysing power of lipase is also shown to be remarkably increased by the addition of castor seed extract. This behavior is ascribed to the accelerating influence of the mineral salts of alkali-metals and proteose contained in the extract; globulin and other coagulable albumins contained also in the castor seed extract, do not appear to exert any influence.
6. The decomposition products of protein, such as leucine and asparagine, are also found to exert a pronounced stimulative effect on lipase action.

January 15, 1912.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
VOL. V., No. 4.

Published September, 1912.

The Action of Lipase on Oxidised and Polymerised Oils.

By

Yoshio Tanaka, *Kōgakuhakushi.*

The author has discovered that lipase hydrolyses oxidised and polymerised oils more slowly than the raw oil.

The Action of Lipase on Oxidised Oil.

The resistance of insolated oil to the action of lipase may be seen in the following experiment.

Experiment 1.

Twenty-five gram portions of several oils which had been exposed to direct sunlight for 25 days and become somewhat rancid, were severally triturated with one gram of pressed castor seed together with 3.0 c.c. of N/10 sulphuric acid and 4.0 c.c. of water, and allowed to hydrolyse at 38°C. Each test was carried out side by side with the corresponding raw oil and the amounts of decomposition effected in 2 and 17 hours were determined, with the following results.[1] The author determined also the iodine values of the raw and the insolated oils, which are also appended:

1 The amount of oil decomposed was approximately calculated from the acid value and the mean molecular weight of the oil in each case.

Oil.	Per cent. of oil decomposed in		Iodine value.
	2 hours.	17 hours.	
Tsubaki oil	58.7	98.5	81.5
Tsubaki oil, exposed ...	49.0	78.8	72.6
Soja bean oil...	50.0	96.0	125.0
Soja bean oil, exposed...	34.6	41.2	98.0
Linseed oil	45.6	—	176.0
Linseed oil, exposed ...	12.8	—	145.3

From these results, it will be seen that fatty oils exposed to the

With the Compliments of the Director of the Engineering College.

It has been observed by Marx, Schmid, Mayrhofer, Nagel and Scala that rancid oil contains aldehydes and similar substances. Hence, the retardation of the lipolytic hydrolysis of the insolated oils used which were somewhat rancid, may have been due to the presence of aldehydic substances which were shown by Connstein to have a deleterious influence on lipase. The following experiment will show this.

Experiment 2.

Soja bean oil exposed to direct sunlight for 25 days was purified by distilling with steam and then repeatedly washing with water. 25 grams of the oil thus treated were decomposed by one

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
VOL. V., No. 4.

Published September, 1912.

The Action of Lipase on Oxidised and Polymerised Oils.

By

Yoshio Tanaka, *Kōgakuhakushi.*

The author has discovered that lipase hydrolyses oxidised and polymerised oils more slowly than the raw oil.

The Action of Lipase on Oxidised Oil.

The resistance of insolated oil to the action of lipase may be seen in the following experiment.

of decomposition effected in 2 and 17 hours were determined, with the following results.[1] The author determined also the iodine values of the raw and the insolated oils, which are also appended:

1 The amount of oil decomposed was approximately calculated from the acid value and the mean molecular weight of the oil in each case.

Oil.	Per cent. of oil decomposed in		Iodine value.
	2 hours.	17 hours.	
Tsubaki oil	58.7	98.5	81.5
Tsubaki oil, exposed ...	49.0	78.8	72.6
Soja bean oil...	50.0	96.0	125.0
Soja bean oil, exposed...	34.6	41.2	98.0
Linseed oil	45.6	—	176.0
Linseed oil, exposed ...	12.8	—	145.3

From these results, it will be seen that fatty oils exposed to the atmosphere are decomposed by lipase with greater difficulty than their corresponding raw oils. This is most marked in the case of drying oils and decreases through the classes of semi-drying and non-drying oils. This considerable reduction in the activity of lipase on an insolated oil is always accompanied by a diminution of the iodine value of the oil.

A number of possible causes for this fact may be suggested.

It has been observed by Marx, Schmid, Mayrhofer, Nagel and Scala that rancid oil contains aldehydes and similar substances. Hence, the retardation of the lipolytic hydrolysis of the insolated oils used which were somewhat rancid, may have been due to the presence of aldehydic substances which were shown by Connstein to have a deleterious influence on lipase. The following experiment will show this.

Experiment 2.

Soja bean oil exposed to direct sunlight for 25 days was purified by distilling with steam and then repeatedly washing with water. 25 grams of the oil thus treated were decomposed by one

gram of pressed castor seed as before, with the following result; the comparative result with insolated oil not treated is also given:

Oil.	Per cent. of oil decomposed in two hours.
Exposed soja bean oil	35.0
Exposed soja bean oil, purified	39.0

The increased hydrolysis in the case of the purified oil was probably due to the removal of aldehydic substances from the oil. However, the presence of aldehydic substances can not be recognised as the only cause of the retardation of the lipolytic action on the exposed oil, as is proved by the fact that lipolysis of the purified exposed oil is much slower than that of the raw oil.

Moreover, further experiments have shown that, even an oil which had been exposed to direct sunlight for a comparatively short time and was not found to be rancid, was also decomposed by lipase with less rapidity than the raw oil, as may be seen in the following experiment.

Experiment 3.

Twenty-five grams of soja bean oil exposed to sunlight for five days were decomposed by one gram of pressed castor seed as before, the comparative experiment with the raw oil being also carried out; the following results were obtained:

Oil.	Per cent. of oil decomposed in two hours.
Raw oil	49.8
Exposed oil	40.0

The author found also that a sample of oil exposed to the action of light alone, as in the case of oil in a sealed glass bottle, was decomposed by lipase as easily as the unexposed oil, as may be seen in the following experiment.

Experiment 4.

Twenty-five grams of soja bean oil exposed to sunlight in a sealed glass bottle for three months were decomposed by one gram of pressed castor seed, together with 3 c.c. of N/10 sulphuric acid and 4 c.c, of water for one hour at 38°C., the comparative experiment with unexposed oil being carried out in parallel. The iodine value was also determined. The results obtained are given below:

Oil.	Per cent. of oil decomposed in one hour.	Iodine value.
Soja bean oil, unexposed	29.3	12.50
Soja bean oil, exposed to sunlight in sealed bottle	29.0	12.45

A similar experiment was made with linseed oil, which was exposed to sunlight without the access of air for one month; the results may be found in the following table:

Oil.	Per cent. of oil decomposed in one hour.	Iodine value.
Linseed oil, unexposed	45.6	176.0
Linseed oil, exposed to sunlight in sealed bottle...	45.8	176.5

From these results, it seems likely that light alone produces no effect on the chemical composition of oil and consequently the

action of lipase is unaffected by it. This result is in accord with Ritsert's observation that the exposure of oils to the effect of light alone is incapable of producing any change.

The author also investigated the lipolysis of an oxidised oil, prepared by passing a current of air through linseed oil for 50 hours at room temperature, the procedure being carried out in the dark.

Experiment 5.

Twenty-five grams of such a blown oil prepared as above were decomposed by one gram of pressed castor seed, the experiment with raw linseed oil being also made for comparison. The iodine value was also determined and is appended in the following table showing the results:

Oil.	Per cent. of oil decomposed in three hours.	Iodine value.
Raw linseed oil	58.0	176.0
Blown linseed oil	28.5	162.1

These results show that lipase acts very slowly on the oxygen-absorbed oil. In this experiment, the decrease in iodine value of the blown oil can probably not be ascribed to such a change as polymerisation but rather to the absorption of oxygen. Although there may be some difference between the slow oxidation by insolation and the more rapid oxidation by blowing in air, it seems probable that the effect of atmospheric oxygen on the unsaturated glycerides is to produce some oxidised substances which are hardly decomposable by lipase, and light accelerates this effect of oxidation.

The author investigated the iodine and saponification values

of the fatty acid resulting from hydrolysing the blown linseed oil and compared them with those of the fatty acid contained in the remaining neutral oil. The blown oil was prepared as above and washed with water, the "lipase powder"[1] used being washed out with ether to remove the fatty matter.

Experiment 6.

One hundred grams of the blown oil were hydrolysed by a quantity of the ether-washed "lipase powder," corresponding to four grams of the original powder, and 30 c.c. of water for five hours at 38°C. The resulting oil containing about 48% of free fatty acid was treated as follows to separate the neutral oil from the free fatty acid. Thirty grams of the decomposed oil were dissolved in 200 c.c. of petroleum ether, to which was added 150 c.c. of absolute alcohol, and titrated with alcoholic potash, using phenolphthalein as indicator. Then were added equal parts of water and alcohol and shaken to separate the solution of soap in 50 per cent. alcohol from the solution of neutral oil in petroleum ether. These solutions were shaken repeatedly, the soap solution with fresh petroleum ether and the oil solution with fresh 50 per cent. alcohol. The fatty acids were obtained from both clear solutions respectively. The iodine and saponification values of these acids were as follows:

Fatty acids from	Iodine value.	Saponification value.
Hydrolysed glycerides	178	199
Besidual glycerides	159	190

[1] This journal, p. 125.

Thus the fatty acids from the residual glycerides have lower iodine and saponification values than those from the hydrolysed glycerides. From these results it may be suggested that the oxidised glycerides which have higher molecular weight owing to the absorption of oxygen, are more resistant to lipase than are the original glycerides.

As one of the oxidised products, the presence of a class of fatty acids which are termed "oxidised" acids by Lewkowitch and characterised by their insolubility in petroleum ether, has been shown by Fahrion. But the quantity of "oxidised" acids in the blown oil was too small to produce such a remarkable retardation of lipolytic hydrolysis of the oil as that described in Experiment 5. The author assumes moreover the presence of other kinds of oxidised products which are also hardly decomposed with difficulty by lipase.

The action of lipase on fatty oils oxidised by heating in access of air or oxygen was found to be more easily influenced.

Experiment 7.

About 100 grams of oil were heated in a porcelain basin for one hour at 140°C. being stirred constantly. The oils thus obtained were washed with water. Twenty-five grams each of the resulting oils were triturated with one gram of pressed castor seed, 3 c.c. of N/10 sulphuric acid and 4 c.c. of water, and allowed to hydrolyse for one hour at 38°C. respectively. Comparative experiments with the corresponding raw oils were also carried out. The results obtained were as follows:

Oil.	Per cent. of oil decomposed in one hour.
Tsubaki oil...	36.2
Tsubaki oil, heated	32.0
Olive oil	35.6
Olive oil, heated	29.0
Soja bean oil	29.0
Soja bean oil, heated...	18.0
Perilla oil	35.3
Perilla oil, heated	20.8
Linseed oil...	32.1
Linseed oil, heated	16.0
Tung oil	30.4
Tung oil, heated	10.2

As can be seen from the figures in the table, the lipolysis of oils heated in access of air is more difficult than in the case of the corresponding raw oils. This is chiefly due to the formation of some oxidised glycerides. But in this case it is necessary to pay attention to the occurence of a certain amount of polymerisation and the action of lipase on the polymerised oil.

The Action of Lipase on Polymerised Oil.

Experiment 8.

Linseed oil was heated at 150°C. for two hours in a current of nitrogen. The lipolytic hydrolysis of the resulting oil and its iodine value were compared with those of the raw linseed oil, with the following results, the hydrolytic procedure being as before:

Oil.	Per cent. of oil decomposed in		Iodine value.
	one hour.	three hours.	
Raw linseed oil	31.7	58.8	176.0
Linseed oil, heated ...	28.8	54.0	172.5

Thus, the oil heated in the presence of nitrogen hydrolyses less rapidly than the raw oil and this is most likely due to polymerisation occurring to some extent.

That the polymerised glycerides are more slowly hydrolysed by lipase than the original ones may also be seen from the following experiment. The polymerised oil used was prepared by heating linseed oil for thirty minutes at 170°—180°C., without access of air.

Expeirment 9.

One hundred grams of the polymerised linseed oil were hydrolysed by about three grams of the ether-washed "lipase powder" and 25 c.c. of water for five hours. The resulting fatty matter containing about 40 per cent. of free fatty acid was treated as in Experiment 6 to separate neutral glycerides from free fatty acids, and their iodine and saponification values were determined respectively:

Fatty acids from	Iodine value.	Saponification value.
Hydrolysed glycerides	173.8	198.1
Residual glycerides	160.5	189.4

From these results, it is suggested that the glycerides of polymerised fatty acids which have higher molecular weight due to polymerisation are more resistant to lipase than the raw oil.

Summary.

1. It is shown that Lipase acts with less rapidity upon oxidised oil, prepared by insolation or by blowing air, than the raw oil. This reduction of the activity of lipase on oxidised oil is most marked in the case of drying oil, and it decreases through the classes of semi-drying and non-drying oils.

2. The slowness of the lipolysis of oxidised oil is due to the presence of some oxygen-absorbed products which are decomposable by lipase with less rapidity and which have probably a larger molecular weight due to the absorption of oxygen.

3. The lipolysis of rancid oil is also slow, possibly on account of the presence of the oxidised products named, and of aldehydic substances, the latter injuring the activity of the lipase.

4. An oil oxposed to sunlight, while protected from contact with the air, does not show any retardation of its lipolytic hydrolysis, showing that light alone produces no effect on the chemical composition of oil.

5. Heated oil which has been prepared by heating in a current of nitrogen, is more difficult of hydrolysis by lipase than the raw oil, showing that the polymerised products of glycerides are with difficulty decomposable by the enzyme.

January 15, 1912.

ON THE DETERMINATION OF ACTUAL STRESSES IN A METALLIC BRIDGE.

BY

I. HIROI,
C. E., Kogakuhakushi.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
Vol. V. No. 5.

Published November, 1913.

On the Determination of Actual Stresses in a Metallic Bridge.

By

I. Hiroi, *C. E., Kogakuhakushi*

Professor of Civil Engineering.

The exact determination of stresses as they actually occur in a bridge under load, in spite of its importance in designing a new structure, as well as in the examination of an existing one has but rarely been attempted. The reason for this lies mainly in difficulties attending the work of measurement on the one hand, and the uncertainties inseparable from mathematical calculations on the other.

While the computation of primary stresses is a comparatively simple work with all kinds of structures, that of secondary stresses generally entails considerable labor and the results obtained are far from being certain, even when their causes are known. The principal cause of this uncertainty lies in the imperfect fulfillment, in the actual structures, of the assumptions on which the calculations are based. The rigidity of construction aimed at in modern railway bridges, especially in those of moderate spans, is conducive to the production of high secondary stresses. To measure such stresses, under a rapidly moving load, with the means available at present would be next to an impossibility, owing to the vibration set up by the load.

In order to obtain, by calculation, the secondary stresses due to rigidity of joints, various methods have been advanced. The author, in the course of his lectures, has given equations for longitudinal and transverse rigidity of joints, and in his "Statically-Indeterminate Stresses in Frames Commonly used for Bridges," has, in Chapter VII, otherwise deduced the same equations. In order to put to practical test, the reliability of these formulas, the model of a Warren truss shown in Fig. 1. Plate I, was constructed, and the stresses produced in its members by certain loading were measured by means of dial strain-meters, having a magnification of 400. The material used for the model was a mild steel, with modulus of elasticity of about 30,000,000 lbs. per square inch. The joints were fully riveted, thus making them quite as rigid as are found in real structures of the kind. The ensemble of the test is shown in Fig. 2. Pl. I.; the load was applied, as shown, by means of hydraulic jack, provided with pressure-gauge. The strains were measured uniformly on a length of 8 inches, and since one division on the dial indicated the linear movement of 1/5000 inch of the connecting bar, it corresponds to an intensity of stress of

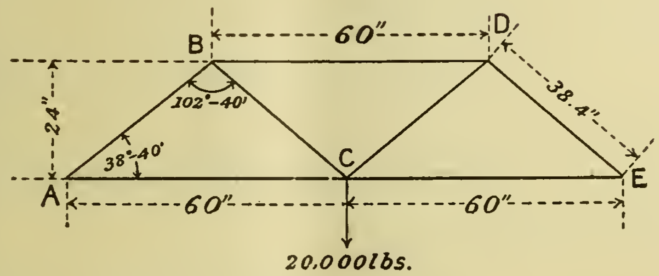
$$\frac{1}{5000}\times\frac{1}{8}\times 30{,}000{,}000=750 \text{ lbs. per sq. in.}$$

While there was no difficulty in estimating the reading of the needle to 1/10 of one division on the dial, i. e. to 1/50000 inch in length of the bar, it was hardly of any value, considering the effects of various disturbing causes and the degree of accuracy attained by the workmanship of the apparatus.

Calculation of Stresses.

On loading the truss model, with a central load of 20,000 lbs., we get, by calculation, the following primary stresses and their intensities:

Fig. 1.



Member.	Primary Stress in lbs.	Cross-section in sq. in.	Intensity in lbs. per sq. in.
AB	$-10{,}000\times1.6=-16{,}000$	3.50	$-$ 4,570
BC	$+10{,}000\times1.6=+16{,}000$	1.17	+13,680
AC	$+10{,}000\times1.25=+12{,}500$	3.50	+ 3,570
BD	$-10{,}000\times2.5=-25{,}000$	3.50	$-$ 7,140

These stresses cause, then, the following changes in lengths, assuming E=30,000,000 lbs. per sq. in.

Member.	Length.	Δl.	$\frac{\Delta l}{l}$
AB	38.4 in.	$-.00584$ in.	$-.000152$
BC	38.4 ,,	$+.01751$,,	$+.000456$
AC	60.0 ,,	$+.00714$,,	$-.000119$
BD	60.0 ,,	$-.01428$,,	$-.000238$

Since, in triangle ABC

$$\Delta A=\frac{\Delta a}{a}(\cot B+\cot C)-\frac{\Delta b}{b}\cot C-\frac{\Delta c}{c}\cot B$$

$$\Delta B=\frac{\Delta b}{b}(\cot A+\cot C)-\frac{\Delta a}{a}\cot C-\frac{\Delta c}{c}\cot A$$

$$\Delta C=\frac{\Delta c}{c}(\cot A+\cot B)-\frac{\Delta c}{c}\cot B-\frac{\Delta b}{b}\cot A$$

we get, on the assumption of freedom from constraints at the panel-joints, the following imaginary angular changes in the truss:

$$\Delta BAC = .000456(1.25-.2248)$$
$$-.000119\times 1.25-.000152\times .2248=+.0002850$$

$$\Delta ABC = .000119(1.25+1.25)$$
$$+.000152\times 1.25-.000456\times 1.25= \quad -.0000825$$

$$\Delta BCA = -.000152(1.25-.2248)$$
$$+.000456\times .2248-.000119\times 1.25=-.0002021$$

$$\Delta CBD = .000456(1.25-.2248)$$
$$+.000456\times .2248+.000238\times 1.25=+.0008674$$

$$\Delta BCD = -.000238(1.25+1.25)$$
$$-.000456\times 1.25-.000456\times 1.25= \quad -.0017350$$

and for the moment-equations,

$$6E\Delta BAC=\frac{L_{ac}}{I_{ac}}(2M_{ac}-M_{ca})-\frac{L_{ab}}{I_{ab}}(2M_{ab}-M_{ba})$$

$$6E\Delta ABC=\frac{L_{ab}}{I_{ab}}(2M_{ba}-M_{ab})-\frac{L_{bc}}{I_{bc}}(2M_{bc}-M_{cb})$$

$$6E\Delta BCA=\frac{L_{bc}}{I_{bc}}(2M_{cb}-M_{bc})-\frac{L_{ac}}{I_{ac}}(2M_{ca}-M_{ac})$$

$$6E\Delta CBD=\frac{L_{bc}}{I_{bc}}(2M_{bc}-M_{cb})-\frac{L_{bd}}{I_{bd}}(2M_{bd}-M_{db})$$

$$6E\Delta BCD=\frac{L_{cd}}{I_{cd}}(2M_{cd}-M_{dc})-\frac{L_{cb}}{I_{cb}}(2M_{cb}-M_{bc})$$

Since $L_{ac}=L_{bd}=60$ in., $L_{ab}=L_{bc}=38.4$ in., $I_{ab}=I_{ac}=I_{bd}=6.7$ in.^{4} and $I_{bc}=I_{cd}=1$ in^4,

we get

$$+51,300=\frac{60}{6.7}(2M_{ac}-M_{ca})-\frac{38.4}{6.7}(2M_{ab}-M_{ba})$$

$$-14,850=\frac{38.4}{6.7}(2M_{ba}-M_{ab})-38.4(2M_{bc}-M_{cb})$$

$$-36,378=38.4(2M_{bc}-M_{cb})-\frac{60}{6.7}(2M_{bd}-M_{db})$$

$$-312\,300=38.4(2M_{cd}=M_{dc})-38.4(2M_{cb}-M_{bc})$$

Further, we have

$$M_{ab}+M_{ac}=0$$

$$M_{ba}+M_{bc}+M_{bd}=0$$

and from symmetry,

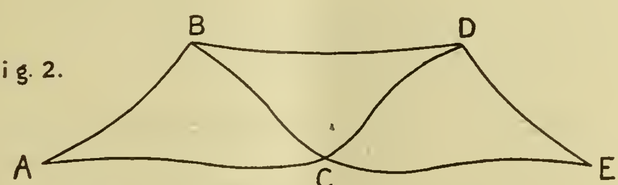
$$M_{bd}=-M_{db} \qquad M_{cd}=-M_{cb} \qquad M_{dc}=-M_{bc}$$

Combining these equations, we get the following values of moments :

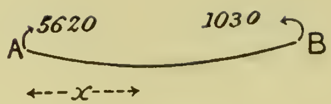
$M_{ab}=-\ 5,620$ in.-lbs.
$M_{ac}=+\ 5,620$ „ „
$M_{ba}=+\ 1,030$ „ „
$M_{bc}=-\ 3,480$ „ „
$M_{ca}=+13,360$ „ „
$M_{cb}=+\ 3,260$ „ „

Applying these moments to the ends of respective members, by paying attention to the signs (+for anticlockwise motion), we find the distortions of the truss members to be as shown exaggerated in Fig. 2.

Fig. 2.



The moment (m) and mean fibre-stress in the flange (f) at any point of each member, are then obtained in the following manner:



$$m = 5620 + \frac{1030 - 5620}{38.4} x$$

$$f = -4570 \pm \frac{m}{6.7} \times 1.44$$

$$m = -5620 + \frac{5620 + 13360}{60} x$$

$$f = +3570 \pm \frac{m}{6.7} \times 1.44$$

$$m = +3480$$

$$f = -7140 \pm \frac{3480}{6.7} \times 1.44 = \begin{cases} -7870 \\ -6390 \end{cases}$$

$$m = -2450 + \frac{2450 + 3260}{38.4} x$$

$$f = +13680 \pm \frac{m}{1} \times 1.56$$

Note, in the above, the signs for moments made +, when causing compression on the upper fibres of the members, in the respective positions they occupy in the truss.

The Measurement of Flange-Stresses.

The flange-stresses were measured only at those points found to be suited for the attachment of the strain-meter.

In AB and DE, their middle points were chosen, where according to the foregoing equations,

$$m = 5620 + \frac{1030 - 5620}{38.4} \times 19.2 = +3{,}330 \text{ in.-lbs.}$$

$$f = -4570 \pm \frac{3330}{6.7} \times 1.44 = \begin{cases} -5{,}285 \text{ lbs. per. sq. in.} \\ -3{,}855 \quad " \quad " \quad " \quad " \end{cases}$$

In AC and CE, also, the middle points were taken where, again,

$$m = -5620 + \frac{5620 + 13360}{60} \times 30 = +3870 \text{ in.-lbs.}$$

$$f=+3570\pm\frac{3870}{6.7}\times 1.44=\begin{cases}+4400 \text{ lbs. per. sq. in.}\\ +2740 \text{ ,, ,, ,, ,,}\end{cases}$$

In BD, the points chosen were 15.6 inches from B and D; although any other point should have given the same results, as the moment and consequently the flange-stresses ought to be uniform throughout the length, being -7890 and -6390 in-lbs, as already found.

No measurements were made in BC, owing to the form of the member not being adapted for fixing the instrument.

The following table shows the comparison of flange-stresses obtained by measurement and by calculation.

Member.	Flange.	Intensity of stresses in lbs. per sq. in. By measurement.		Intensity of stresses in lbs. per sq. in. By calculation.	
AB and DE	Upper	-5400	$=-4800\pm600$	$-4570\pm715=$	-5285
	Lower	-4200			-3855
AC and CE	Upper	$+2300$	$=+3100\pm800$	$+3570\pm830=$	$+2740$
	Lower	$+3900$			$+4400$
BD	Upper	-7900	$=-7250\pm650$	$-7140\pm750=$	-7890
	Lower	-6600			-6390

Each of the measured stresses is the mean of the measurements made at the four edges of each of the sections chosen.

While the agreement between the calculated and the measured stresses is far from being complete, it is, in the mind of the writer, close enough to establish the reliability of the method of calculation followed, knowing, as he does, the imperfect workmanship of the instrument used and considering the various disturbing causes which were unavoidable.

Several other measurements were made at points nearer to the panel points; these naturally gave comparatively lower flange-stresses, owing to the presence of connection-plates. It was also found that the distribution of stress in one and the same flange

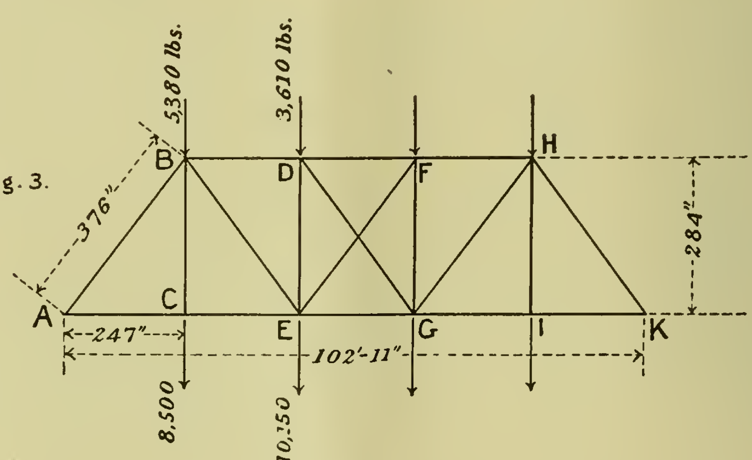
varied to a certain extent; in one case, the difference was as much as 50% of the lesser, owing, no doubt, to structural eccentricity at the joint.

Calculation of Stresses in a Railway Bridge.

The structure chosen for investigation is a riveted bridge, with a clear span-length of 100 feet, typical of those found on lines of the Imperial Government Railways. Its general details are shown on Pl. II. The load taken consists of 2 engines, each weighing 240,000 lbs. followed by a uniform load of 3,000 lbs. per foot of track, which is apparently the load for which the bridge was designed.

The general dimensions and the distribution of the dead load are as shown in Fig. 3

Fig. 3.



Primary Stresses.—From the data above given, are obtained the following primary stresses and their intensities on effective cross-sections, of which those arising from the live-load are increased according to the common impact-formula, $\frac{300}{l+300}$, for the dynamic effects.

Member.	Effective cross section. sq. in.	Dead-load stress. lbs.	Live-load stress. lbs.	Impact.	Total stress. lbs.	Intensity. lbs. per sq. in.
AB	12	+24,100	+ 75,100	.75	+155,500	+12,960
EG	17	+36,100	+113,600	„	+234,900	+13,640
BD	21	−36,100	−113,600	„	−234,900	−11,190
DF	21	−36,100	−113,600	„	−234,900	−11,190
AB	30	−36,700	−114,300	.75	−236,700	− 7,890
BC	8	+ 8,500	+ 55,200	.88	+112,300	+14,040
BE	8	+18,300	+ 69,300	.82	+144,400	+18,050
DE	11	− 3,600	− 25,800	.87	− 51,800	− 4,710
DG	5	0	+ 34,300	.87	+ 64,100	+12,820

For the greatest bending moments in the floor-system,

Stringer	11,500	172,500	.94	346,200	ft.-lbs.
Floor-beam (Interm.)	27,600	290,500	.88	573,700	„

The flange-stresses in the stringer and floor-beam would then be

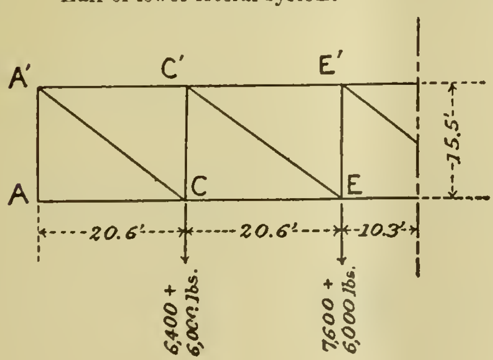
$$\text{Stringer} \qquad \frac{346{,}200\times 12}{8{,}080}\times 19.4 = 9{,}980 \text{ lbs. per sq. in.}$$

$$\text{Floor-beam} \qquad \frac{573{,}700\times 12}{15{,}664}\times 25.0 = 10{,}990 \text{ „ „ „ „}$$

Wind Stresses.—For calculating the wind stresses, a pressure of 30 lbs per sq. ft. acting on the exposed surfaces of the bridge and the train will be assumed, as being the pressure at which light trains are liable to be overturned; only the pressure acting on the train will be considered as a moving load.

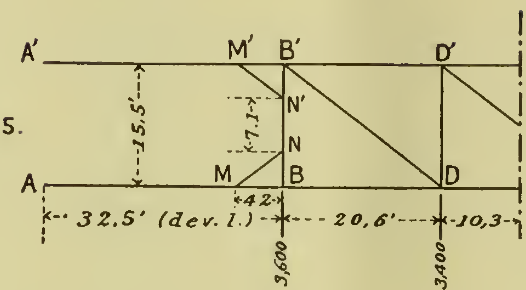
Half of lower lateral-system.

Fig. 4.



Half of upper lateral-system.

Fig. 5.

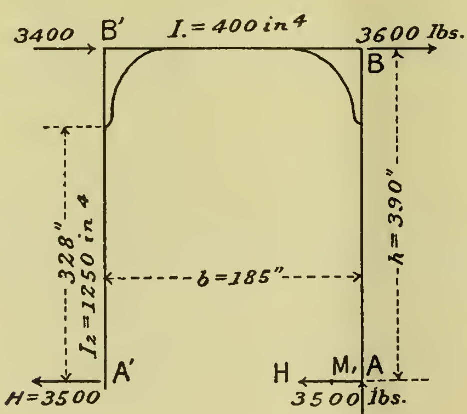


These pressures cause the following stresses in the chords as members of the upper and lower lateral systems, each system considered to be independent of one another.

Member.	Effective cross section. sq. in.	Stress. lbs.	Intensity. lbs per sq. in.
$A'\ C'\ (AC)$	12	$-34{,}500$	$-2{,}880$
$C'\ E'\ (CE)$	12	$-52{,}600$	$-4{,}380$
		$+34{,}500$	$+2{,}880$
$E'\ G'\ (EG)$	17	$\pm 52{,}600$	$\pm 4{,}380$
$B'\ D'\ (BD)$	21	$-\ 4{,}500$	$-\ 210$
$D'\ F'\ (DF)$	21	$\pm\ 4{,}500$	$\pm\ 210$

In the portal-brace, if the lower ends of the end-posts are assumed to be hinged, the stresses in the latter would be

Fig. 6.



Direct stress (V)

$$=\frac{7000\times 390}{185}=-14{,}800\text{ lbs.}$$

Bending moment (M)

$$=3500\times 328=-1{,}148{,}000$$

in.-lbs.

Whereas, if the end-posts were fixed at their lower ends,

$$M_1 = \frac{h(bI_2+3hI_1)}{bI_2+6hI_1}H = 234 \times 3{,}500 = +809{,}000 \text{ in.-lbs.}$$

$$M = M_1 - 328\,H = -339{,}000 \text{ in.-lbs.}$$

$$V = \frac{(390-234)7000}{185} = -5{,}900 \text{ lbs.}$$

The actual stress in AB would probably be about the mean of the two preceeding ones, viz:

$$\text{Direct stress} = \quad 10{,}000 \text{ lbs.}$$

$$\text{Bending moment} = \begin{cases} -743{,}500 \text{ in.-lbs.} \\ +404{,}500 \quad ,, \quad ,, \end{cases}$$

causing, in the end-post, the extreme fibre-stress of

$$-\frac{10{,}000}{30} - \frac{743500 \times 9.5}{1250} = -5980 \text{ lbs. per sq. in.}$$

The horizontal component of the direct-stress amounts to $\pm 9{,}600$ lbs.

The transfered load which is

$$\overset{\text{sq. ft}}{200} \times \overset{\text{lbs}}{30} \times \overset{\text{ft}}{10} \div \overset{\text{ft}}{15.5} = 3370 \text{ lbs. per panel,}$$

produces the following stresses in the chords and end-posts:

$A'\ E'\ (AE)$	$\pm$ 6,700
$E'\ G'\ (EG)$	$\pm$10,100
$A'\ B'\ (AB)$	$\pm$10,500

The transfered load, also causes the following bending moments:

	Bending moment.	Flange-stress.
Stringer	30,900 ft.-lbs.	890 lbs. per sq. in.
Floor-beam	20,400 ,, ,,	400 ,, ,, ,, ,,

The greatest tension, due to the assumed wind-pressure, found in the lower chord and end-post of the lea-side truss may therefore amount to:

AC	$(9600+\ 6{,}700)\ \dfrac{1}{12}=1360$	lbs per sq. in.
CE	$(34{,}500+\ 9{,}600+\ 6{,}700)\ \dfrac{1}{12}=4230$	„ „ „ „
EG	$(52{,}600+\ 9{,}600+10{,}100)\ \dfrac{1}{17}=4650$	„ „ „ „
AB	$(5930+10{,}500)\ \dfrac{1}{30}=6330$	„ „ „ „

Calculation of Secondary Stresses.

Of the various causes of secondary stresses, only the longitudinal thrust of moving loads, and the eccentricity and rigidity of joints will be considered, as others are either inconsidersble in their amounts or uncertain in their actions.

Stresses due to the longitudinal thrust.—The braking force transmitted through the floor-system to the lower chord would probably never exceed

$$390{,}000\times 0.2-11{,}000=28{,}000 \text{ lbs.}$$

The lateral bending moment caused by this force in the floor-beam would then be

$$\frac{28{,}000}{6}\times\frac{1}{2}\times 5.25=12{,}250 \text{ ft.-lbs.}$$

causing an extreme fibre-stress of

$$\frac{12250\times 12}{162}\times 6=\pm 5{,}400 \text{ lbs. per sq. in.}$$

while the direct stress in the lower chord, at the end panel, due to the same cause, would be

$$28{,}000\times\frac{1}{2}=14{,}000 \text{ lbs.}$$

Stresses due to the eccentricity in joints.—In the truss itself there is hardly any eccentricity in joints, worth considering. The eccentricity in the connection of the lateral system amounts to 8 inches in the upper and 3 to 18 inches in the lower; the last amount being found at the end-joint, were the latter free of constraint, would cause extreme fibre-stresses of

$$34500 \times 18 \times \frac{5.5}{350} = \pm 9760 \text{ lbs. per sq. in. in wind-side chord.}$$

$$34500 \times 3 \times \frac{5.5}{350} = \pm 1630 \text{ ,, ,, ,, ,, ,, lea-side ,,}$$

As it is, however, not more than one-half of these amounts would really be active in those members.

Stresses due to the rigidity of joints.—For calculating the secondary stresses due to the longitudinal rigidity of the panel joints, in order to simplify the work, an equivalent uniform load (w) for max. moment at centre will be used.

Then, since

$$w = \frac{113600 \times 23.7 \times 8}{(102.9)^2} = 2{,}035 \text{ lbs. per ft.,}$$

for the case of full loading, we get the following stresses and specific changes of lengths, assuming $E = 30{,}000{,}000$ lbs. per sq. in.

Member.	Length in in.	Gross Section in sq. in.	Moment of Inertia in in.^{4}	Total Stress in lbs.	$\frac{\Delta l}{l}$
AC	247	15	313	+152,000	+.000338
EG	247	21	375	+227,900	+.000364
BD	247	21	469	−227,900	−.000364
DF	247	21	469	−227,900	−.000364
AB	376	30	641	−231,300	−.000257
BC	284	10	37	+ 87,300	+.000291
BE	376	11	70	+115,800	+.000351
DE	284	11	70	− 3,600	−.000011
DG	376	6	9	0	0

The last column of the preceeding table gives the following angular changes and the corresponding moment-equations:

$$6E.\Delta BAC=+113{,}400=\frac{247}{313}(2M_{ac}-M_{ca})-\frac{376}{641}(2M_{ab}-M_{ba})$$

$$6E.\Delta ABC=+\ 93{,}600=\frac{376}{641}(2M_{ba}-M_{cb})-\frac{284}{37}(2M_{bc}-M_{cb})$$

$$6E.\Delta ACB=-207{,}000=\frac{284}{37}(2M_{cb}-M_{bc})-\frac{247}{313}(2M_{ca}-M_{ac})$$

$$6E.\Delta BCE=+\ 13{,}860=\frac{247}{313}(2M_{ce}-M_{ec})-\frac{284}{37}(2M_{cb}-M_{bc})$$

$$6E.\Delta CBE=-\ \ 1{,}980=\frac{284}{37}(2M_{bc}-M_{cb})-\frac{376}{70}(2M_{be}-M_{eb})$$

$$6E.\Delta BEC=-\ 11{,}860=\frac{376}{70}(2M_{eb}-M_{be})-\frac{247}{313}(2M_{ec}-M_{ce})$$

$$6E.\Delta BDE=+186{,}840=\frac{247}{469}(2M_{db}-M_{bd})-\frac{284}{70}(2M_{de}-M_{ed})$$

$$6E.\Delta DBE=-\ 74{,}700=\frac{376}{70}(2M_{be}-M_{eb})-\frac{247}{469}(2M_{bd}-M_{db})$$

$$6E.\Delta BED=-111{,}960=\frac{284}{70}(2M_{ed}-M_{de})-\frac{376}{70}(2M_{eb}-M_{be})$$

$$6E.\Delta FDE=+\ 76{,}500=\frac{284}{70}(2M_{de}-M_{ed})-\frac{247}{469}(2M_{df}-M_{fd})$$

$$6E.\Delta DEG=-\ 73{,}440=\frac{247}{375}(2M_{eg}-M_{ge})-\frac{284}{70}(2M_{ed}-M_{de})$$

Farther, we have

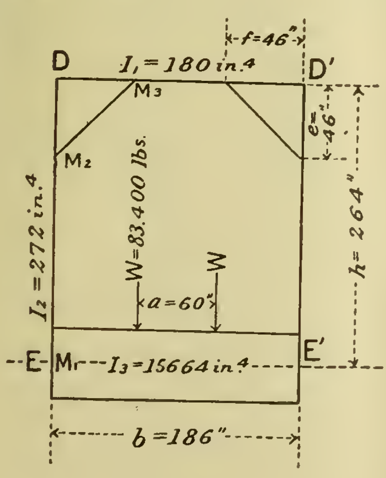
$$M_{ab}+M_{ac}=0$$
$$M_{ba}+M_{bc}+M_{be}+M_{bd}=0$$
$$M_{ca}+M_{cb}+M_{ce}=0$$
$$M_{db}+M_{de}+M_{df}=0$$
$$M_{ec}+M_{eb}+M_{ed}+M_{eg}=0$$
$$M_{ge}=-M_{eg}$$
$$M_{fd}=-M_{df}$$

These equations furnish the following values of moments:

Member.	End-Moment. in.-lbs.	Section-Mod. in.³	Extreme Fibre-stress. lbs. per sq. in.
AB	$M_{ab}=-$ 70,000	143	$-$ 490
	$M_{ba}=-$ 29,100	80	
AC	$M_{ac}=+$ 70,000	50	+1,540
	$M_{ca}=+$ 77,200		
BC	$M_{bc}=-$ 13,800	9	+1,790
	$M_{cb}=-$ 16,100		
BD	$M_{bd}=+$ 47,500	86	$-$1,320
	$M_{db}=+$113,600		
BE	$M_{be}=+$ 4,600	14	+ 490
	$M_{eb}=+$ 6,800		
CE	$M_{ce}=-$ 61,100	50	+1,220
	$M_{ec}=+$ 39,200		
DE	$M_{de}=-$ 16,600	14	$-$1,190
	$M_{ed}=-$ 10,300		
DF	$M_{df}=-$ 97,000	220	$-$ 440
	$M_{fd}=+$ 97,000		
EG	$M_{eg}=-$ 35,700	62	+ 580
	$M_{ge}=+$ 35,700		

For other positions of the load, the amounts of secondary moments will naturally differ, but those in the chords and larger web members as found above would not be exceeded.

Fig. 7.



As to the secondary stresses, due to the transverse rigidity of joints, commencing at *DE*, we have for the cross-frame *DED'E'* shown in Fig. 7 the following expression for the moments:

$$M_1 = -\frac{\left\{\dfrac{h-e}{3I_2} - \dfrac{eh(b-2f)}{2I_2(h-e)^2}\right\}M_2 - \dfrac{(b^2-a^2)}{8I_3}W}{\dfrac{b}{2I_3} + \dfrac{2(h-e)}{3I_2} + \dfrac{e^2(b-2f)}{2I_1(h-e)^2}} = 38{,}800 \text{ in.-lbs.}$$

$$M_2 = -\frac{\dfrac{h-e}{6I_2} - \dfrac{eh(b-2f)}{2I_1(h-e)^2}}{\dfrac{h}{3I_2} + \dfrac{h^2(b-2f)}{2I_1(h-e)^2}} M_1 = -3{,}800 \text{ in.-lbs.}$$

The extreme fibre-stress, caused by these moments, in DE or $D'E'$ will then be

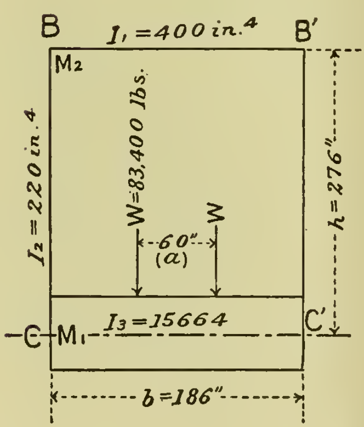
$$\left(38{,}800 - \frac{38800+3800}{218} \times 26\right)\frac{5.5}{272} = \pm 680 \text{ lbs. per sq. in.}$$

At BC, the cross-frame being as shown in Fig. 8, we have:

$$M_1 = -\frac{\dfrac{h}{3I_2}M_2 - \dfrac{(b^2-a^2)}{8I_3}W}{\dfrac{2h}{3I_2} + \dfrac{b}{2I_3}} = 29{,}1000 \text{ in.-lbs.}$$

$$M_2 = -\frac{\dfrac{h}{6I_2}}{\dfrac{h}{3I_2} + \dfrac{b}{2I_1}} M_1 = -9400 \text{ in.-lbs.}$$

Fig. 8.



and for the consequent extreme fibre-stress in the post,

$$BC(B'C')\left(29{,}100 - \frac{29100+9400}{276} \times 26\right)\frac{5.5}{220} = \pm 630 \text{ lbs. per sq. in.}$$

The combinations of the several stresses, thus far calculated, viz: those due to dead and live loads and the second-

ary stresses caused by the rigidity of joints (Case I) and for the simultanrous occurrence of all the stresses considered (Case II) lead to the following results:

CASE I.

Member.	Intensity of stress lbs. per sq. in. Primary.	Secondary.	Total.
AC	+12,960	+1,540	+14,500
CE	+12,960	+1,220	+14,180
EG	+13,820	+ 580	+14,400
BD	−11,200	−1,320	−12,520
DF	−11,200	− 440	−11,640
AB	− 7,890	− 490	− 8,380
BC	+14,000	+1,790+630	+16,420
BE	+18,050	+ 490	+18,540
DE	− 4,710	−1,190−680	− 6,500
DG	+12,800	——	+12,800
Stringer	9,980	——	9,980
Floor-beam	10,990	——	10,990

CASE II.

Member.	Intensity of stress in lbs. per sq. in. Primary and Secondary.	Wind.	Long. Thrust.	Eccentricity.	Total.
AC	+14,500	+1,360	+1,170	0	+17,030
CE	+14,180	+4,230	+ 930	+ 800	+20,140
EG	+14,400	+4,650	+ 700	+1,200	+20,950
AB	− 8,380	−6,330	———	——	−14,710
Stringer	9,980	890	——	——	10,870
Floor-beam	10,990	400	5,400	——	16,790

Case I is ordinary, while Case II is extraordinary; the latter may or may not occur in the life of a bridge.

The intensity of stress found in Case I is, with few exceptions, under 16,000 lbs. per sq. in. which is about the limit allowed, in

the best practice, for the structural steel. The web members, BC and BE, are, in cross sections, not quite up to those of the rest of the members, and consequently, would be the first ones to require reinforcing, when the rolling load exceeds the one the structure is calculated for. In Case II, for which the allowable limit may be safely raised up to three-quarters of the elastic limit of the material, which latter should not be less than 30,000 lbs. per sq. in., making the allowable limit 22,500 lbs per sq. in., the greatest intensity is found to be under 21,000 lbs. per sq. in. There may be found, in the details and subsidiary members, stresses, especially local ones, which might exceed those computed for the main members; but it may be stated that the structure is, as a whole, one well proportioned; besides, having certain provision for possible future increase in the amount of rolling load.

It is sincerely to be wished, that every Railway Administration would institute like investigations, carrying them out more thoroughly than has been attempted in this, for all the structures on its lines. Accidents arising from faulty designs or excessive loading might then be entirely averted.

In conclusion, the writer desires to express his appreciation of the valuable assistance, rendered him by Assist.-Prof. Y. Nagayama in carrying out the foregoing measurements and computations.

Plate I

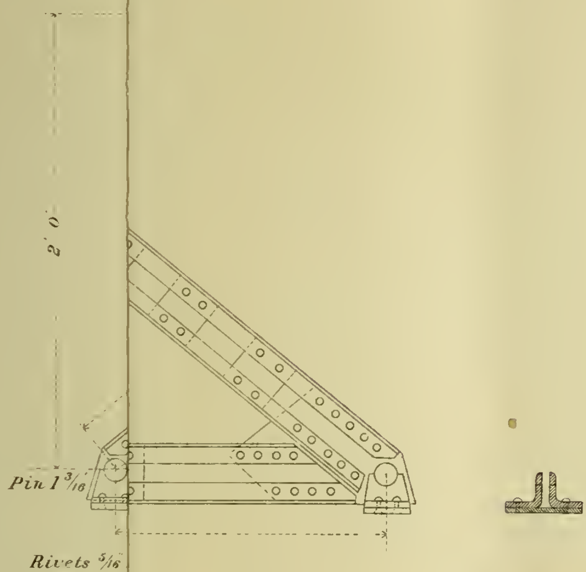


Fig. 1.

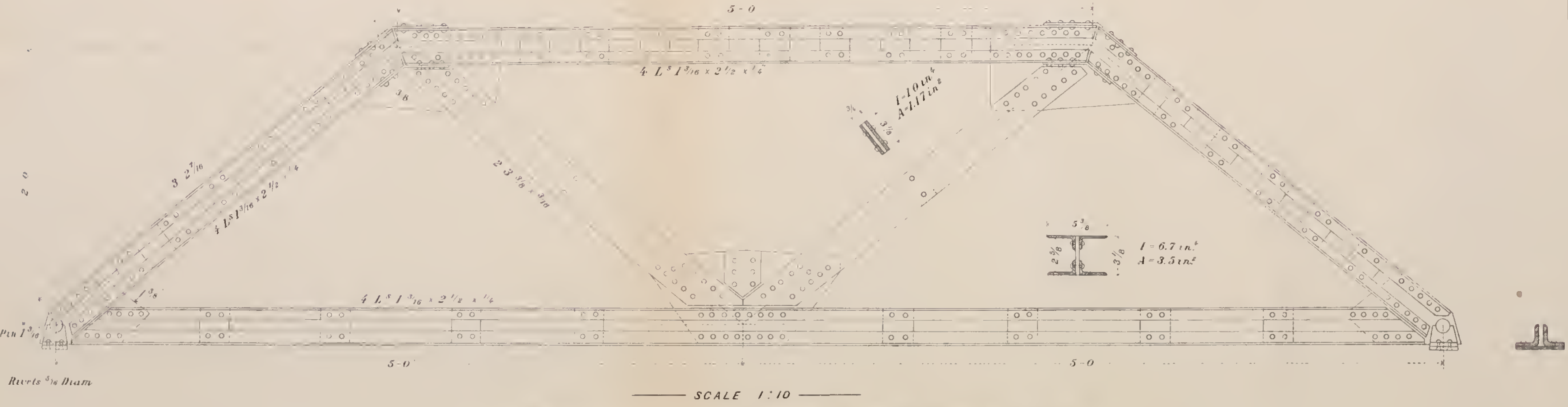
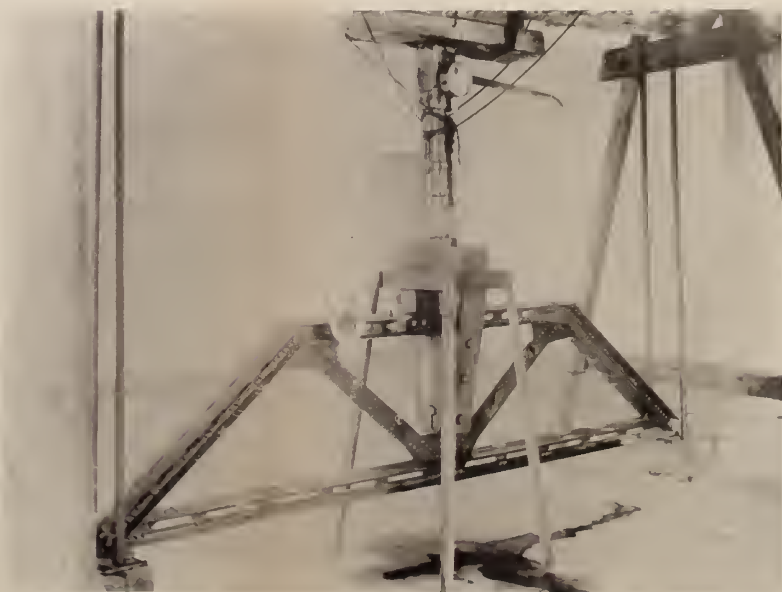

5 - 0
4 L^s 1 3/16 x 2 1/2 x 1/4
I = 1.0 in.4
A = 1.17 in.2
I = 6.7 in.4
A = 3.5 in.2
4 L^s 1 3/16 x 2 1/2 x 1/4
Pin 1 3/16
5 - 0
5 - 0
Rivets 5/16 Diam
SCALE 1:10

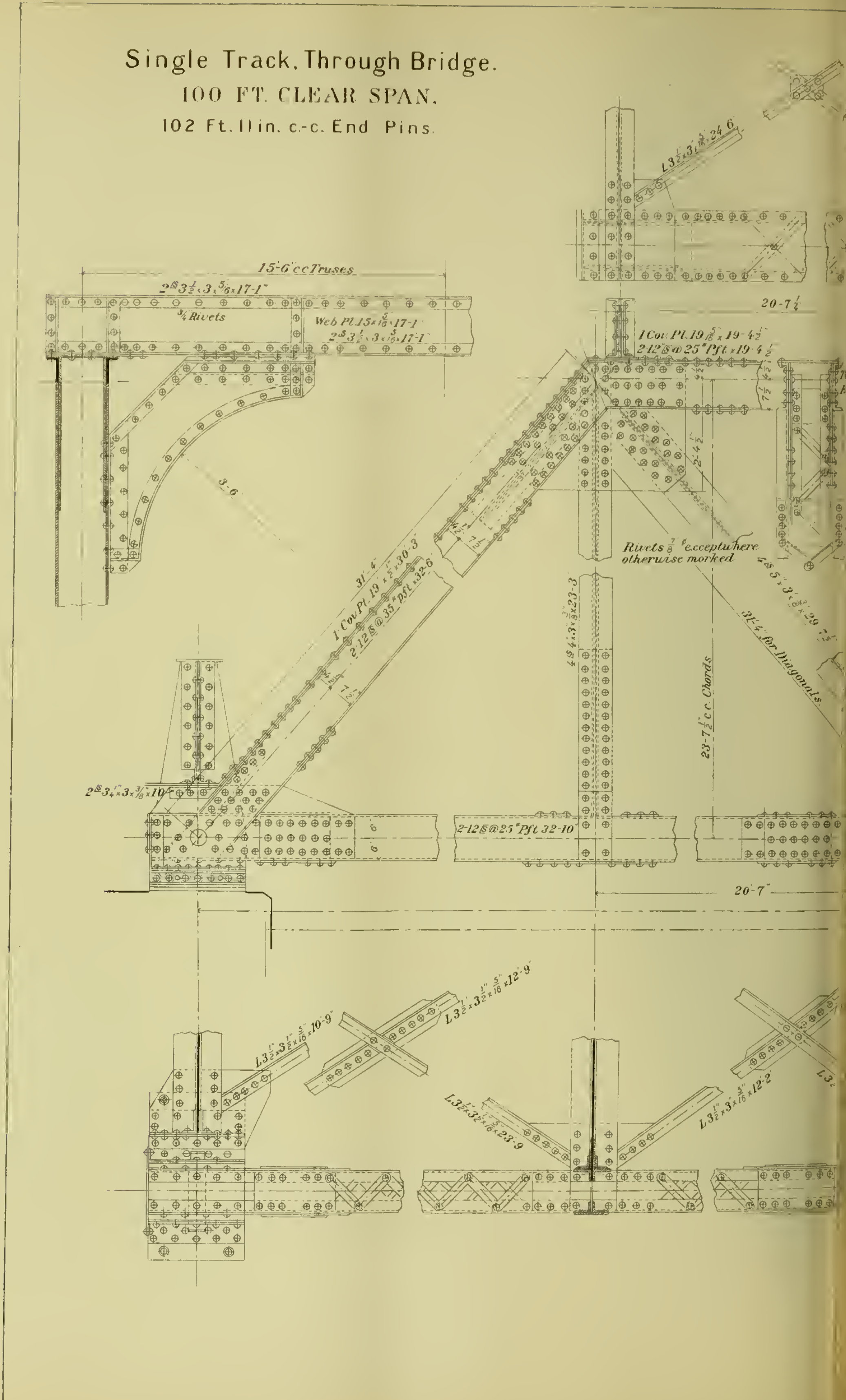
Fig. 2.

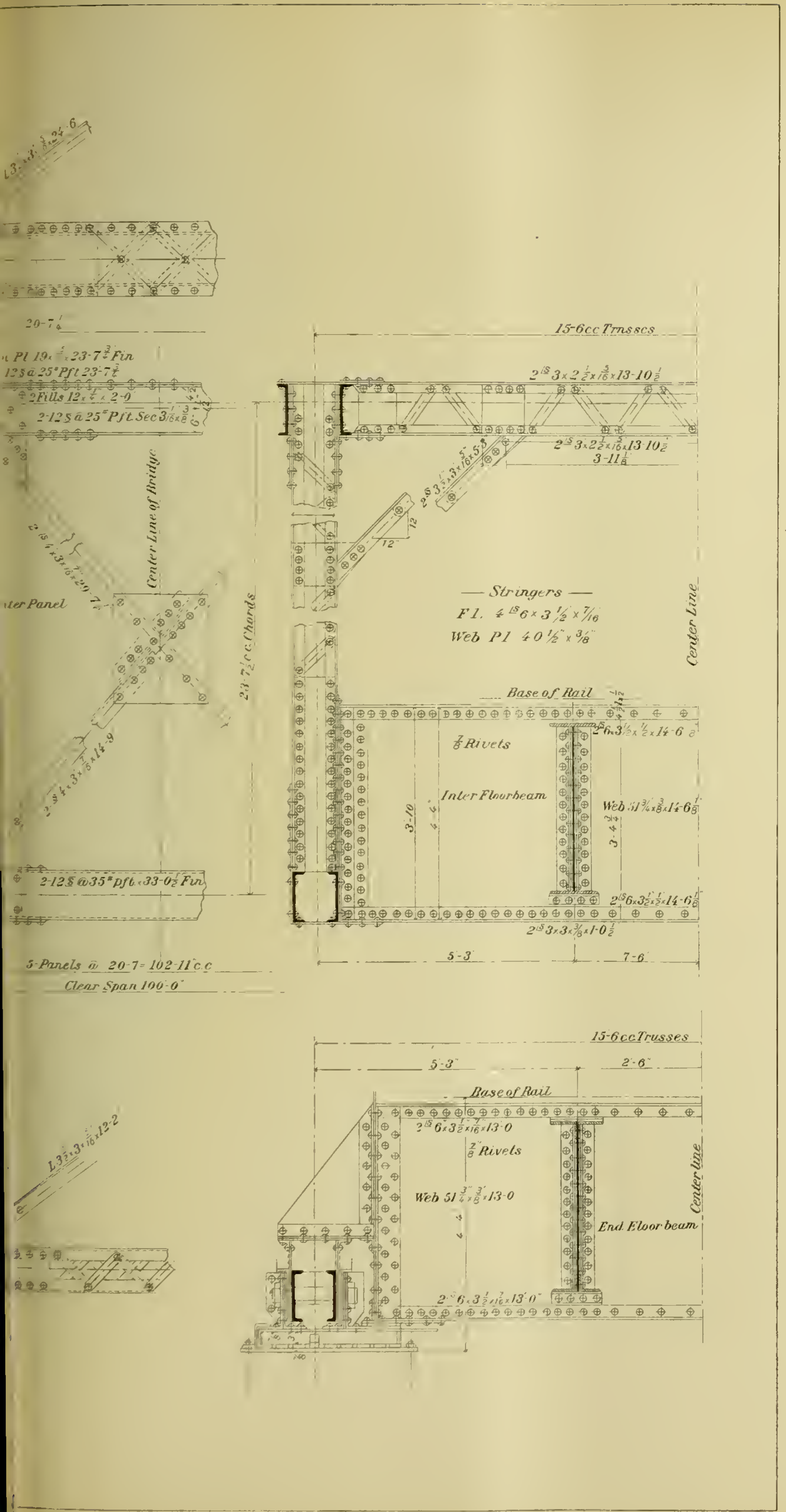


Single Track, Through Bridge.

100 FT. CLEAR SPAN.

102 Ft. 11 in. c.-c. End Pins.




15-6 cc Trusses
Stringers
Fl. 4 ∟s 6 x 3 1/2 x 7/16
Web Pl 40 1/2" x 3/8"
Center Line
Base of Rail
7/8 Rivets
Inter Floorbeam
Center Line of Bridge
Inter Panel
23-7 1/2 c.c. Chords
5-Panels @ 20-7 = 102-11 c.c
Clear Span 100-0"
5-3"
7-6
2-6"
End Floor beam
Center line
Base of Rail
7/8 Rivets

ON SHEARING STRESS IN A SHIP'S STRUCTURE.

BY

K. SUYEHIRO,
Kogakuhakushi.

THE JOURNAL OF THE COLLEGE OF ENGINEERING
TOKYO IMPERIAL UNIVERCITY, TOKYO, JAPAN.
Vol. V. No. 6.

Published November, 1913.

On Shearing Stress in a Ship's Structure.

By

K. Suyehiro, *Kōgakuhakushi.*

The present subject has not been, so far, treated with the attention it deserves. Prof. Jenkins is, perhaps, the only writer who deals with it fully, whilst a few years ago, Dr. Bruhn added a very brief note on the subject in a paper on the stress at discontinuities and showed a rough sketch of the distribution of shearing stress in a ship's section. So far as the maximum shearing stress is concerned, the opinions of these two investigators were same and no naval architect would doubt the validity of their methods of calculation. But, with regard to the distribution of shearing stresses, their conclusions differed in many respects. Since then, no one has tried to advance this subject.

My intention is to throw some light, on the darkness in which this subject at present lies.

I. Distribution of shearing stress in a ship's section.

In treating such a practical subject, one must not forget to take the basis of argument in analogous cases which may be solved analytically or otherwise determined experimentally in a universally accepted way.

Usually, this subject is treated on the assumption that the shearing stress in a ship's section would be distributed as in the

section of a solid rectangular beam. But it seems, that there is not much similarity between a solid beam, and a hollow beam, like a ship, because, in the former, the unbalanced force due to the variation of bending moment, is in equilibrium with the shearing force distributed in a horizontal section whilst in the latter, this mode of equilibrium may be different.

The general feature of the distribution of shearing stresses in a hollow beam can be seen from that in a thin hollow cylindrical beam.

(1). Shearing stress in a thin hollow circular cylindrical beam.

Taking the central axis of the cylinder as the axis of z, a horizontal line drawn through the centre as the axis of y, and a line drawn vertically-downward through the centre as the axis of x (see fig. 1), the Saint Venant's "flexure function" is as follows:—

$$\phi = -(\tfrac{3}{4}+\tfrac{1}{2}\sigma)\left\{(a_0^2+a_1^2)x+\frac{a_0^2a_1^2x}{x^2+y^2}\right\}+\tfrac{1}{4}(x^3-3xy^2)+c \dots \quad (1)$$

where σ = Poisson's ratio.

a_0 = External diameter.

a_1 = Internal diameter.

c = Constant.

Now the component shearing stresses are given by

$$\left.\begin{aligned} X_z &= -\frac{F}{2(1+\sigma)I}\left\{\frac{\partial\phi}{\partial x}+\tfrac{1}{2}\sigma x^2+(1-\tfrac{1}{2}\sigma)y^2\right\} \\ Y_z &= -\frac{F}{2(1+\sigma)I}\left\{\frac{\partial\phi}{\partial y}+(2+\sigma)xy\right\} \end{aligned}\right\} \dots\dots\dots \quad (2)$$

where F = Shearing force.

I = Moment of inertia of section.

Substituting (1) in these equations, and transforming into polar co-ordinates, we get

$$X_z = -\frac{F}{2(1+\sigma)I}\left[-(\tfrac{3}{4}+\tfrac{1}{2}\sigma)\left\{(a_0^2+a_1^2)-\frac{a_0^2a_1^2(\cos^2\theta-\sin^2\theta)}{r^2}\right\} + r^2\cos^2\theta(\tfrac{3}{4}+\tfrac{1}{2}\sigma)+r^2\sin^2\theta(\tfrac{1}{4}-\tfrac{1}{2}\sigma)\right]$$

$$Y_z = -\frac{F}{2(1+\sigma)I}\left\{(\tfrac{3}{4}+\tfrac{1}{2}\sigma)\frac{2a_0^2a_1^2}{r^2}+r^2(\tfrac{1}{2}+\sigma)\right\}\cos\theta\,\sin\theta.$$

In the case where the thickness of shell $(a_0-a_1=t)$ is small, the means of these component stresses taken over the thickness are

$$\bar{X}_z = \frac{F}{1}r^2\sin^2\theta$$

$$\bar{Y}_z = -\frac{F}{I}r^2\sin\theta\cos\theta$$

where $\qquad r =$ the mean radius.

Therefore, substituting the value of $I=\pi r^3 t$,

$$\bar{X}_z = 2\frac{F}{2\pi rt}\sin^2\theta$$

$$\bar{Y}_z = -\frac{F}{2\pi rt}\sin 2\theta$$

Thus, finally

$$\left.\begin{aligned}\bar{X}_z &= 2\tau\sin^2\theta\\ \bar{Y}_z &= -\tau\sin 2\theta\\ R &= \sqrt{\bar{X}_z^2+\bar{Y}_z^2} = 2\tau\sin\theta\end{aligned}\right\}\cdots\cdots\cdots\cdots (3)$$

$$\frac{X_z}{Y_z} = -\tan\theta$$

where $\qquad \tau =$ the mean shearing stress over the section.

$\qquad R =$ the resultant shearing stress.

Now there are two formulae for practical calculation, one of them being

$$R_1 = \frac{F.\,m}{2\times\text{horizontal thickness}\times I}$$ (See Prof. Jenkin's paper read before the Institution of Naval Architects, 1890).

and the other

$$R_2 = \frac{Fm}{2tI}$$ (See Prof. Biles', "The design and construction of ships," vol. I, page 265).

where $m =$ the moment of area outside of the part under consideration, about neutral axis $= 2r^2t.\sin\theta$.

Substituting the values of I and m,

$$\left.\begin{aligned} R_1 &= \tau \times 2\sin^2\theta \\ R_2 &= \tau \times 2\sin\theta \end{aligned}\right\} \qquad (4)$$

Comparing (3) with (4), it will be seen that R is equal to R_2, and that the resultant shearing stress (R) is nearly in the direction of the tangent to the periphery.

Therefore, in virtue of the equality of shearing stresses in perpendicular planes, it may be inferred that the shearing force in hollow beams which counterbalances the unbalanced force due to the variation of bending moment, will be in the longitudinal plane normal to the periphery and the approximate value of such shearing stress given by

$$R = \frac{F.m}{2t.I} \qquad (5)$$

(2). Distribution of shearing stress in a ship's section.

Under the assumption that the above inference holds in a ship's section, I have calculated the shearing stresses in an existing steel steamer. The amount of shearing stresses in various parts is shown graphically in fig. 2, in which the intensity of stress is drawn perpendicularly to each member on one side, irrespective of the sign of stresses.

The calculations were made in the following way:—

(*a*) Calculations for side plating are self evident.

(*b*) Deck platings. For deck platings, m was taken to be

double the moment of area of the platings included between the point under consideration and the centre line (referring to fig. 3, $m=2\times$moment of OA about neutral axis) and t the thickness of the plating.

(c) Bilge platings. In calculation of shearing stress in bilge plating, t is usually assumed, such that not only the outside plating, but also all vertical members as margin plate and centre girder which interesect with the horizontal line drawn through the point under consideration, would offer the resistance to the unbalanced force due to the variation of bending moment. But, as the inner bottom has no connection to the upper part of the ship to resist the unbalanced forces, unbalanced forces in all the bottom members lying between the bilges (*i.e.*, all longitudinal members constituting double bottom) will be kept in equilibrium by the shearing force in the bilge plating only.

Thus, the shearing stress at the point B was calculated, by taking m to be the sum of the moments of sectional area of all longitudinal members composing the bouble bottom, and t the thickness of the bilge plating at the same point.

(d) Double bottom. The shearing stresses in vertical members were calculated without difficulty. The total unbalanced force in that part of double bottom lying above a horizontal section (say HL) can only be kept in equilibrium by the shearing force in all vertical members of double bottom. Therefore, in calculating the stress, m was taken to be the moment of all double bottom members lying above the horizontal line HL and t the collective thickness of all vertical members.

It may easily be observed, that the shearing stress in these vertical members and that in outside platings are of opposite sign,

and that the algebraic sum of all vertical shearing forces worked out from these shearing stresses is equal to the shearing force in the section due to the load.

Shearing stress in horizontal members, including the outer bottom, would be distributed in a very complicated manner, and its exact calculation would be impossible. But the following method may give some approximation.

In fig. 3, let XYZW be the central part of a double bottom, and t' and t the thickness of the inner and outer bottoms respectively. Then, in order that this part be in equilibrium in fore and aft direction.

$$\frac{\mathrm{F}}{\mathrm{I}} m = 2\tau' t' + 2\tau t$$

Where τ' and τ are shearing stresses at X and Z respectively.

And, in order that the resultant unbalanced force due to the varying bending moment, and the resultant shearing force in the part under consideration be in same straight line (*i.e.* for non-existence of a couple).

$$\frac{\mathrm{F}}{\mathrm{I}} i = 2\tau' t' y' + 2\tau t y$$

Where $i=$ moment of inertia of the area WXYZ about the neutral axis.

y and $y'=$ the distance of Z and X below the neutral axis respectively.

From these two equations, the shearing stresses in the inner and outer bottoms were found.

(3). Experiments with wooden beams.

For the purpose of verifying the method of calculation for bilge platings, I made experiments with two wooden beams. The section of the wooden beams was as shown in fig. 4. These beams

were built of wooden battens which were securely cemented together.

The beams were supported at both ends with a span of 24 inches and loaded at centre. With a load of 1232 lbs. for one and 1210 lbs. for the other, fracture began to appear in top and bottom seams, while there was no sign of weakness in the seams at the middle of depth.

The shearing stresses at the top and bottom seams as calculated under my assumption was 316 lbs./□″, while the same calculated by the ordinary method (taking full sectional area cut off by the horizontal plane through the seams) was 40 lb./□″. The shearing stress in the seams at the neutral axis was 339 lb./□″. Theaefore, if the ordinary method were correct, the beams would have been sheared at the neutral axis. But the experiments showed otherwise. My argument, therefore, is not disproved by the experiment.

(4). General Conclusion.

It is a well known phenomenon, that in not very rare cases, the seams of bilge plating of iron ships as well as of wooden ships are subjected to the severest stress. This has been, so far, attributed to the shearing stress which appears in case of heavy rolling, when the neutral axis might happen to pass through bilge plating. But, according to my experience, the weakness in bilge plating has sometimes been observed even in boats of limited service, which have never been subjected to heavy rolling.

From the stress diagram in fig. 2, it will be seen that the shearing stress in bilge plating does not fall of sensibly. Moreover this plating—especially at the seams—is generally subjected

to corrosion on the inside and to wearing on the outside. These two causes may explain the weakness in bilge plating.

It will be observed that the heavier the double bottom is constructed, the severer will the bilge plating be strained. Therefore, it seems that some distinction for the strength of bilge plating must be made in one way or another, between ships having double bottom and those with ordinary floor.

Also, it will be seen that in the inner bottom and in the main deck plating, the shearing stress is comparatively low. Therefore, double rivetted seams may not be necessary for them. The British Corporation Rules have already given practical attention to this consideration.

Needless to say, these inferences have been arrived at from a pure theoretical point of view. In actual ships, the mode of distribution of stresses would be very complicated and moreover a part of the shearing force would be shared by transverse members. The true quantitative values of shearing stresses can be found only by actual observations in a ship. I desire to emphasize the necessity of actual measurements of strain in ships for the further develpment of naval architecture.

II. Approximate Formula for Maximum Shearing Stress.

In estimating the maximum shearing stress in a plate girder, civil engineers generally calculate it, as if the whole shearing force were uniformly distributed over the web. I apply the same principle for estimating the maximum shearing stress in a ship.

Consider an ideal ship having hollow rectangular section as shown in fig. 5. For such a ship.

$$\frac{m}{\mathrm{I}}=\frac{ay+\frac{1}{2}a'y}{2(ay^2+\frac{1}{3}a'y^2)}$$

where $2a$ = the sectional area of the top or bottom member.

$2a'$ = the sectional area of the vertical member on one side.

Therefore,

$$\frac{\mathrm{m}}{\mathrm{I}}=\frac{a+\frac{1}{2}a'}{2y(a+\frac{1}{3}a')}=\frac{1}{2y}\left(1+\tfrac{1}{2}\frac{a'}{a}\right)\left(1+\tfrac{1}{3}\frac{a'}{a}\right)^{-1}=\mathrm{C}\frac{1}{2y}$$

$\therefore$

$$\text{Max shearing stress}=\frac{\mathrm{F}.m}{2t.\mathrm{I}}=\mathrm{C}\frac{\mathrm{F}}{2y.2t}$$

$$=\mathrm{C}\frac{\text{shearing stress}}{2\times\text{thickness of side plating}\times\text{depth of ship.}}$$

where C is a constant depending upon the ratio $\frac{a'}{a}$

In the case where a' is less than or nearly equal to a (this is the case for ships of ordinary construction), the constant C will be very near to 1. This I have tested by taking the following examples from actual ships, with the results given in the following table.

Ships.	Items.	Max. shearing stress. By exact formula.	Max. shearing stress. By approximate formula with C = 1.	Coefficient C.
Tenyo maru	Transpacific liner	F×0.00156	F×0.00110	1.43
530′×65′×45′.	Transatlantic liner	F×0.00149	F×0.00124	1.20
Niigata Maru.	Cargo steamer	F×0.00397	F×0.00326	1.22
Kushiro Maru.	Mixed steamer......	F×0.00592	F×0.00443	1.34
Kasuga.	Armoured cruiser...	F×0.00228	F×0.001	1.31
Mikasa.	Battle ship	F×0.00144	F×0.00113	1.27
Chitose.	2nd class cruiser ...	F×0.00400	F×0.00237	1.61
Otowa.	3rd class cruiser ...	F×0.00476	F×0.00303	1.57
Uji.	Gunboat...............	F×0.01190	F×0.0103	1.15
Harusame.	T.B. Destroyer......	F×0.02250	F×0.0194	1.16

Thus, for actual ships the constant C varies in a narrow range 1.2–1.6. Therefore, this formula may be used with fair approximation for estimating the maximum shearing stress in a ship.

In concluding this paper, I have to express my thanks to Mr. Takahashi, a student of the Engineering College, who has helped me in calculating the values of the constant C for several ships.

DISTRIBUTION OF SHEARING STRESSES IN A SHIP'S SECTION

279-0"×39'-6"×23'-3" STEEL STEAMER.

Draught	19'-9"
Displacement	4735 tons
Maximum S.F.	590 tons

SCALE

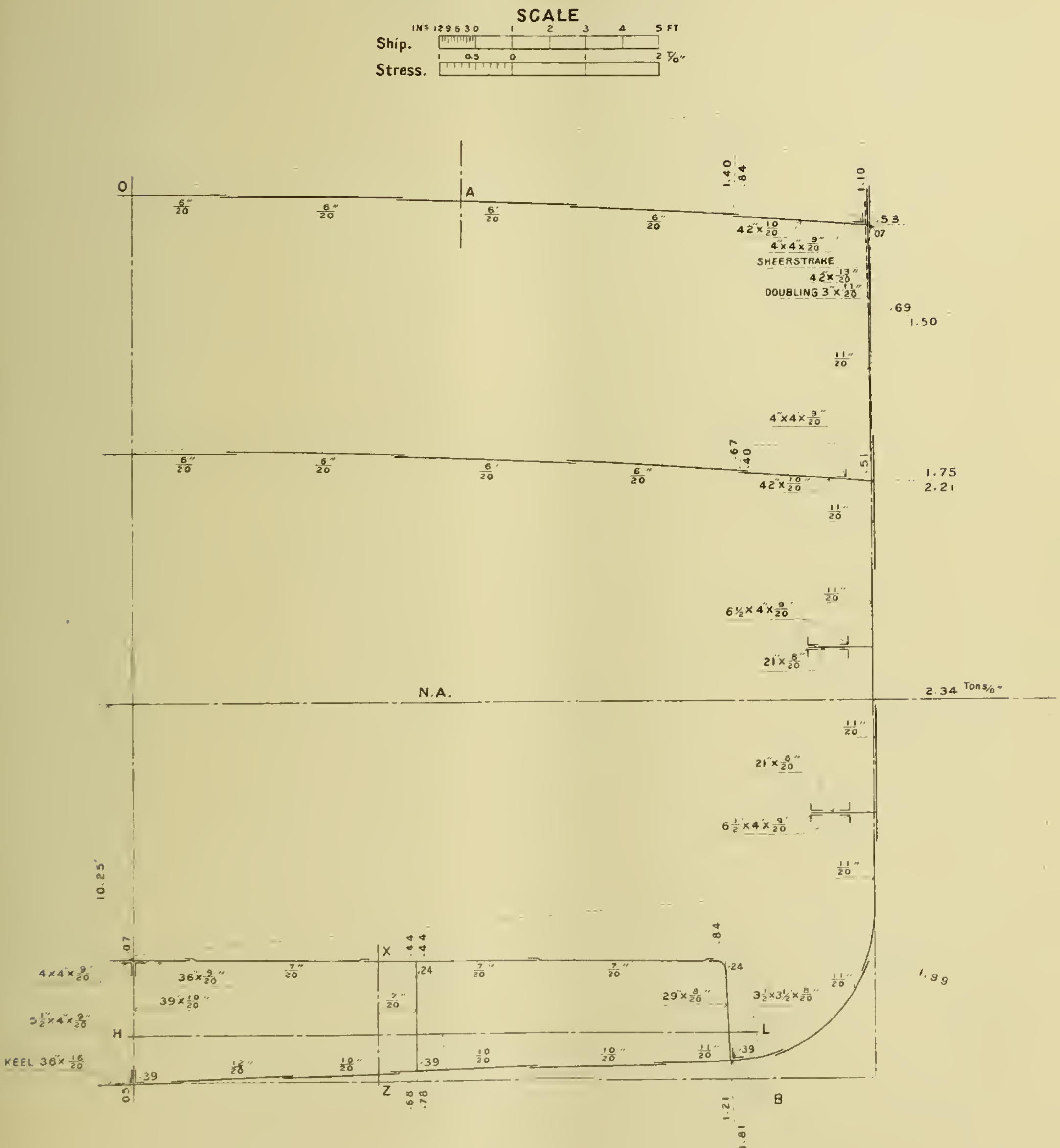


Fig. 2.

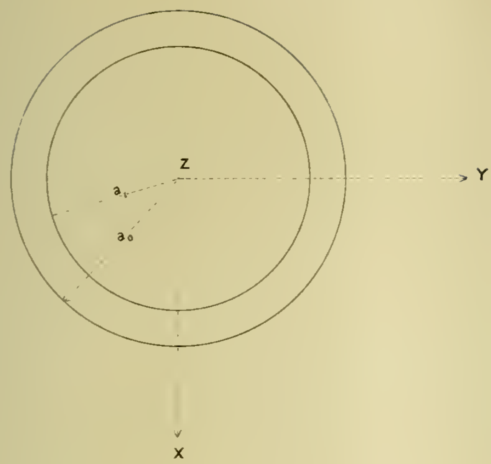


Fig. 1.

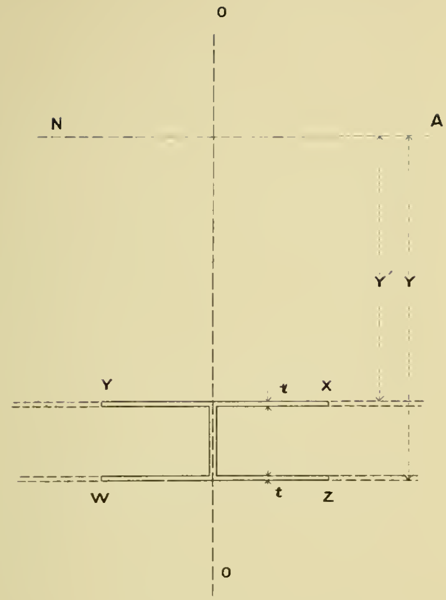


Fig. 3.

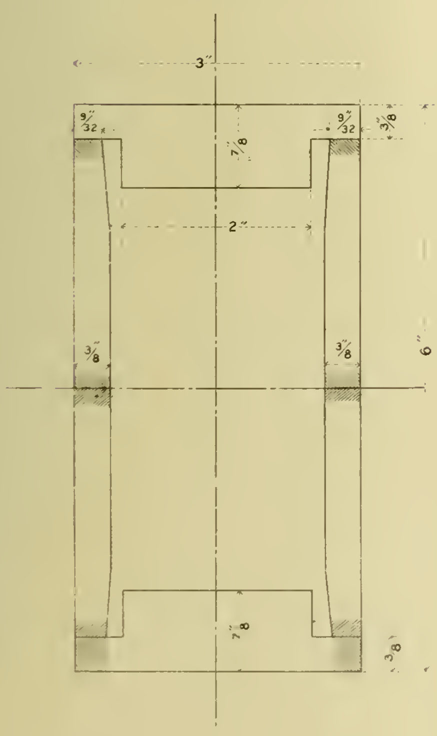


Fig. 4.

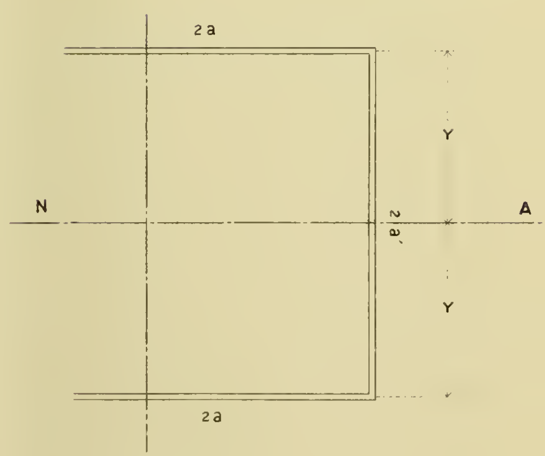


Fig. 5.

On the Fineness of the 800 Standard Silver Bar.

CONTENTS.

		Page.
I.	Introduction.	191.
II.	Fineness of fillet.	192.
III.	Fineness of blank.	196.
IV.	Method of sampling the coin.	197.
V.	Blank and scissel.	199.
VI.	Loss in annealing.	201.
VII.	Conclusions.	202.

APPENDIX.

1.	Ratio of blank to scissel.	204.
2.	Rolling mills.	208.
3.	Internal Fineness of 50-Sen Silver Bar.	211.
	Technical terms of the mint.	212.

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
Vol. V. No. 7.

Published April, 1914.

On the Fineness of the 800 Standard Silver Bar.

By

Osamu Yamagata, *Rigakushi*,

Superintendent of Operative Department, Imperial Mint, Osaka.

I. INTRODUCTION.

The nature of segregation of silver in silver-copper alloys has several times been the subject of useful investigations. In 1890, Dr. Koga and the author* studied the fineness of the One Yen silver coin of 900 standard, and determined a method of taking an assay sample from that coin. This method of sampling has since been adopted for similar coins at certain mints, as at London and Bombay.

Messrs. Milne and Bourke† made similar investigations on the Straits Settlements' Dollar and confirmed our experiments. Their researches were extended to the Rupees of 916.6 standard, cut out of the fillet in two rows, and showed that a circular piece punched from the center of the coin may be considered a sufficient assay sample for practical purpose.

It is the object of the present paper to describe certain investigations on 50-Sen silver bars of 800 standard, which may be of interest in the operation of the mint.

---

* Y. Koga and O. Yamagata, "On the Fineness of One Yen Silver Coin," Memoires of The Science Department, Tokyo University, 1890.

† A Milne, *Lieut. Colonel, I.M.S.* and J. J. Bourke, *Captain, I.M.S.* "The Method of Sampling Silver Coins in Indian Mints," Bombay Mint, 1904.

A brief statement of the method of coining 50-Sen pieces may not be out of place here. About 200 kg. of silver alloy melted in a crucible is ladled out and poured into iron moulds, granulated assay samples being taken from the top and the middle parts of the pot. Two bars are usually cast from one ladle. The top end, measuring 2 to 3 cm., is cut off from the bar, which is then gradually rolled down to the thickness of the coin, being once annealed when it has been reduced to about one half of the original thickness. Blanks are cut out of the fillet in two rows, and are rimmed, annealed and pickled before being sent to the coining press.

TABLE I.

Dimensions of 50-Sen Silver Bar, Fillet and Blank.

	Length.	Breadth or Diameter.	Thickness.
	m.	mm.	mm.
Bar	0.584	49.20	11.88
Fillet	3.480	56.60	1.73
Blank	—	27.27	1.73

II. FINENESS OF FILLET.

Two bars *A* and *B* cast out of one ladle, at the middle part of 185 kg. pot, weighed 3.50 and 3.41 kg. respectively, They were rolled down to the thickness of the coin, without cutting off their top ends.

The two fillets were cut at top-end, top, middle and bottom into seven equal strips, *a* to *g*, the strips *a* and *g* being subdivided into halves. The results of assays of all these strips are given in TABLE II, the pot assays being 798.4 and 798.5 and the ladle assay 798.6.

Table II.

Fineness of Fillets.

		Top End.	Top.	Mid.		Bot.
		←15 cm.→	←171 cm.→	←161 cm.→	←10 cm.→	
Fillet A	*a*	797.4	796.5 797.6	797.1 797.6	798.0	797.6 798.2
	b	800.1	797.2	797.6	798.1	797.9
	c	801.2	799.0	798.7	798.9	797.8
	d	800.8	800.0	800.5	799.1	797.8
	e	799.9	800.4	798.9	799.2	797.6
	f	799.2	798.6	797.8	798.5	797.6
	g	797.7	797.7 796.3	798.0 797.7	798.0	798.1 797.6
Average		799.5	798.5	798.4	798.5	797.8

		←15 cm.→	←171 cm.→	←171 cm.→		
Fillet B	*a*	799.2	797.0 797.6	797.4 797.2		797.5 797.9
	b	801.7	799.8	797.6		797.6
	c	804.0	800.8	798.5		797.4
	d	803.4	801.1	801.5		797.5
	e	801.8	799.6	799.8		797.8
	f	800.1	797.9	797.6		797.9
	g	797.2	797.6 796.6	798.1 797.3		797.7 797.3
Average		801.1	799.1	798.6		797.7

The fillet *A* was further assayed in the same way, at nine other equidistant parts, and the results together with those given in the above table are shown by Fig. 1, (Plate I.) which is drawn on the

supposition that the fillet was restored to the original form of the bar. From this figure, the curves of fineness of the bar may be drawn, as shown in Fig. 2, Plate I.

Further experiments were made on the five bars, *C* to *G*, from different parts of different charges of the pot. Pot and ladle assays for these bars are given in table III.

Table III.

Pot and Ladle Assays.

Bars	*C*	*D*	*E*	*F*	*G*
Pot Assay	798.8	799.0	799.0	799.0	798.7
Ladle Assay	798.9	798.9	799.1	799.0	799.3

These bars, with the top ends cut off, were rolled in the usual way; and assays were made on nine equal strips taken from top, middle and bottom of the finished fillets, with the results shown in table IV.

Table IV.

Fineness of Fillet at Top, Middle and Bottom.

		Top.	Mid.	Bot.
C First Bar from the First Charge.	*a*	797.4	797.7	798.1
	b	798.5	798.3	798.4
	c	799.2	799.1	798.4
	d	800.3	800.4	798.3
	e	801.0	801.9	798.5
	f	800.8	799.6	798.5
	g	799.6	798.6	798.3
	h	798.2	798.3	798.5
	i	797.4	798.0	798.1
	Average	799.2	799.1	798.3

D	*a*	797.6	798.1	797.6
First Bar from the Second Charge.	*b*	798.1	797.8	798.2
	c	799.3	799.0	798.0
	d	800.4	801.9	798.1
	e	801.2	802.3	798.0
	f	801.0	800.3	798.0
	g	799.7	798.7	798.0
	h	798.0	798.1	798.0
	i	797.5	797.7	798.0
	Average	799.2	799.3	798.0

E	*a*	797.4	797.3	798.3
Bar from Middle Part of the Second Charge.	*b*	798.1	797.7	798.5
	c	799.0	798.9	798.2
	d	800.0	802.2	797.8
	e	800.6	802.2	798.4
	f	800.0	799.5	798.4
	g	799.5	798.1	798.4
	h	798.2	798.1	798.6
	i	797.3	797.3	797.9
	Average	798.9	799.0	798.3

F	*a*	797.8	797.7	797.7
Last Bar from the Second Charge.	*b*	798.6	798.2	799.3
	c	800.1	798.6	799.5
	d	801.3	804.7	799.5
	e	801.9	802.1	799.3
	f	801.0	802.2	799.2
	g	799.2	799.0	799.1
	h	798.1	797.7	799.6
	i	797.3	797.9	799.0
	Average	799.5	799.8	799.1

G *Last Bar from the Third Charge.*	*a*	798.2	797.8	798.6
	b	799.1	798.4	799.3
	c	801.0	798.8	798.8
	d	801.8	801.6	798.8
	e	801.0	803.4	798.8
	f	800.1	799.9	799.2
	g	799.2	798.7	798,7
	h	797.8	798.6	799.0
	i	797.3	798.1	798.4
	Average	799.5	799.5	798.8

III. FINENESS OF BLANK.

The fineness of the blanks cut out of the three fillets, *C*, *D* and *E*, was calculated by Milne and Bourke's method, that is, by adding the products of the area of blank segments into the fineness of the respective zones. Area of a segment $= \frac{r^2}{2}\left(\frac{\theta\pi}{180} - \sin\theta\right)$, where $\theta =$ number of degrees of the angle subtended by the arc of the segment at the center of the circle with radius r. When $r = 1$. the area of the segments in Fig. 3 will be as shown in Table V.

Table V.

Area of Segments of Blank.

Segments.	Area.	Proportion.
a or *i*	0.295	0.09
b or *h*	0.795	0.25
c or *g*	0.917	0.29
d or *f*	0.812	0.26
e	0.323	0.11
πr^2	3.142	1.00

Fig. 3.

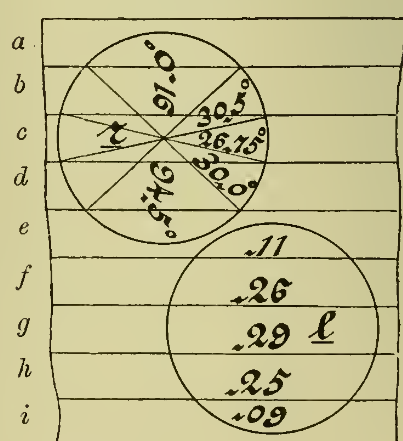


TABLE VI.

Calculation of Fineness of Blanks R & L from Top Part of the Fillet C.

Blank *R*.			Blank *L*.	
a	797.4 × .09 = 71.8		*e*	801.0 × .11 = 88.1
b	798.5 × .25 = 199.6		*f*	800.8 × .26 = 208.2
c	799.2 × .29 = 231.8		*g*	799.6 × .29 = 231.9
d	800.3 × .26 = 208.1		*h*	798.2 × .25 = 119.5
e	801.0 × .11 = 88.1		*i*	797.4 × .09 = 71.8
	Total 799.4			Total 799.5

It may seem rather rough to take the fineness of the zone directly for that of the segment. But the assumption was thought sufficient for practical purpose, considering that the deficiency in segments *d* and *e* may be supplied by the surplus in *b* and *a*. The slight difference of thickness of a fillet at its different parts, treated in Appendix 2 (p. 210), is neglected here and in the following calculations.

Fineness of the blanks cut from top and middle parts of the fillets, *C*, *D* and *E*, calculated in the same way, is given in Table VIII. (p. 199)

IV. METHOD OF SAMPLING THE COIN.

Our method of sampling One Yen silver coin has also been applied, at this Mint, to 50–Sen silver coin. The sample consists of three equal circular punchings taken from the angles of an equilateral triangle described in a concentric circle of five sevenths of the coin's diameter.

To confirm this method, the fineness of the top and middle parts of the fillet *C* are represented by steps in Fig. 4, (plate II) from which the curves of fineness were drawn as shown by thick broken lines.

The approximate fineness of a circular punching may be found by drawing a longitudinal line from its center to the curve of fineness. By taking the average fineness of three punchings in four sets, *a*, *b*, *c* and *d*, the fineness of the blanks, *R* and *L*, at top and middle of the fillet, is obtained as follows :

TABLE VII.

Fineness of Samples.

Blank.	Punchings.	Top.	Middle.
R	a′ or d‴	798.2	798.0
	a″ or d″	800.4	800.6
	a‴ or d′	799.5	799.5
	Average	799.4	799.4
	b′ or c‴	798.5	798.2
	b″ or c″	800.7	801.2
	b‴ or c′	798.9	798.7
	Average	799.4	799.4
L	a′	801.0	800.6
	a″	798.4	798.3
	a‴	798.4	798.3
	Average	799.3	799.1
	b′ or d‴	800.8	800.2
	b″ or d″	797.9	798.2
	b‴ or d′	799.6	798.6
	Average	799.4	799.0
	c′	800.7	799.2
	c″	797.7	798.2
	c‴	800.7	799.2
	Average	799.7	798.9

Average fineness of the blanks from the fillets, *D* and *E*, was computed in the same way, and all these results, compared with

the calculated total fineness, as well as with the fineness of the center of the blank, are shown in Table VIII.

Table VIII.

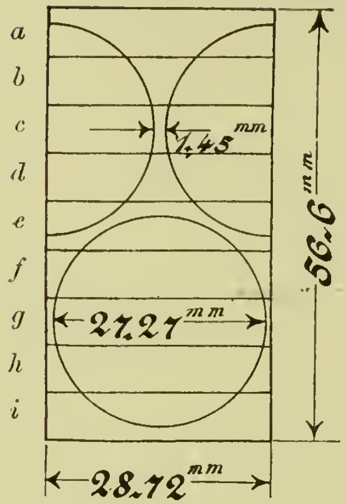
Fineness of the Sample, Calculated Total Fineness and Fineness of Blank Center.

Fillet.	Blank.		Calculated Total Fineness.	Sample Fineness.				Fineness at Center of Blank.
				a	b	c	d	
C	Top	*R*	799.4	799.4	799.4	799.4	799.4	799.2
		L	799.5	799.3	799.4	799.7	799.4	799.6
	Mid.	*R*	799.4	799.4	799.4	799.4	799.4	799.1
		L	799.1	799.1	799.0	798.9	799.0	798.6
D	Top	*R*	799.3	799.1	799.0	799.0	799.1	799.0
		L	799.6	799.1	799.2	799.1	799.2	799.5
	Mid.	*R*	799.7	799.8	799.4	799.4	799.8	798.9
		L	799.3	799.1	799.1	798.9	798.9	798.2
E	Top	*R*	799.1	799.4	799.2	799.2	799.4	799.3
		L	799.2	799.1	798.9	799.8	798.9	799.7
	Mid.	*R*	799.7	800.0	799.7	799.7	800.0	799.0
		L	798.8	799.3	799.3	799.1	799.3	798.7

V. BLANK AND SCISSEL.

The ratio of the blank to the scissel in a 50–Sen fillet is shown in Fig. 5, where the breadth of fillet is 56.6 mm, the diameter of blank is 27.27 mm., and the smallest margin left between the adjacent blanks is 1.45 mm.

Fig. 5.



Area of scissel is obtained by subtracting the area of the two blanks from the area of this piece of the fillet, and area of scissel in a strip by subtracting the area of segment of the blank in the strip from the area of that strip.

TABLE IX.

Area of Blank and Scissel.

Denomination.	Area in sq. mm.
Fillet	$56.6\,(27.27+1.45) = 1625.5$
Two Blanks	$2\pi\,(13.635)^2 = 1168.2$
Scissel	$1625.5-1168.2 = 457.3$
Strip	$1625.5 \div 9 = 180.6$

Now taking the average of fineness of top and middle of the three fillets, *C*, *D* and *E*, at each of the nine strips, we get an ideal fillet. From this fineness and the proportional area of blank and scissel in each strip, the fineness of the blank and scissel was calculated, with results shown in Table X, area of segment of the blank having been calculated from the proportional area of Fig. 3. (p. 196)

TABLE X.

Calculation of Fineness of Blank and Scissel.

Strip.	Area in sq. mm.		Proportion of Area.		Fineness of Ideal Fillet.	Proportion of Fineness.	
	Blank.	Scissel.	Blk.	Scl.		Blank.	Scissel.
a	64.2	116.4	.039	.072	797.6	31.11	57.43
b	151.9	28.7	.093	.018	798.1	74.22	14.37
c	169.4	11.2	.105	.006	799.1	83.91	4.79
d	146.0	34.6	.090	.021	800.8	72.07	16.82
e	105.2	75.4	.065	.047	801.5	52.10	37.67
f	146.0	34.6	.090	.021	800.2	72.02	16.80
g	169.4	11.2	.105	.006	799.0	83.89	4.79
h	151.9	28.7	.093	.018	798.1	74.22	14.36
i	64.2	116.4	.039	.072	797.5	31.10	57.42
Total	1168.2	457.2	.719	.281	7191.9	574.63	224.46
					÷9	÷.719	÷.281
Average Fineness					799.1	799.2	798.5

VI. LOSS IN ANNEALING.

A certain quantity of 50–Sen blanks taken from daily products were cleaned from oil and dirt, then annealed in a gas furnace and pickled in hot dilute sulphuric acid, in the usual way. Average loss of weight in cleaning was 0.25, and in annealing and pickling, 0.50 per mil. The latter loss may be regarded as consisting of copper only, which was oxidised in fire and dissolved by acid, disregarding an insignificant quantity of the oxygen in copper oxide previously formed in the annealing of the bar. By this surface decomposition, the blank is enriched in silver by 0.4 per mil, thus,

$$\frac{800}{1000-0.5}=\frac{800.4}{1000}.$$

VII. CONCLUSIONS.

1. The fineness of a bar is highest near its central longitudinal line and gradually lowers towards both sides. Tables II & IV.

2. Top end of a bar is considerably rich and irregular in fineness. At least 2 cm. of the top should be cut off from the bar. Table II and Fig. 1.

3. Bottom of a bar is considerably poor, but regular in fineness. A small portion of the bottom should be better cut off from the finished fillet. Table II & IV.

4. Average fineness at all the remaining parts of the fillet is practically uniform, and conforms also to the pot assay. Fig. 1, and Table II, III and IV, *C, D & E*.

5. The fineness of the bar cast from the very bottom of a pot is irregular and considerably higher than the pot assay. Table IV, *F & G*.

6. Our method of sampling a coin by three punchings, as applied to 50–Sen silver, gives better results than the Indian method of sampling by a central punching. Table VIII. Moreover, when repetition of assay is required, the second or the third sample can be readily taken by our method, while by the Indian method, it is necessary to roll out the annulus, clip it into fine bits and take the mean of duplicate assays of the fragments.

7. Average fineness at different stages of 50–Sen coinage is as follows. Tables III & X.

Pot Assay.	Fillet.	Scissel.	Blank.	Coin.
798.9	799.1	798.5	799.2	799.6

Assays of silver of the above experiments were made by Gay Lussac's volumetric method using Stas pipette, in the usual way. These assays were mostly done by Messrs. B. Imada and K.

Kishimoto, *Assistant Assayers*, under supervision of Dr. Y. Koga, *Superintendent of Assay Department* ; to all them are due the author's hearty thanks.

APPENDIX.

1. On the Ratio of Blank to Scissel.

The proportion of coins turned out of the bar depends chiefly upon the ratio of the blank to the scissel. Blanks are cut out of the fillet in one or more rows, as shown by Figs. 6, 7 & 8.

Fig. 6.

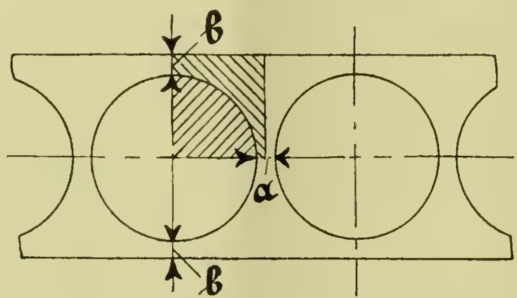


Fig. 7.

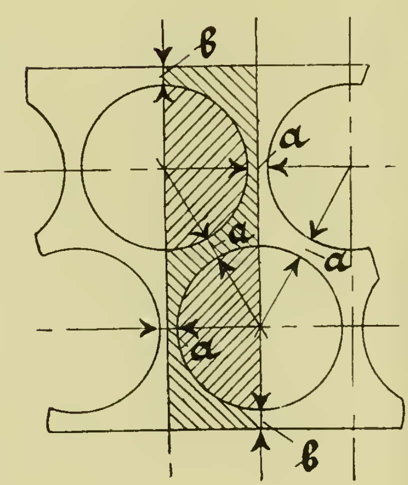
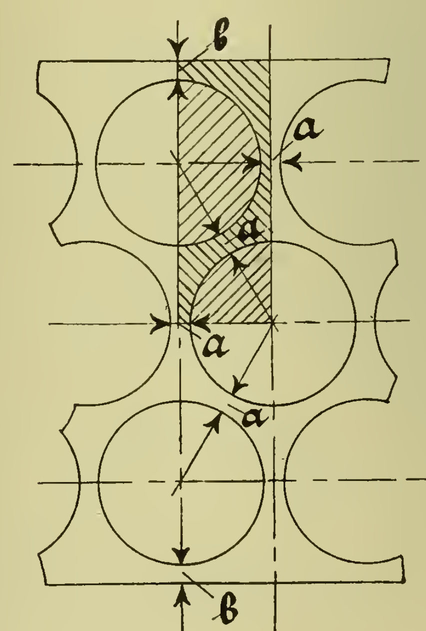


Fig. 8.



Let r=radius of the blank,

a=shortest margin left between the adjacent blanks, and

b=shortest margin left between the blank and the fillet edge.

The ratios of blank to scissel are :

for one row, Fig. 6,

$$\frac{\pi r^2}{4} : \left(r+\frac{a}{2}\right)\left(r+b\right)-\frac{\pi r^2}{4},$$

for two rows, Fig. 7,

$$\pi r^2 : \left(r+\frac{a}{2}\right)\left[2r+2b+\sqrt{\left(2r+a\right)^2-\left(r+\frac{a}{2}\right)^2}\right]-\pi r^2$$

and for three rows, Fig. 8,

$$\frac{3\pi r^2}{4} : \left(r+\frac{a}{2}\right)\left[r+b+\sqrt{\left(2r+a\right)^2-\left(r+\frac{a}{2}\right)^2}\right]-\frac{3\pi r^2}{4}.$$

Suppose $r=1$, and a & $b=0$, the percentage of blank and scissel will be as follows :

	One Row.	Two Rows.	Three Rows.
Blank	78.54	84.18	86.24
Scissel	21.46	15.82	13.76

From the above figures, it is evident that the proportion of the out-put of blanks is increased by : (1) increasing the diameter of the blank without changing the margins a and b, (2) increasing the number of rows of blanks cut in a fillet, and (3) diminishing the margins, a and b.

Silver blanks, larger than about 35 mm. in diameter, should be cut in one row, in order to keep the fineness within the legal remedy. Even gold bars, when enlarged in width, have the homogeneity of fineness impaired. The author has declined to cut 20-Yen gold blanks of 28.8 mm. diameter in two rows, having found that the range of fineness of the coins, above or below the

standard, was considerably enlarged, as compared with those cut in one row, although the fineness never exceeded the legal limit.

20–Sen silver blanks of 20.3 mm. diameter, 800 fine, are cut in two rows. The legal remedy of fineness being 3 per mil, they can not be cut in three rows, because the central row will be too rich in silver, as shown in Table II & IV. 10–Sen silver blanks of 17.6 mm. diameter, 720 fine, are cut in three rows, although the legal remedy of fineness is also 3 per mil, because these bars are very homogeneous, the composition being very near to Levol's alloy.

The bars of bronze or nickel bronze, of which legal remedy of fineness is usually neglected, may have their breadth extended as far as the capacity of the rolling mill allows, and the blanks may be cut in as many rows as are suitable for the punching press. 5–Sen nickel bronze and 1–Sen bronze are cut in three rows, and 5–Rin bronze in four rows.

The margins a and b can be minimized by using an accurate cutting machine, upon bars of excellent quality. Some bars containing certain impurities break at their sides, resulting in fillets with serrated edges. This is often the case with 5–Sen and 10–Sen bars, and sometimes with 900 gold and 800 silver. These bars are usually made extra-wide, in order to provide for occasional occurrence of the serrations. One of the most effective methods to overcome this difficulty, without changing the quality of the alloy, is to change the construction of the mould. The improved mould consists of two grooved pieces, united together as shown by Fig. 10, instead of one grooved piece lapping the plane face of the other, as Fig. 9 of the older type.

Transversal Section of 50–Sen Mould.

Fig. 9.
Old Type.

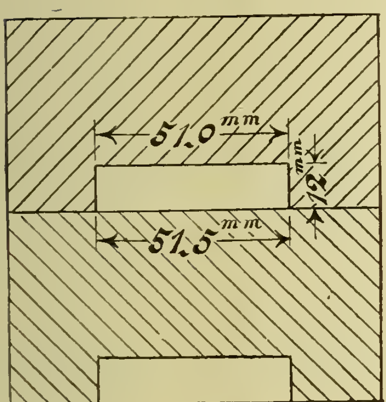
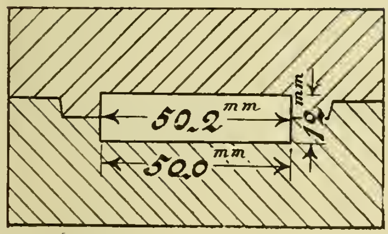


Fig. 10.
New Type.



When the new type of mould was first applied to the 5–Sen nickel, in 1890, non-brittleness of the bar was attributed to non-access of air to the edges while casting, and consequent less oxidation and less sudden cooling of these parts. It was next applied to the gold bar, in 1897, with the object of minimizing its breadth, and the resulting fillet edges were smoother than the old bars. In 1910 the new type was applied to the 10–Sen, whereby the edge breakage so often met with in the fillet has been almost entirely avoided. 50–Sen mould has been lately altered in the same way, slightly reducing its breadth, as shown in Figs. 9 & 10.

The groove of a mould is usually made slightly obtuse-angled, in order to facilitate the removal of the bar from it, hence the transversal section of the old bar is somewhat truncated-triangular. Both sides, of such a bar will suffer less rolling pressure and consequently undergo less elongation, than its main body, resulting in a zigzag edged fillet. Similar results have been often observed in bars, imperfectly filed at edges. Such irregularity of

parts is greatly reduced in bars cast in the new mould, and this seems to be the chief cause of non-breakage of the edge.

2. On Rolling Mills.

One of the most important pieces of machinery in the mint is the rolling mill, upon the accuracy of whose working chiefly depends the production of coins of good weight. The rolling mill now used in this mint comprises the following kinds, all individually driven by electric motors.

TABLE XI.

Rolling Mill of the Mint.

Mills.		Rolls.				Motors.	
No.	Kind.	Housing.	Diam.	Width.	Rev. P. M.	H. P.	Rev. P. M.
			mm.	mm.			
Gold Room.							
1	Break-down	Single	254.	317.	40.	50.	850.
1	Middling	,,	254.	216.	45.	40.	550.
1	,,	,,	254.	317.	45.	15.	550.
4	Finishing	,,	165.	182.	60–75.	5.	850.
Silver Room.							
1	Break-down	Double	355.	322.	30.	50	800.
1	Middling	,,	254.	317.	40.	30.	1000.
1	,,	,,	254.	317.	54.	10.	700.
3	Finishing	Single	165.	182.	62.	5.	850.
Bronze Room.							
1	Break-down	Double	355.	322.	28.	80.	700.
1	Middling	,,	355.	322.	30.	50.	800.
1	Finishing	,,	355	322.	40.	30.	1000.
1	,,	Single	254.	387.	64.	15.	600.

Rolls of the gold and silver finishing mills are of hardened steel, while all the rest are of chilled cast iron. In these finishing mills, one roller of the pair is geared to the motor, while the other is left idle, revolving by the passage of the fillet.

The electric motor of 220 volt direct current, for each of the above rolling mills, as designed by Prof. E. Aoyagi of the Imperial University, Kyoto, is of cumulative compound wound type with a heavy fly-wheel on its shaft, so constructed that the combination of the compounding and the fly-wheel energy acts as a constant current regulator, confining the current fluctuation and the speed drop within a certain range, against the load fluctuations of the mill. In the 80 and 50 H. P. bronze mills, a second fly-wheel is attached to the intermediate shaft, to provide for violent fluctuations of load.

The break-down and middling mills of the Gold Room are of a new construction. They are coupled to the motors by double reduction gears, with tooth wheels, as in the United States Mints. Steam engines for the silver and bronze mills have been recently replaced by electric motors, the double reduction being made with double-helical wheels and Renold chain wheels (Fig. 17, Plate VI), as in the Royal Mint, London. Other kinds of reduction gears have also been applied to the finishing mills of 165 mm. steel rolls; such as single reduction by worm wheel, and double reductions by Renold chain wheels and by belt wheels. In each case no special difference has been observed in the resulting fillet, as well as in the power consumption, provided the machinery is of equal quality.

The dimensions of a 50–Sen fillet, after each passage between different rolls, as measured by Mr. S. Yamada, *Accountant, Rolling Room,* are given in Table XII; the original bar, weighing 3403

grams, and the finished fillet, 3401 grams, are illustrated by Figs. 11 and 12 (Plate III), respectively.

TABLE XII.

Dimensions of a 50–Sen Fillet.

Roll.		No. of Passage.	Dimensions after each Passage.					
			Length.		Breadth.		Thickness.	
No.	Diam.		Actual.	Dif.	Actual.	Dif.	Actual.	Dif.
	mm.		mm.	mm.	mm.	mm.	mm.	mm.
Original Bar.			620.4		49.80		12.16	
1.a	355.	1.	685.0	64.6	50.60	.80	10.81	1.35
		2.	774.0	89.0	51.40	.80	9.46	1.35
		3.	830.0	56.0	52.00	.60	8.22	1.24
		4.	982.0	152.0	52.50	.50	6.66	1.56
		5.	1236.0	254.0	53.40	.90	5.22	1.44
Anneald.								
1.b	355.	6.	1382.0	146.0	53.50	.10	4.65	.57
		7.	1629.0	247.0	53.90	.40	3.93	.72
		8.	1971.0	342.0	54.40	.50	3.24	.69
		9.	2545.0	574.0	55.00	.60	2.49	.75
2.a	254.	10.	2883.0	338.0	55.04	.04	2.19	.30
2.b	254.	11.	3194.0	311.0	55.08	.04	1.97	.22
3.a	254.	12.	3358.0	164.0	55.11	.03	1.87	.10
3.b	254.	13.	3495.0	137.0	55.14	.03	1.80	.07
		14.	3587.0	92.0	55.16	.02	1.74	.06
4.	165.	15.	3665.0	78.0	55.18	.02	1.71	.03
5.	165.	16.	3692.0	27.0	55.20	.02	1.69	.02

A slight depression is usually found in the longitudinal middle parts of a bar, cast in a mould with plane faces, as in Fig. 11. Such depression is obliterated by passing the bar between the break-down rolls. By the succeeding passages, the fillet is made

slightly thicker at the longitudinal middle parts, gradually thinning towards both sides ; but the average of thickness of any transversal section of the finished fillet is uniform throughout the whole length, except 2 or 3 cm. at both ends, where the thickness decreases towards the extremities, as shown in Fig. 12.

3. Internal Fineness of 50-Sen Silver Bar.

To study the condition of segregation of silver in the interior of a coinage bar, a portion of 50-Sen silver bar was carefully cut cross-wise from its middle part. This sample piece weighed 70.5 grams and was of the dimensions as shown in Fig. 13, Plate IV. This piece, with its top upwards, was placed between two plane blocks of hardened steel, and was given a number of heavy blows by means of a screw press, being annealed several times during the process, in a covered crucible filled with bone-black, to prevent oxidation of its surface. The oval flat piece thus obtained was cut into 45 parts as shown in Fig. 14, in which the results of assays made, at Assay Department, on all these parts are also shown. From these results curves of fineness were deduced as in Fig. 15, which shows approximately the segregation of silver in a cross section of 50-Sen silver bar. (See Fig. 2, Plate I).

General average of the above assays is 799.46 against 799.1, the ladle assay of the bar ; this difference may be due to the low fineness of the remaining cuttings not assayed, which amounted to nearly one third of the whole sample. It is also noticeable that the higher fineness predominates near the one side of the bar ; this may be due to the higher temperature of the mould on that side, and consequent slower solidification of the alloy.

It will be seen from the above results (Fig. 14 & 15) that it is

almost impossible to determine the fineness of 800 standard coinage bar by the cutting assay-pieces. For this reason assay is usually made on a sample dipped out of the thoroughly mixed molten alloy (before being cast into the bar) and granulated by pouring into water.

TECHNICAL TERMS USED AT THE MINT.

Bar. Long rectangular piece of coinage alloy, cast in a mould.
Syn. *Ingot* in U. S. A.

Fillet. The bar after having been passed between the rolls.
Syn. *Strip* in U. S. A.

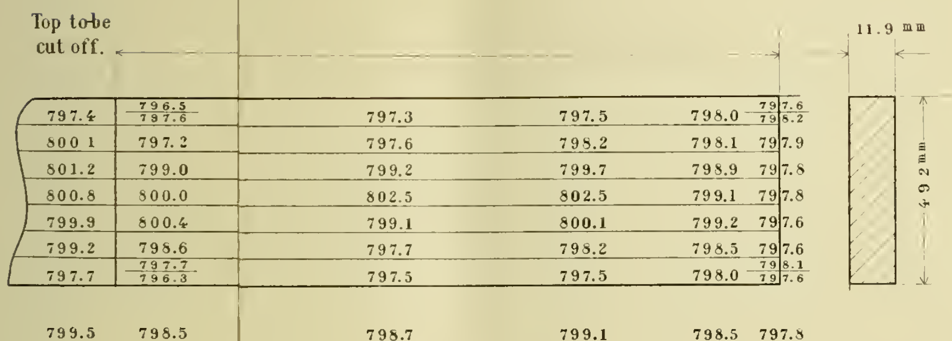
Blank. Circular disc cut out from the fillet.
Syn. *Planchet* in U. S. A.

Scissel. The remaining part of the fillet after the blanks have been cut out.
Syn. *Clippings* in U. S. A.

The Imperial Mint, Osaka.
September, 1913.

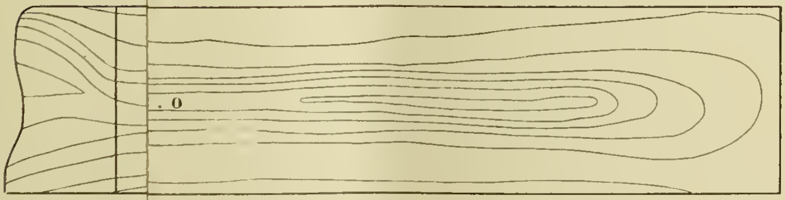
Plate I.

A.



Top to be cut off.
11.9 mm
49 2 mm
797.4 796.5/797.6 797.3 797.5 798.0 797.6/798.2
800 1 797.2 797.6 798.2 798.1 797.9
801.2 799.0 799.2 799.7 798.9 797.8
800.8 800.0 802.5 802.5 799.1 797.8
799.9 800.4 799.1 800.1 799.2 797.6
799.2 798.6 797.7 798.2 798.5 797.6
797.7 797.7/796.3 797.5 797.5 798.0 798.1/797.6
799.5 798.5 798.7 799.1 798.5 797.8

Bar.



. 0

Fig. 1. Fineness of 50-Sen Silver Bar, A.

Top to be cut off.

568 mm 11.9 mm 49.2 mm

797.4	796.5 797.6	797.3	797.1	796.9	797.4	797.7	797.1 797.6	797.6	797.2	797.4	797.1	797.5	798.0	797.6 798.2
800.1	797.2	798.0	798.2	797.7	797.5	798.0	797.6	797.9	798.0	797.5	797.6	798.2	798.1	797.9
801.2	799.0	799.6	799.9	798.1	797.9	798.5	798.7	798.8	799.1	798.9	799.2	799.7	798.9	797.8
800.8	800.0	801.1	800.5	800.5	799.9	801.2	800.5	801.6	801.6	801.6	802.5	802.5	799.1	797.8
799.9	800.4	799.2	798.4	801.1	799.8	800.8	798.9	798.7	799.2	799.2	799.1	800.1	799.2	797.6
799.2	798.6	797.8	797.7	797.9	798.1	798.2	797.8	798.2	797.8	798.1	797.7	798.2	798.5	797.6
797.7	797.7 796.3	796.7	797.1	797.3	797.2	797.4	798.0 797.7	797.6	797.5	797.5	797.5	797.5	798.0	798.1 797.6
799.3	798.5	798.5	798.4	798.5	798.3	798.8	798.4	798.6	798.6	798.6	798.7	799.1	798.5	797.8

Fig. 2. Curve of Fineness of the above Bar.

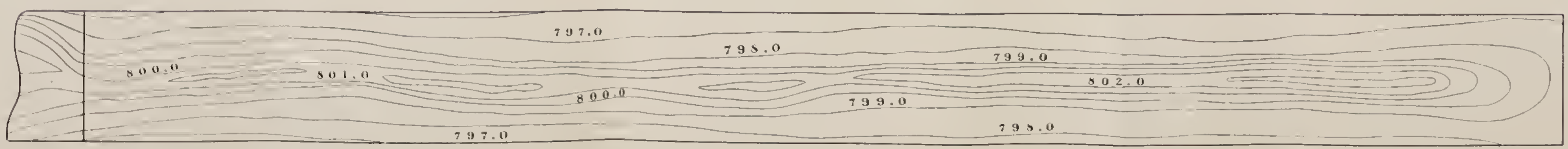


Fig. 4. Curve of Fineness of the Fillet, *C*.

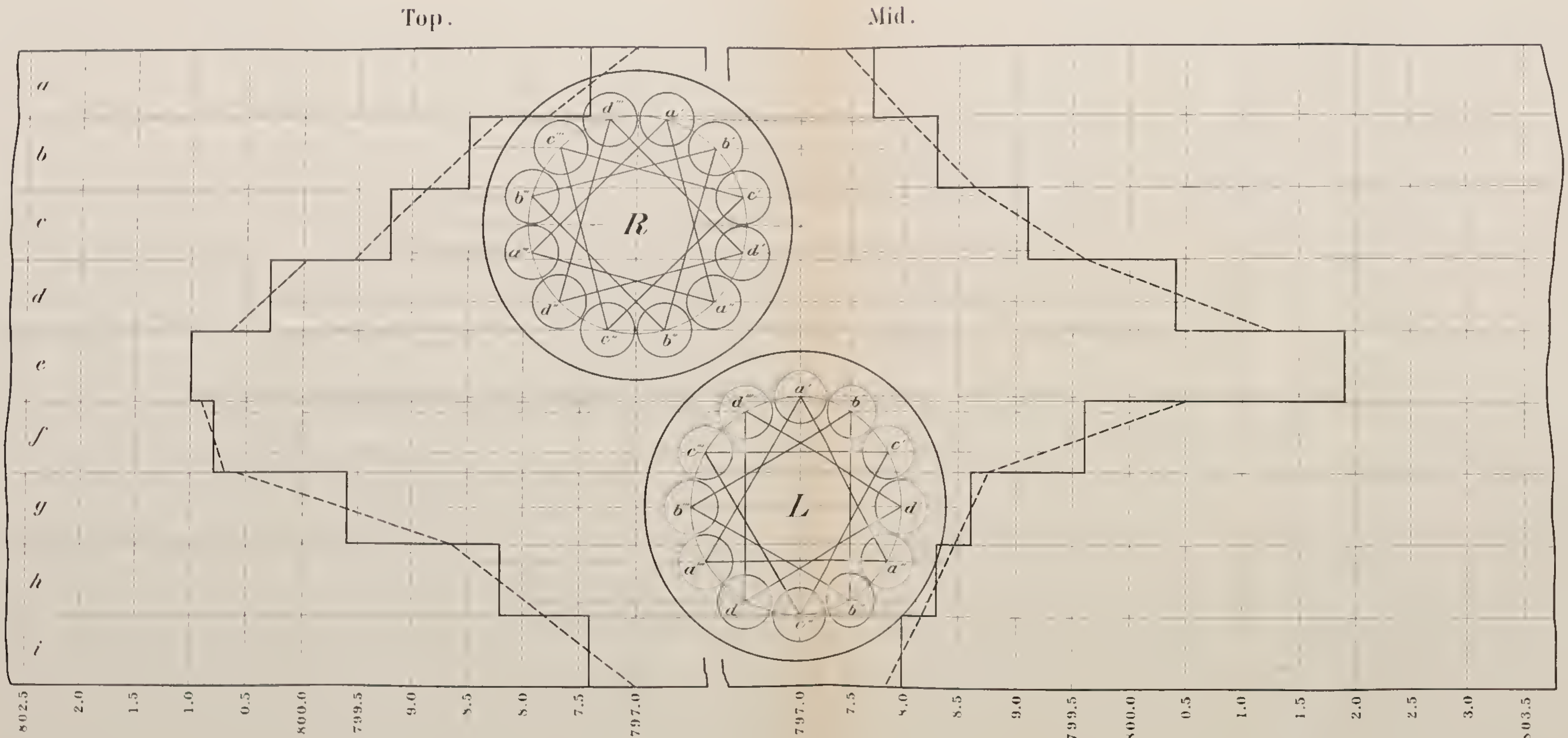


Fig. 11. 50 Sen Silver Bar.

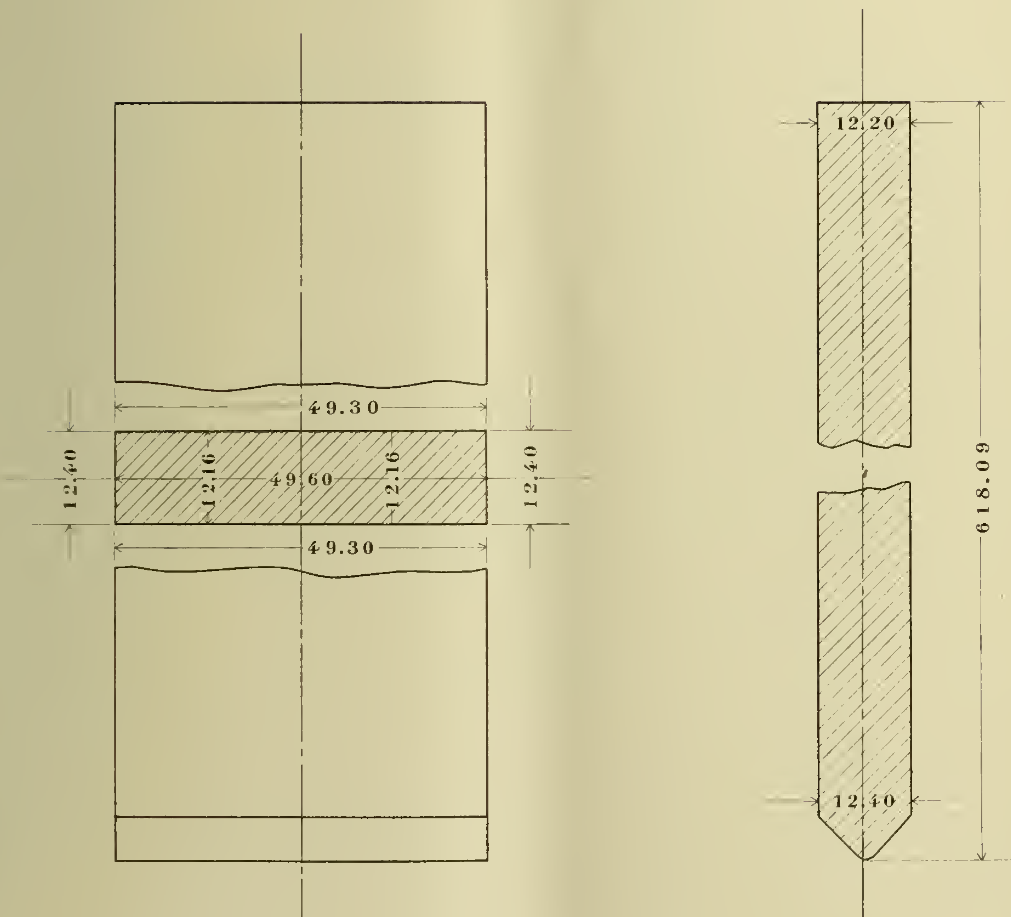


Fig. 12. Finished Fillet of the above Bar.

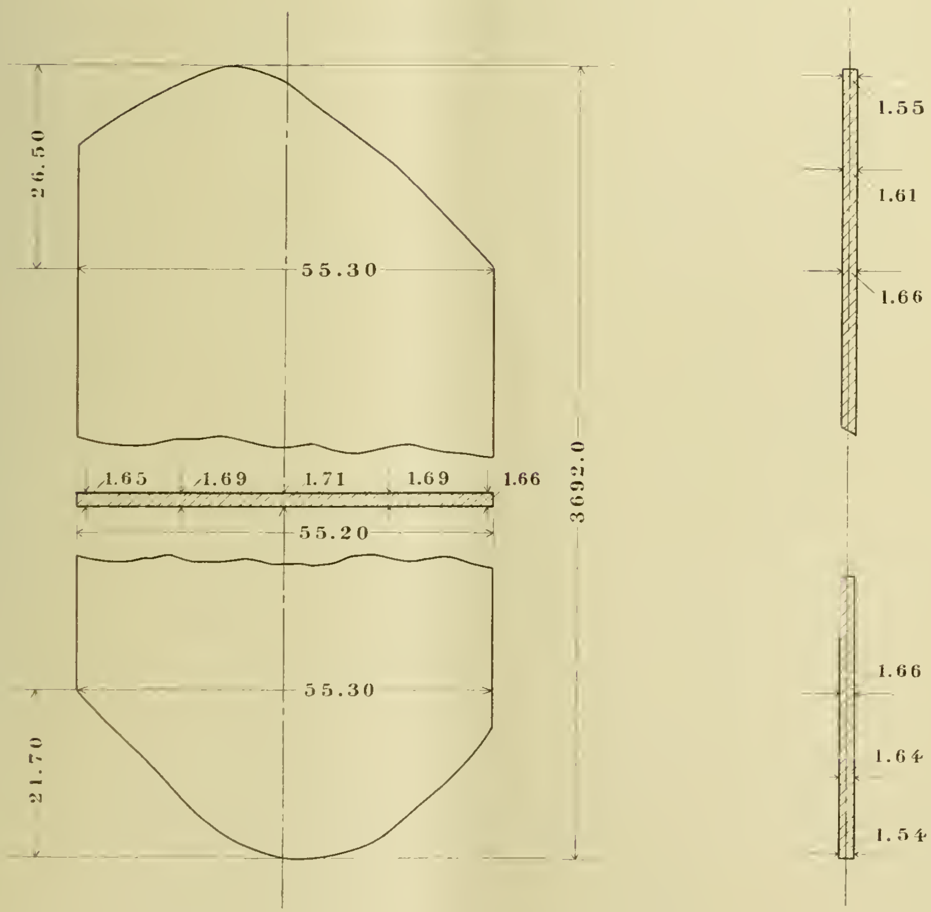


Fig. 13. Sample Piece cut cross-wise from 50-Sen Silver Bar.

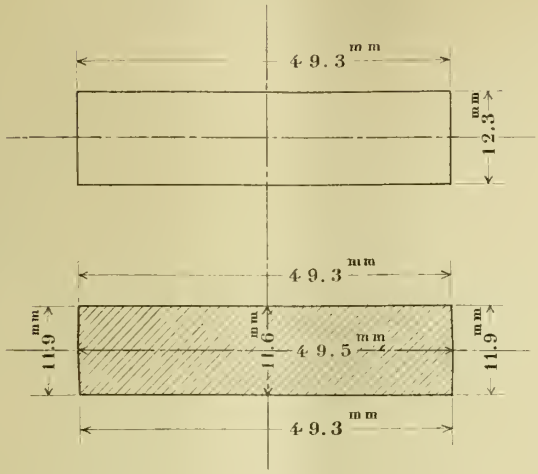
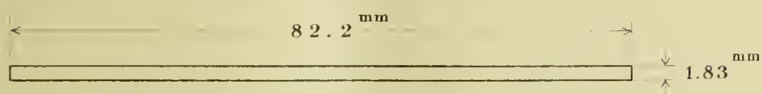


Fig. 14. The above Piece flattened and assayed at different parts.



57.6 mm

								Average.
		796.7	798.8	801.5	801.2	797.6		799.2
	798.7	798.9	800.3	803.6	801.3	799.3	797.3	799.9
	798.3	799.8	801.0	803.8	801.5	799.7	798.2	800.3
	797.5	799.7	800.5	803.3	801.3	800.1	798.2	800.1
	798.3	799.1	800.1	802.2	801.1	799.8	798.2	799.8
	796.9	798.6	799.0	800.9	800.5	799.4	797.4	799.0
		796.9	797.4	798.5	796.8	796.8		797.3
Average.	797.9	798.5	799.6	802.0	800.5	799.0	797.9	

Fig. 15. Section of 50-Sen Silver Bar. showing segregation of silver.

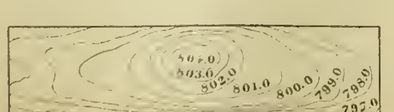


Fig. 16. GOLD & SILVER MELTING ROOM. Imperial Mint, Osaka.
Showing Gas Furnaces, Moulds, Bars, &c.

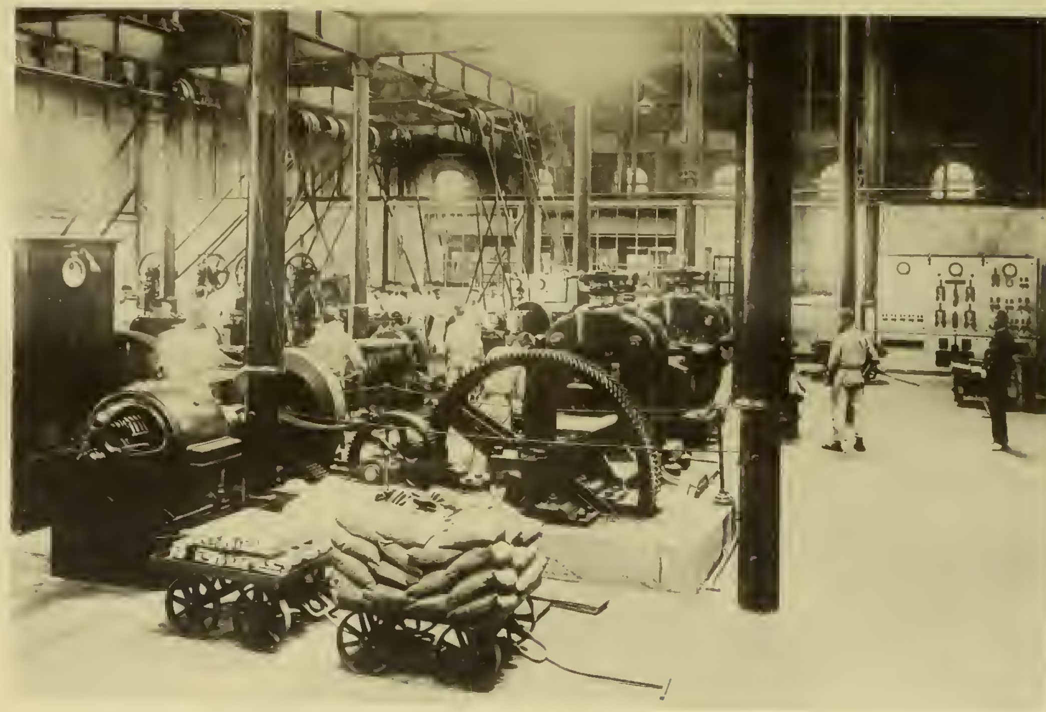


Fig. 17. SILVER ROLLING ROOM, Imperial Mint, Osaka. Showing Break-down Mill, with Reduction Gears uncovered.

THE RELATION BETWEEN THE HORSE POWER AND THE WEIGHT OF AN ENGINE.

COLUMN OF UNIFORM STRENGTH.

ERRATA.

Page.	Line.	for	read
209	20	VI	VII
210	2	III	III & IV
211	10	IV	V

Kogakuhakushi.

THE RELATION BETWEEN THE HORSE POWER AND THE WEIGHT OF AN ENGINE.

COLUMN OF UNIFORM STRENGTH.

BY

Prof. A. Inokuty, M. E., *Kogakuhakushi.*

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
Vol. V. No. 8.

Published June, 1914.

The Relation Between the Horse Power and the Weight of an Engine.

By

Prof. A. Inokuty, *M. E., Kogakuhakushi.*

The stress at a point in a section of a machine working at a steady speed is made up of two parts; one part is the stress due to the useful working force transmitted through the section and the other that due to the accelerating force arising from the reciprocation or rotation of the mass of machine parts. The power exerted at any instant by a reciprocating engine is equal to the product of the total useful stress transmitted through a cross section of the working part and the velocity of that section. Divide the mean value of the power in ft. lbs. per sec. for one complete working cycle of the engine by 550 ft. lbs. per sec. ; and the quotient is the so-called horse power of the engine. The accelerating force due to the reciprocation varies conjointly as the mass of the moving part, the linear dimension of the reciprocation or the stroke, and the square of the number of revolutions per unit time. Since the sum of the useful stress and the acceleration stress must necessarily be limited by the safe stress of the material, there exists for a high speed engine of a given type a certain definite speed of rotation for which the horse power is a maximum. The weight of an engine evidently varies as the cube of linear dimension; and it may be shown that the weight per horse power of a high speed engine

increases as the linear dimension increases. These and like points may be discussed in the following manner.

Let

P = the useful load transmitted through a cross section of the working part of the engine, as for example the total initial pressure on the piston, transmitted through the piston rod, connecting rod, and crank.

n = the number of revolutions per unit time.

p = the stress due to P on a particular member such as the piston rod.

p_0 = the stress due to the accelerating force on the same member.

l = the length of the same member or of any other member; for an engine, l may conveniently be taken as the stroke of the piston.

A = the area of the section, at a point of which the stresses p and p_0 act in the same direction.

ρ = the density of the material of which the engine is made; or weight of material per unit of volume.

The stress due to the acceleration or retardation of the reciprocating parts is expressed by

$$p_0 = \text{const.} \times \frac{l \times \text{mass} \times n^2}{\mathrm{A}}$$

But the weight or mass of the engine may be expressed by

$$\begin{aligned} \mathrm{M} &= \text{const.}\,\rho l \mathrm{A} \\ &= c\rho l^3 \quad \text{......................................(1)} \end{aligned}$$

where c is a constant depending on the type of the engine. Hence

$$\begin{aligned} p_0 &= \text{canst.}\,\rho n^2 l^2 \\ &= k_0 \rho n^2 l^2 \quad \text{......................................(2)} \end{aligned}$$

where k_0 is a constant depending on the mode of distribution of the mass of the engine parts. It will be the smaller the better the

design of the engine. Since $n\,l$ is proportional to the linear velocity, the above equation shows that the stress due to acceleration varies as the square of the piston speed. The same relation also holds for the stress due to centrifugal force in a uniformly rotating part such as the rim of a flywheel, it being expressed by $p_0=\rho v^2$, where v is the linear velocity of the rim.

The stress due to the useful load on the engine is given by

$$p=k\frac{\mathrm{P}}{l^2}, \qquad \text{.............................}(3)$$

where k is another constant depending on the mode of action of the load P and also on the form of the engine parts.

Now the safe stress of the material of which the engine is made is given by

$$f=p+p_0=k\frac{\mathrm{P}}{l^2}+k_0\rho n^2l^2 \qquad \text{...........}(4)$$

so that

$$\mathrm{P}=\frac{1}{k}(fl^2-k_0\rho n^2l^4)\,; \qquad \text{.....................}(5)$$

but the horse power of the engine is

$$\mathrm{HP}=\text{const}\ nl.\ \mathrm{P}. \qquad \text{.............................}(6)$$

Therefore

$$\mathrm{HP}=\mathrm{C}(fnl^3-k_0\rho n^3l^5). \qquad \text{.....................}(7)$$

Now considering the speed of rotation and the size of the engine to be quantities independent of each other, the expression above for the horse power becomes maximum in two different cases.

Case 1. Considering engines of different sizes, all running at a given number of revolutions per min., the horse power becomes maximum when

$$\frac{d\mathrm{HP}}{dl}=\mathrm{C}(3fnl^2-5k_0\rho n^3l^4)=0,$$

that is, when

$$\tfrac{3}{5}f=k_0\rho n^2l^2=p_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The corresponding maximum power is

$$max.\,\mathrm{HP}=\tfrac{2}{5}\mathrm{C}\,fnl^3. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Since nl is proportional to the linear velocity we may write $nl=k'v$, where k' is a constant; and the above becomes

$$max.\,\mathrm{HP}=\tfrac{2}{5}\mathrm{C}\,k'fl^2v \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

From (8) it is seen that for maximum power at a given speed of rotation, the engine should be so designed that the stress due to the accelerating force shall be 3/5th of the working stress of the material, the remaining 2/5th only being that due to the useful working load. Eliminating by means of (8) the speed of rotation n from the equation (9), we obtain

$$max.\,\mathrm{HP}=\tfrac{2}{5}\left(\tfrac{3}{5}k_0\right)^{\frac{1}{2}}\mathrm{C}\,\frac{f^{1\frac{1}{2}}l^2}{\rho^{\frac{1}{2}}}, \qquad \ldots\ldots\ldots(11)$$

which is of the same form as the equation arrived at by Mr. Lanchester by the principle of dimension equation.

Case 2. Considering engines of the same dimensions running at different speeds, that which develops the maximum power is given by

$$\frac{d\mathrm{HP}}{dn}=\mathrm{C}(fl^3-3k_0\rho n^2l^5)=0,$$

that is, by

$$\tfrac{1}{3}f=k_0\rho n^2l^2=p_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

The corresponding maximum power is

$$max.\,\mathrm{HP}=\tfrac{2}{3}\mathrm{C}\,fnl^3, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

which differs from (9) only in the coefficient. In this case the

engine should be so designed that the inertia stress shall be 1/3rd of the safe working stress, the remaining 2/3rds being that due to the useful stress. Eliminating by means of (12) the speed of rotation n from the equation (13), we obtain

$$max.\ \mathrm{HP}=\tfrac{2}{3}(\tfrac{1}{3}k_0)^{\frac{1}{2}}\mathrm{C}\frac{f^{1\frac{1}{2}}l^2}{\rho^{\frac{1}{2}}}, \qquad \text{............(14)}$$

a result of the same form as the equation (11) obtained in the Case 1. The ratio of the maximum horse power given by (13) to that given by (9) is $1\frac{2}{3}$, on the supposition that the engines in the two cases are exactly similar. For in the two cases the intensities of useful stresses are limited to $\frac{2}{5}$ and $\frac{2}{3}$ of the safe working stress of the material.

As a problem occurring in actual practice it is however better to state the above result in the following modified form. Suppose that the same amount of horse power is developed by each of two high speed engines, one being designed for maximum horse power at a given speed of rotation and the other for maximum horse power with a given size, and that both engines have the same principal dimensions such as the diameter of cylinder and the stroke of the piston. Then since the total useful stress transmitted through a corresponding cross section must be the same in the two engines, we must have

$$\tfrac{2}{5}f\mathrm{A}_1=\tfrac{2}{3}f\mathrm{A}_2,$$

where A_1 and A_2 are the values of A for the two engines. Hence

$$\mathrm{A}_1=1\tfrac{2}{3}\mathrm{A}_2.$$

and this will be true for all the working parts through which the useful stress is transmitted. Thus the first engine is $1\frac{2}{3}$ times as heavy as the second engine. Dividing (1) by (7), we obtain the mass or weight per horse power,

$$\frac{M}{HP}=\frac{c\rho}{Cn(f-k_0\rho n^2l^2)} \qquad \text{..................(15)}$$

This becomes a minimum when

$$\tfrac{1}{3}f=k_0n^2l^2,$$

which is just the same as equation (12). Thus the condition of maximum horse power expressed by (12) satisfies also the condition of minimum weight per horse power. Combining (15) with (12), we obtain

$$\frac{M}{HP}=\frac{3c}{2C}(3k_0)^{\frac{1}{2}}l\left(\frac{\rho}{f}\right)^{1\frac{1}{2}}. \qquad \text{..............(16)}$$

Hence supposing that the engines are so designed, and worked at such a speed, that the stress due to the accelerating force is $\frac{1}{3}$rd of the working stress of the material, the weight per horse power will be proportional to the linear measurement; in other words, the horse power developed per lb. weight of engines of similar type decreases in the inverse proportion of linear dimension. The larger the engine, the greater is the total horse power, in the duplicate proportion as shown by (11) or (14); but at the same time, the smaller becomes the horse power per lb. weight. Thus arises the following problem. It is required to obtain a certain amount of total horse power with the least weight of the engine. By (14) it is seen that an engine having four equal cylinders can develop just the same horse power as a single cylinder engine of double the linear measurement. But by (1) the weight of the four cylinder engine is given by

$$M=4\times c\rho(\tfrac{1}{2}l)^3=\tfrac{1}{2}c\rho l^3,$$

that is, one half the weight of the single cylinder engine. In general a single cylinder engine and a similar engine having N

equal cylinders each $\frac{1}{\sqrt{N}}$th in linear measurement can develop the same amout of horse power. This follows at once from equation (14); for,

$$max.\ \mathrm{HP}=\mathrm{const.}\times \mathrm{N}\left(\frac{l}{\mathrm{N}^{\frac{1}{2}}}\right)^2=\mathrm{const.}\times l^2.$$

But as regards the total weight of the second engine, we have by (1)

$$\mathrm{M}=\mathrm{N}c\rho\left(\frac{l}{\sqrt{\mathrm{N}}}\right)^3=\frac{1}{\sqrt{\mathrm{N}}}(c\rho l^3), \qquad \ldots(17)$$

so that *the weight decreases in the inverse proportion of the square root of the number of cylinders.* This is a broad governing principle relating to the weight of an engine of a given power. Against it there is however a set back that the efficiency of an engine decreases as the size of the cylinder becomes smaller. In aeronautical engines the weight of the fuel to be carried is thus increased, so that the multiplication of cylinders will be limited by this factor.

Now to examine the materials used for the working parts of an engine. Let f_1 and ρ_1 be the tensile strength and the density of one kind of material, and f_2 and ρ_2 the corresponding quantities for another kind of material. So far as the horse power and weight of an engine are concerned, it is seen from (14) that these materials will be equally good, if

$$f_1^{1\frac{1}{2}}\rho_1^{-\frac{1}{2}}=f_2^{1\frac{1}{2}}\rho_2^{-\frac{1}{2}}$$

that is, if

$$f_2=f_1\left(\frac{\rho_2}{\rho_1}\right)^{\frac{1}{3}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

For example, commercial rolled aluminium for which ρ_2=about .096 lb. per cubic inch may be compared with miled steel of average quality used for the construction of steam boliers. For

the latter metal ρ_1=about 0.283 1b. per cubic inch and average tensile strength, f_1=about 28 tons per square inch. Hence the rolled aluminium would be as good as the boiler steel, if

$$f_2=28\left(\frac{.096}{.283}\right)^{\frac{1}{3}}$$
$$=19.5 \text{ tons per sq. inch.}$$

Rolled aluminium commercially pure has a tensile strength ranging from about $6\frac{1}{2}$ to $10\frac{1}{2}$ tons per square inch, while the hard rolled plates of aluminum alloy prepared by Mr. Yarrow with the addition of 6 per cent. copper and adopted for the construction of a torpedo boat gave 14 to 16 tons per square inch tensile strength. These are not sufficiently high to equal the boiler steel. An alloy of aluminium termed "Duraluminium," invented recently and manufactured by the Electric and Ordnance Accessories Co. of Birmingham is said to contain upwards of 90 per cent of aluminium, and to have a specific gravity of about 2.8 and a melting point of 650° C. The makers state that the tensile strength of the alloy is as high as 40 tons per square inch when the metal is hard and has little elongation; 28 to 30 tons per square inch with 15 per cent. elongation in 2 inch gauge length; and 25 tons per square inch with 20 per cent. elongation in 2 inch length. Specific gravity of 2.8 is equivalent to .101 lb. per cubic inch, so that Duraluminium will be better than the boiler steel, if its tensile strength is higher than

$$f_2=28\left(\frac{.101}{.283}\right)^{\frac{1}{3}}=19.9 \text{ tons per sq. inch,}$$

hence this metal in its weakest and most ductile state is much superier to boiler steel, so far as the horse power and weight of the engine are concerned.

It may be added here that in the adoption of a new material for the construction of an engine or structure, the resistance of it to shearing would probably be another very important quality demanding our careful examination.

Column of Uniform Strength.

By

Prof. A. Inokuty, *M. E., Kogakuhakushi.*

A long straight column bearing an axial compressive load undergoes a lateral deflection when that load exceeds a certain value, and the deflected axis assumes a sine curve if the column is homogeneous and of uniform section and the stress nowhere exceeds the elastic limit. The bending stress thus induced in the material of the column varies from one section to another along the length, being greatest at a section furthest from the original straight axis and zero at a point of inflection. Now the lateral dimensions of a long straight column can be so varied as to make the greatest bending stress for a section constant for the whole length. Such a column, being analogous to a beam of so-called uniform strength, may be termed a column of uniform strength. Formulae for the strength of columns of this form can be obtained theoretically by considering the bending action alone and the result is almost the same as by Euler's formula, the only difference being in the value of the coefficients. For columns of moderate length such as occur in practice, it is necessary to consider the direct compressive stress as well as the bending stress due to the deflection and for this purpose the writer has obtained a set of semi-empirical formulae in the form of Gordon-Rankine formula. These formulae might be useful in designing large columns of approximately uniform strength, as for example shear legs, connecting rods for large engines, &c.

Referring to Fig. I, let a straight bar of great length be acted on by axial forces P, P, tending to compress the bar along the

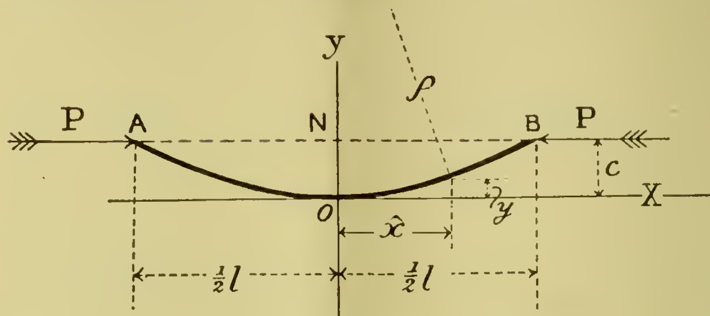


Fig. I.

original straight axis ANB, and let it be bent into a curve AOB, O being the centre of the length and ON=c the maximum deflection at the centre. Since this deflection c is generally a very small quantity compared with the length of the ber l it is accurate enough to assume NA and NB each equal to half the length. Take the tangent at O to the curve of deflection for the axis of x and the normal at O for that of y. At any section indicated by $(x, y,)$ the bending moment is

$$\mathrm{M}=(c-y)\,\mathrm{P}. \quad \text{..............................(1)}$$

The relation between the bending moment and the curvature of the bar is

$$\frac{1}{\rho}=\frac{d^2y}{dx^2}=\frac{(c-y)\,\mathrm{P}}{\mathrm{EI}} \quad \text{......................(2)}$$

and the equation for the strength is

$$(c-y)\,\mathrm{P}=f\frac{\mathrm{I}}{y'}, \quad \text{....................................(3)}$$

in which y' is the distance of the part of the bar furthest from the neutral axis of the section on the tension or compression side, ρ the radius of curvature at the point (x, y), and I the geometrical moment of inertia of the section about its neutral axis perpendicular to the plane of bending. Combining (2) and (3) we obtain

$$\frac{1}{\rho}=\frac{d^2y}{dx^2}=\frac{f}{\mathrm{E}y'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Case 1. *A long straight column of uniform strength, rounded at both ends, having a rectangular section with the depth of side, parallel to the plane of bending, of constant dimension.*

Fig. 2 represents the front and side view of the column of length l under the action of compressive load P, the width of the section at a distance x from the centre being b and the constant depth h as shown in the Fig. Putting $y'=\frac{1}{2}h$ in (4), we obtain

$$\frac{1}{\rho}=\frac{2f}{\mathrm{E}h}.$$

The greatest bending stress f is to remain constant for any section and therefore it is seen that the column bends under the load P into an arc of a circle of radius ρ given by the above relation. If I_0 is the value of I at the centre, we have from (2)

$$\mathrm{P}=\frac{\mathrm{EI}_0}{c\rho}.$$

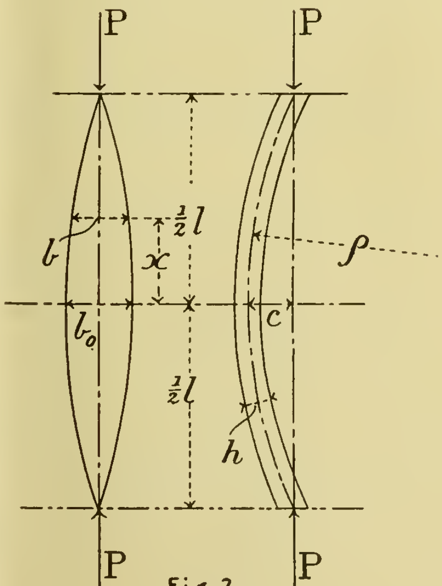


Fig. 2.

Now from a well known property of a circle we have the following relation between a chord and the corresponding versed sine,

$$c(2\rho-c)=(\tfrac{1}{2}l)^2.$$

The deflection c at the centre is a very small quantity and may be neglected in comparison with the radius ρ; we have then

$$c\rho=\tfrac{1}{8}l^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Substituting this in the expression for P above given, we obtain

$$P=8\frac{EI_0}{l^2}, \qquad \text{..............................} (A_1)$$

which is of the same form as Euler's formula for a long column of uniform section rounded at both ends; the only difference is in the coefficient, which in the present result is 8, while in Euler's it is π^2. It may be remarked that in a long column of uniform section if a load P satisfies the condition of equilibrium expressed by Euler's formula, any deflected position is a position of equilibrium under the same load P and consequently the deflection c is an indeterminate quantity. The same thing happens also in the case of a column of uniform strength. Now to find the variable width b of the column, the well known property of a circle above cited may again be made use of; thus

$$(2\rho-y)y=x^2,$$

in which y may safely be neglected in comparison with 2ρ. Then

$$y=\frac{x^2}{2\rho}.$$

Eliminating ρ by means of (5) we have

$$y=\frac{4cx^2}{l^2}.$$

Now from (3) we have

$$(c-y)P=\tfrac{1}{6}fbh^2$$

and

$$cP=\tfrac{1}{6}fb_0h^2.$$

Dividing one by the other,

$$\frac{b}{b_0}=1-\frac{y}{c},$$

which by (6) becomes

$$\frac{b}{b_0}=1-\left(\frac{x}{\frac{1}{2}l}\right)^2. \qquad \text{............................} (7)$$

This is a parabola with its axis at right angles to the length of the column and passing through the centre. Each of two parabolic

arcs forming the outline of the column may be approximated by two straight lines symmetrically sloping towards the ends of the column and touching the parabolic arc. Of all such sets of straight lines, that set which requires the least amount of material for the column is the following:—Set off a width equal to $1\frac{1}{4}b_0$ at the centre and a width equal to $\frac{1}{4}b_0$ at each end of the column. Join the ends of these widths by straight lines, which will touch the parabolic arcs at $x=\frac{1}{4}l$, the width at that section being equal to $\frac{3}{4}b_0$.

The result of this Case I for columns of rectangular section may be easily extended to columns of any section, such as hollow rectangle, symmetrical double tee section, cruciform section, &c. The constant dimension h must then be regarded as the depth of side, parallel to the plane of bending, of a circumscribing rectangle.

Case 2. *A long straight column of uniform strength, rounded at both ends, the dimension of the section perpendicular to the plane of bending being constant.*

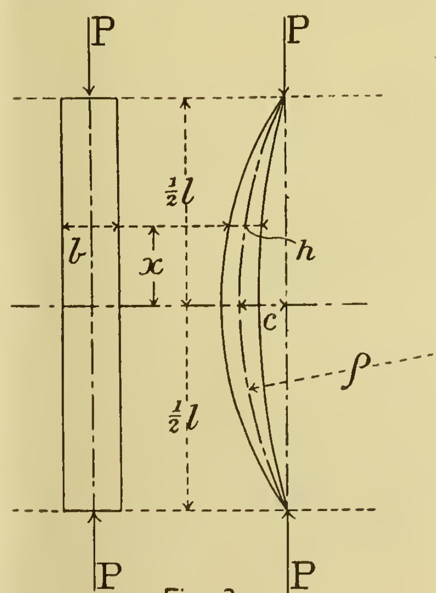


Fig. 3.

Referring to Fig. 3, the width b of a section is now constant while the depth h is variable, so that the depth at the centre is distinguished by h_0. The variable moment of inertia of a section at distance x can be written as follows:—

$$I=\frac{h^3}{h_0^3}I_0.$$

By substitution in (2) we obtain

$$\frac{d^2y}{dx^2}=\frac{(c-y)P}{EI_0}\left(\frac{h_0}{h}\right)^3.$$

On making x and y equal to zero simultaneously, this becomes

$$\frac{1}{\rho_0}=\frac{c\mathrm{P}}{\mathrm{EI}_0}. \qquad\qquad (8)$$

in which ρ_0 is the radius of curvature of the elastic curve assumed by the axis of the column, at the centre of the length. The above equation for the second differential coefficient of y may then be written in the form

$$\frac{d^2y}{dx^2}=\frac{c-y}{c\rho_0}\left(\frac{h_0}{h}\right)^3. \qquad\qquad (9)$$

Substituting the value of I in (3) we have

$$(c-y)\mathrm{P}=\frac{2f\mathrm{I}_0}{h_0}\left(\frac{h}{h_0}\right)^2.$$

On making x and y equal to zero simultaneously, this becomes

$$c\mathrm{P}=\frac{2f\mathrm{I}_0}{h_0},$$

which being substituted in the above equation gives

$$\frac{c}{c-y}=\left(\frac{h_0}{h}\right)^2. \qquad\qquad (10)$$

Combining (9) and (10) we obtain

$$\frac{d^2y}{dx^2}=\frac{1}{\rho_0}\,\frac{c^{\frac{1}{2}}}{(c-y)^{\frac{1}{2}}}.$$

Integrating and noting that dy/dx is zero at the origin, we have

$$\left(\frac{dy}{dx}\right)^2=\frac{4c^{\frac{1}{2}}}{\rho_0}\left[c^{\frac{1}{2}}-(c-y)^{\frac{1}{2}}\right].$$

Integrating again we obtain

$$\frac{2}{3}\left[c^{\frac{1}{2}}-(c-y)^{\frac{1}{2}}\right]^{\frac{1}{2}}\left[2c^{\frac{1}{2}}+(c-y)^{\frac{1}{2}}\right]=\frac{c^{\frac{1}{4}}}{\rho_0^{\frac{1}{2}}}x. \qquad\qquad (11)$$

This is the equation of the elastic curve assumed by the axis of the column in Case II. Now c is the value of y when x is equal to $\frac{1}{2}l$. Substituting these values in (11), there is obtained an equation of condition that the column may deflect under a load P and keep an equilibrium in a curved form expressed by (11),

$$c\rho_0=\frac{9}{64}l^2 \qquad\qquad (12)$$

Fliminating $c\rho_0$ between this and (8), there results

$$P = \frac{64}{9}\frac{EI_0}{l^2}, \quad \text{.............................}(A_2)$$

which is again of the same form as Euler's formula. The coefficient $\frac{64}{9}$ is equal to 0·721 times π^2, so that the strength of a column having the form stated in Case II is 72 per cent of the strength of a calumn of uniform section and of the same material, the form and the area of the section at the centre being the same.

Eliminate y and ρ_0 from (11) by means of (10) and (12); we then obtain

$$\frac{x}{\frac{1}{2}l} = \tfrac{1}{2}\left(1-\frac{h}{h_0}\right)^{\frac{1}{2}}\left(2+\frac{h}{h_0}\right). \quad \text{...........}(13)$$

This is the equation of the curve which the outline of the column must have in order that it may be of uniform strength. The form of this curve is somewhat like a blunt ellipse; it can be drawn easily by setting off a few values of $x/\frac{1}{2}l$ and the corresponding values of h/h_0

$\frac{h}{h_0}$	0.95	0.90	0.75	0.50	0.25
$\frac{x}{\frac{1}{2}l}$	0.330	0.459	0.688	0.884	0.974

The curved outline of the column may be approximated by two pairs of straight lines touching the curve and sloping towards the ends of the column symmetrically about the cross section at the centre. Of all trangents that can be drawn thus, those which require the least amount of material for the column are the following:— At the centre of the length set off a width equal to $1.143\,h_0$ and at each end set off a width equal to $0.616h_0$. Straight lines joining the ends of these widths form the tangents sought. Those touch the curve (13) at a distance $\frac{1}{4}l$ from the centre and the value of h at that section is given by the equation

$$h^3+3h_0h^2-3h_0^3=0,$$

which by solution gives $h=0.879h_0$. The widths above given have been calculated by

$$(0.8794+\tfrac{1}{2}m)h_0$$

and $$(0.8794-\tfrac{1}{2}m)h_0,$$

in whith $m=\dfrac{4\times(1-0{\cdot}8794)^{\frac{1}{2}}}{3\times0.8794}\,h_0.$

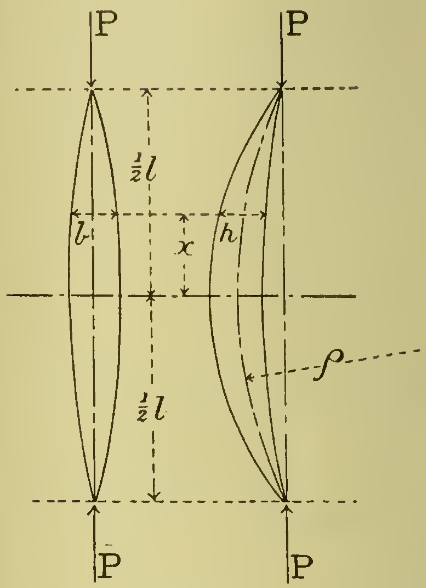


Fig. 4.

Case 3. *A long straight column of uniform strength, with both ends rounded, the cross sections being similar figures.*

The variable moment of inertia of a section distant x from the centre may be written

$$I=\left(\frac{h}{h_0}\right)^4 I_0.$$

Substituting this in (2) we have

$$\frac{d^2y}{dx^2}=\frac{(c-y)P}{EI_0}\left(\frac{h_0}{h}\right)^4.$$

When $x=0$, $y=0$ and $h=h_0$ and therefore

$$\frac{1}{\rho_0}=\frac{cP}{EI_0}. \qquad \text{.................................(14)}$$

Hence we obtain

$$\frac{d^2y}{dx^2}=\frac{c-y}{c\rho_0}\left(\frac{h_0}{h}\right)^4. \qquad \text{........................(15)}$$

Substituting in (3) the value of I we have

$$(c-y)P=\frac{2fI_0}{h_0}\left(\frac{h}{h_0}\right)^3.$$

When $x=0$, $y=0$ and $h=h_0$ and therefore

$$cP=\frac{2fI_0}{h_0}.$$

Hence we obtain

$$\frac{c-y}{c}=\left(\frac{h}{h_0}\right)^3. \quad \text{..................................(16)}$$

Eliminating h between (15) and (16) we have

$$\frac{d^2y}{dx^2}=\frac{1}{\rho_0}\,\frac{c^{\frac{1}{3}}}{(c-y)^{\frac{1}{3}}}.$$

Integrating and observing that $dy/dx=0$ when $y=0$, we obtain

$$\left(\frac{dy}{dx}\right)^2=9A^2\left[c^{\frac{2}{3}}-(c-y)^{\frac{2}{3}}\right],$$

in which $3A^2=c^{\frac{1}{3}}/\rho_0$. Now writing $z^3=c-y$ and $a^3=c$, the equation becomes

$$\left[(a^2-z^2)^{\frac{1}{2}}-\frac{a^2}{(a^2-z^2)^{\frac{1}{2}}}\right]dz=A\,dx.$$

Integrating and observing that when $x=0$, y is also nothing and z becomes equal to $c^{\frac{1}{3}}=a$, we obtain

$$\tfrac{1}{2}z\sqrt{a^2-z^2}+\tfrac{1}{2}a^2\cos^{-1}\frac{z}{a}=Ax, \quad \text{......................(17)}$$

which is the equation of the elastic curve assumed by the axis of the column in Case III. Now c is the value of y when $x=\frac{1}{2}l$, that is, when $z=0$. Substitution of this relation between the constants in (17) gives an equation of condition that the column may deflect under a load P and keep in equilibrium in a curved form expressed by the equation (17), thus

$$c\rho_0=\frac{4}{3\pi^2}l^2.$$

Substituting this in (14) we obtain

$$P=\tfrac{3}{4}\pi^2\frac{EI_0}{l^2} \quad \text{..................................}(A_3)$$

which is again of the same form as Euler's formula. A column of uniform strength having cross sections similar, is therefore $\frac{3}{4}$ times as strong as a column of uniform section made of the same material,

the form and dimensions of the section at the centre being the same.

The equation (16) can be written in the form

$$\frac{z}{a}=\frac{h}{h_0}.$$

Substituting this value of z and the value of ρ_0 above found in the equation (17), we obtain

$$\frac{\pi}{2}\frac{x}{\frac{1}{2}l}=\frac{h}{h_0}\sqrt{1-\frac{h^2}{h_0^2}}+\cos^{-1}\frac{h}{h_0}, \qquad \text{.........(18)}$$

which is the equation of the curve to be given to the outline of the column in Case III in order that it may be of uniform strength. Assume that

$$h=h_0\cos\tfrac{1}{2}\theta,$$

then the above equation becomes

$$\frac{x}{\frac{1}{2}l}=\frac{1}{\pi}(\sin\theta+\theta).$$

The curved outline of the column can be easily drawn by setting off the values of h and x calculated by these equations and given in the following Table:—

θ in degrees	$\frac{x}{\frac{1}{2}l}=\frac{1}{\pi}(\sin\theta+\theta)$	$\frac{h}{h_0}=\cos\frac{1}{2}\theta$
0	0.000	1.000
20	0.220	0.985
40	0.427	0.940
60	0.609	0.866
80	0.758	0.766
100	0.869	0.643
120	0.942	0.500
140	0.982	0.342
160	0.998	0.174
180	1.000	0.000

As in the foregoing Cases the curved outline of the column may be approximated by two pairs of straight lines touching the curve and sloping towards the ends of the column symmetrically about the cross section at the centre. Writing for shortness $\cdot h$ and $\cdot x$ for h/h_0 and $x/\frac{1}{2}l$, let the equation of a tangent be

$$\cdot h = c - m \cdot x, \quad \text{.................................(19)}$$

touching the curve at the point $(\cdot x_c, \ \cdot h_c)$ so that

$$\cdot h_c = c - m \cdot x_c \quad \text{.................................(20)}$$

and

$$\pi \cdot x_c = sin\theta_c + \theta_c, \quad \text{.................................(21)}$$

in which θ_c is given by $\cdot h_c = cos\frac{1}{2}\theta_c$.

The values of dx/dh found from (18) and (19) are equal at the point of contact and therefore

$$\pi\sqrt{1 - \cdot h_c^2} = 4m \cdot h_c^2. \quad \text{.................................(22)}$$

The volume of the column of approximately uniform strength is

$$V = const.(c^2 - cm + \tfrac{1}{3}m^2).$$

The condition of the least value of V leads to the equation

$$3 \cdot x_c(2c - m) = 3c - 2m. \quad \text{.................................(23)}$$

It is not possible to determine the quantities $\cdot x_c$, $\cdot h_c$, c, and m by the direct solution of the last four equations. The following method of solution by approximation may however be employed. Let a curved outline of the column be drawn to scale by the help of the foregoing Table. An inspection of this curve shows that when the volume V is least, $\cdot x$ will be not far from 0·4 or 0·5. Now calculate the numbers in the following Table of the values of $\cdot$V for three assumed values of θ, which roughly corresponds to x=0·4, 0·5, and 0·6.

Values of $\cdot V$

Angle θ assumed	$\cdot x = \frac{1}{\pi}(\sin\theta+\theta)$	$\cdot h = \cos\frac{1}{2}\theta$	$m = \frac{\pi}{2}\frac{\sin\frac{1}{2}\theta}{1+\cos\theta}$	$c = \cdot h + m\cdot x$	$\cdot V = c^2 - cm + \frac{1}{3}m^2 = (c-\frac{1}{2}m)^2 + \frac{1}{12}m^2$
37°18′	.400114	.947490	.279770	1.059430	.852086
47°40′	.500122	.914725	.379293	1.104418	.848796
58°56′	.600061	.870642	.509686	1.176485	.871072

The abscissa of the point of contact for the minimum value of $\cdot V$ is then given by

$$\cdot x_c = \frac{(\cdot x_2^2 - \cdot x_3^2)\cdot V_1 + (\cdot x_3^2 - \cdot x_1^2)\cdot V_2 + (\cdot x_1^2 - \cdot x_2^2)\cdot V_3}{2\{(\cdot x_2 - \cdot x_3)\cdot V_1 + (\cdot x_3 - \cdot x_1)\cdot V_2 + (\cdot x_1 - \cdot x_2)\cdot V_3\}} = 0.4630,$$

from which we obtain

$$\theta_c = 43^\circ\ 44', \qquad \cdot h_c = 0{\cdot}928, \qquad m = 0{\cdot}340,$$

$$c = 1{\cdot}085, \text{ and } \cdot h_1 = c - m = 0{\cdot}746$$

The geometrical construction of the approximate form of outline is as follows:—At the centre of the length set off a width equal to 1.085 h_0 and at each end set off a width equal to 0·746 h_0. Straight lines joining the ends of these widths form the tangents sought. These lines touch the curve (18) at distance $0{\cdot}463 \times \frac{1}{2}l$ from the centre, the width at that section being 0·928 h_0.

The equations of condition of stability for columns of uniform strength already given by the formulae (A_1), (A_2), and (A_3) may be adapted to the well-known semi-empirical expression for the strength of columns of uniform section, first proposed by Tredgold, revived by Lewis Gordon, and afterwards modified by Prof. Rankine, so as to be applicable to columns of any form of section.

These formulae of Rankine for columns with both ends rounded, with both ends flat, and with one end rounded and the other end flat, are respectively

$$\left.\begin{aligned} P&=\frac{fA}{1+4a\frac{l^2}{k^2}},\\ P&=\frac{fA}{1+a\frac{l^2}{k^2}},\\ \text{and}\quad P&=\frac{fA}{1+2a\frac{l^2}{k^2}}. \end{aligned}\right\} \text{.........................(R)}$$

In these eqations, P is the total breaking load on the column; f an experimental costant of strength for each kind of materials in units of force per unit area; a another experimental constant, being an abstract number; A the sectional area of the column; l the length; and k the least radius radius of gyration of the section. It may be remarked here that the formulae quoted by Prof. Unwin in his Machine Design as empirical rules suggested by Grashof are of the form

$$P=\frac{fAI}{\frac{Al^2}{C}+I}$$

and this is just the same as the first of the Rankine formulae (R); for, I/A is equal to k^2 and $4a$ may be written for the reciprocal of the constant C.

Values of the Constants f and a.

	Breaking f in lbs. per sq. in. excepting Grashof's values	$\frac{1}{a}$, (abstract number)	Authority
Wrought iron	36,000	36,000	Rankine.
Cast iron	80,000	6,400	
Dry timber	7,200	3,000	

	Breaking f in lbs. per sq. in. excepting Grashof's values	$\frac{1}{a}$, (abstract number)	Authority
Mild steel	42,000	36,000	Prof. Cotterill.
Wrought iron.........	10,000	22,400	Grashof. These f's are for greatest safe load.
Steel	12,000	20,000	
Cast iron	3,000	40,000	
	12,000	9,600	
Timber	900	6,000	
	500	10,400	
Mild steel	48,000	30,000	Prof. Fidler from experiments of Mr. Christie.
Hard steel	70,000	20,000	

Of the two values of P obtained from Grashof's constants, the lesser is to be adopted. For a very long and slender column, a "theoretical" value of the constant $4a$ in the first of the formulae (R) is f/π^2E, so that the ratio of the experimental to the theoretical value of the constant is $4\pi^2Ea/f$. Now the following assumption is very likely to be approximately correct:—That for a column of uniform strength and of moderate length compared with the sectional dimensions, an empirical formula of the same form as Rankine's still holds good and that the same value of the ratio above mentioned likewise obtains in that formula. The result of this assumption is given in the following Table:—

Columns with both Ends Rounded.

Form and kind of column		Theoretical formula	Semi-empirical formula	Theoretical value of coef ficient $4a$
Of uniform section		$P=\pi^2\frac{EI}{l^2}$	$P=\frac{fA}{1+4a\frac{l^2}{k^2}}$	$\frac{f}{\pi_2 E}$
Of uniform strength	Width of section parallel to the plane of bending consstant	$P=8\frac{EI}{l^2}$	$P=\frac{fA_0}{1+\frac{\pi^2 a}{2}\frac{l^2}{k^2}}$	$\frac{f}{8E}$
	Width of section perpendicular to the plane of bending constant	$P=\frac{64}{9}\frac{EI}{l^2}$	$P=\frac{fA_0}{1+\frac{9\pi^2 a}{16}\frac{l^2}{k_0^2}}$	$\frac{9f}{64E}$
	Sections similar figures	$P=\frac{3\pi^2}{4}\frac{EI}{l^2}$	$P=\frac{fA_0}{1+\frac{16a}{3}\frac{l^2}{k_0^2}}$	$\frac{4f}{3\pi^2 E}$

The last three of the empirical equations in the third column of the above Table give the relation subsisting between the length of the column and the dimensions of the section at the centre of its length, for a given breaking load P. A_0 and k_0 are the area and radius of gyration of the said section.

For the finding of the relation between the dimensions of any section and the distance x of that section from the centre the following process may be employed. The equation (7) may be written thus:

$$\frac{I}{I_0}=\frac{l^2-(2x)^2}{l^2}$$

Substituting this in (A_1) we have

$$P=8\frac{EI}{l^2-(2x)^2}.$$

Hence the first of the three empirical equations above referred to takes a more general form

$$P=\frac{fA}{1+\frac{\pi^2 a}{2}\frac{l^2-(2x)^2}{k^2}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (B_1)$$

for a column of uniform strength, rounded at both ends, having a rectangular section with the depth of side, parallel to the plane of bending, constant. See Fig. 5, Case I. This formula (B_1) applies equally well to columns of any form of

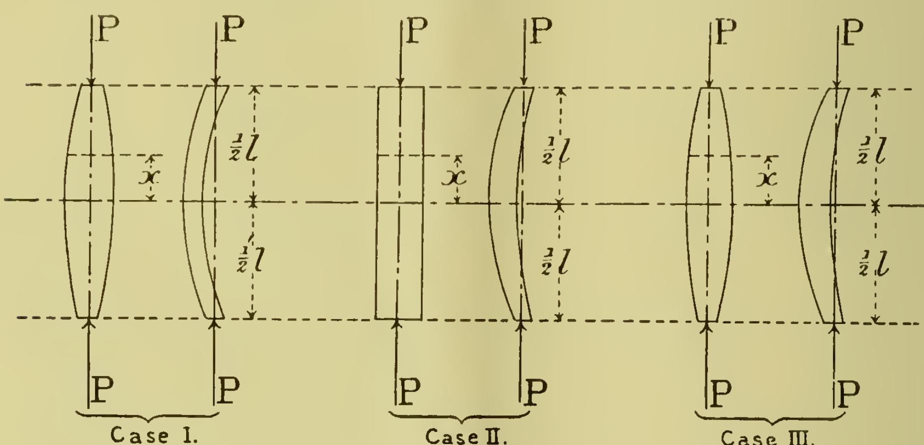


Fig. 5.

section, provided that the dimensions parallel to the plane of bending remain constant. It may be noticed that at each end of the column, x becomes $\frac{1}{2}l$ so that the formula (B_1) reduces itself to the usual simple rule $P=fA$ for a short compression block.

The equation (13) may be written in the form

$$\frac{h^2}{h_0^2}\left(\frac{3}{4}+\frac{h}{4h_0}\right)=\frac{l^2-(2x)^2}{l^2}. \quad \ldots\ldots\ldots\ldots\ldots (13')$$

The factor within the brackets on the left hand side of this equation is not much different from unity, as shown below.

$\frac{h}{h^0}$	1.0	0.8	0.6	0.4
$\frac{3}{4}+\frac{h}{4h_0}$	1.00	0.95	0.90	0.85

Since in Case 2. the width of a section perpendicular to the plane of bending is constant, we have $b=b_0$ so that

$$\frac{h^2}{h_0^2}=\frac{bh^2h_0}{b_0h_0^3}=\frac{k_0kA}{k_0^2A_0}.$$

Hence (13′) becomes

$$\frac{k_0kA}{k_0^2A_0}=\frac{l^2-(2x)^2}{l^2}, \text{ nearly.}$$

which being substituted in the equation (A_2) gives

$$P=\frac{64}{9}\frac{Ek_0kA}{l^2-(2x)^2}.$$

The second of the three empirical equations above referred to therefore takes a more general form

$$P=\frac{fA}{1+\frac{9\pi^2a}{16}\frac{l^2-(2x)^2}{k_0k}}, \quad \text{......................}(B_2)$$

for a column of uniform strength, rounded at both ends, the dimensions of the section perpendicular to the plane of bending being constant. See Fig. 5, Case 2. As in the previous case the formula (B_2) reduces itself at each end of the column to the usual simple rule $P=fA$ for a short compression block.

Now take the equation (18). Expand each term on the right hand side in a series, square the resulting equation, and re-arrange the terms. We have then

$$n^3\phi(n)=\frac{l^2-(2x)^2}{l^2}, \quad \text{......................................}(18')$$

in which

$$\phi(n)=\frac{4}{\pi}\left[\left(\frac{2}{3}+\frac{1}{5}n^2+\frac{3}{28}n^4+\frac{5}{75}n^6+\frac{35}{704}n^8+\ldots\ldots\right)\right.$$
$$\left.-\frac{1}{\pi}\left(\frac{4}{9}n^3+\frac{4}{15}n^5+\frac{12}{175}n^7+\ldots\ldots\right)\right]$$

and n stands for h/h_0. The function $\phi(n)$ is not much different from unity, as shown below.

$n=\frac{h}{h_0}$	1.0	0.8	0.6	0.4
$\phi(n)$	1.000	0.954	0.913	0.873

since
$$n^3=\frac{h^3}{h_0^{\,3}}=\frac{h_0h^3}{h_0^{\,4}}=\frac{k_0kA}{k_0^{\,2}A_0},$$
the equation (18′) becomes
$$\frac{k_0kA}{k_0^{\,2}A_0}=\frac{l^2-(2x)^2}{l^2}, \text{ nearly.}$$
Substituting this in (A_3) we abtain
$$P=\frac{3\pi^2}{4}\frac{Ek_0kA}{l^2-(2\times)^2}.$$

The third of the three empirical equations, above referred to therefore takes a more general from
$$P=\frac{fA}{1+\frac{16a}{3}\frac{l^2-(2x)^2}{k_0k}}, \quad \text{..................}(B_3)$$
which likewise reduces itself at each end of the column to the simple rule $P=fA$ for a short compression block. The above formura (B_3) is for a column of uniform strength, rounded at both ends, having cross sections of similar figures. See Fig. 5, Case III.

In the application of the formulae (B_1), (B_2), or (B_3) to the design of a column, the dimensions of the section at each end can be found by the simple rule $A=fA$; making x in the formula equal to zero, the dimensions of the section at the centre can be found in the same way as by Rankine's formula; and for other sections a few values of k lying between k_0 and the k at each end may be assumed, and the corresponding positions of the sections can be found by the solution of the formula for x, thus

$$\frac{x}{\frac{1}{2}l}=\sqrt{1-\frac{2k^2}{\pi^2 al^2}\left(\frac{fA}{P}-1\right)}, \quad \text{(B}'_1\text{)}$$

$$\frac{x}{\frac{1}{2}l}=\sqrt{1-\frac{16k_0k}{9\pi^2 al^2}\left(\frac{fA}{P}-1\right)}, \quad \text{(B}'_2\text{)}$$

$$\frac{x}{\frac{1}{2}l}=\sqrt{1-\frac{3k_0k}{16al^2}\left(\frac{fA}{P}-1\right)}, \quad \text{(B}'_3\text{)}$$

In a hollow cylindrical column under the action of an axial load there occurs secondary flexure tending to cause the buckling or wrinkling of the cylindrical shell and when the ratio of the diameter to the thickness is large, this secondary flexure becomes more predominant than the primary flexure of the column as a whole and failure may take place under a load very much less than that indicated by Gordon-Rankine formula. Prof. W. E. Lilly of Trinity College, Dublin, carried out a series of experiments on a large number of small steel columns with rounded ends and from the results drew very important conclusions bearing on the point above mentioned. (Proceedings of the Inst. Mech. Eng., London, June 1905.) The columns tested were mild steel tubes ranging from $\frac{3}{8}''$ to $1''$ in external diameter and similar to those used in cycle construction.

Prof. Lilly's experiments show that the value of f in the Gordon-Rankine formula depends on the failure of the tube by secondary flexure and that this secondary flexure does not depend on the length of the column, but only on the thickness and the radius of gyration, the relation of f to these quantities being given by the equation

$$f=\frac{F}{1+a_1\left(\frac{k}{t}\right)^2}, \quad \text{(L}_1\text{)}$$

where F = the compressive strength of the material,

a_1 = an experimental constant

$=\frac{1}{60}$ for mild steel,

t = the thickness of the tube in the same units as the radius of gyration k.

When $\frac{k}{t}$ approaches the limiting value of 0·5 for a round solid bar, f is sensibly equal to F. For wrought iron the compressive strength F may be assumed at 24 or 25 tons per sq. inch and in Hodgkinson's experiments from which Rankine derived the constants in his formula, the average ratio d/t was approximately 16, that is, $k/t=5{\cdot}5$ to $6{\cdot}0$. Substituting these values in the above formula and using $a_1=\frac{1}{0}$, f becomes 34000 to 37000 lbs. per sq. inch, which agrees very well with Rankine's value 36000 lbs. per sq. inch. Hence the same value of a_1 may be employed both for mild steel and for wrought iron.

It is also shown by Prof. Lilly's experiments that for hollow circular columns there exists the most economical values of the ratios k/t and l/k expressed by the following relation:—

$$\left(\frac{l}{k}\right)^2=1000\left(\frac{k}{t}-\frac{1}{2}\right), \quad \text{.........................}(L_2)$$

in which the fraction $\frac{1}{2}$ on the right hand side may be neglected in comparison with large values of $\frac{k}{t}$. Then

$$k^3=\frac{l^2t}{1000}. \quad \text{...................................}(L_3)$$

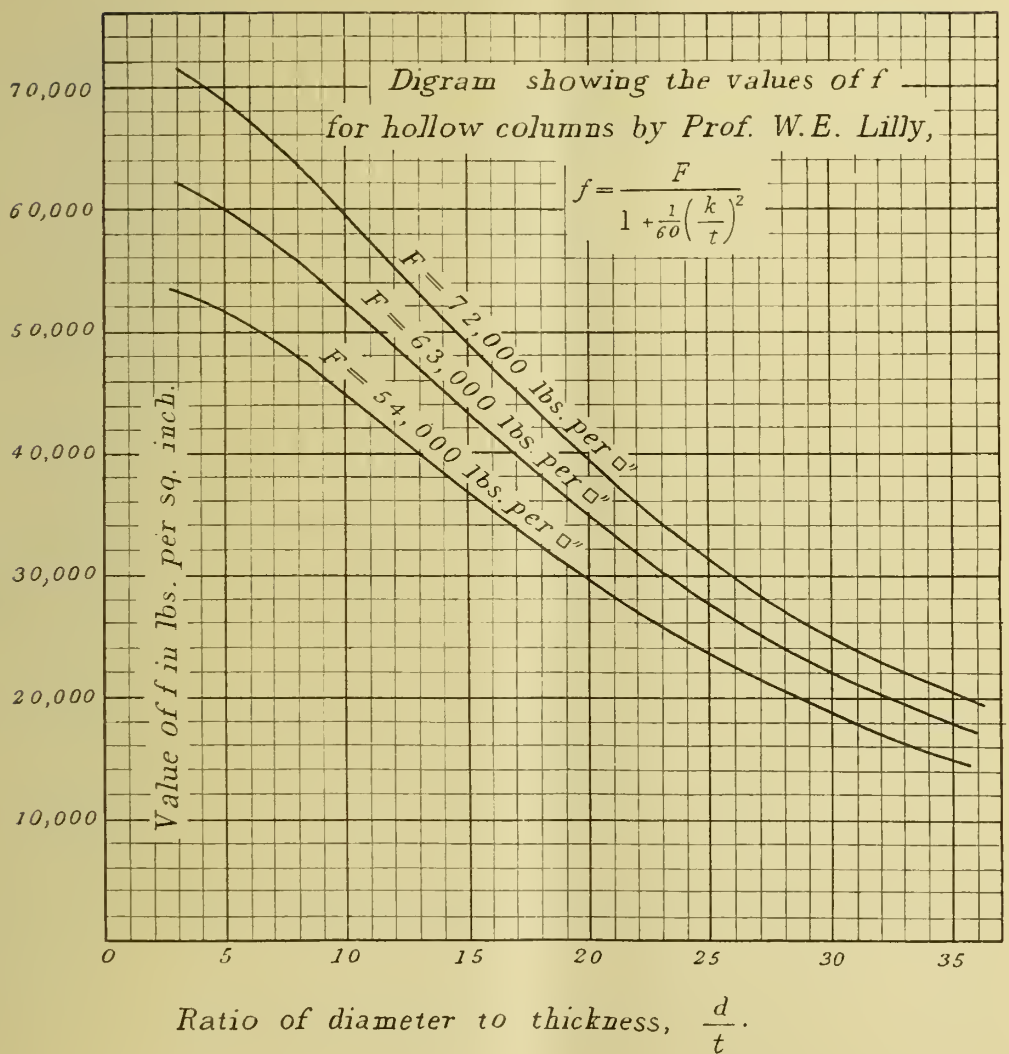
For sections other than hollow circle, the above may be written in the form

$$k^2=\frac{l\sqrt{A}}{94.3}, \quad \text{................................}(L_4)$$

which however requires to be verified by experiments.

COLUMN OF UNIFORM STRENGTH.

BY Prof. A. INOKUTY.



Values of f by Prof. Lilly's Formula.

$\left(\frac{k}{t}\right)^2$	$\frac{d}{t}$	*Value of* $f=\dfrac{F}{1+\frac{1}{60}\left(\frac{k}{t}\right)^2}$ *for* F *equal to*		
		72000 *lbs. per* □″	63000 *lbs. per* □″	54000 *bls. per* □″
120	31.0	24,000	21,000	18,000
84	25.9	30,000	26,300	22,500
60	21.9	36,000	31,500	27,000
42.9	18.5	42,000	36,800	31,500
30	15.5	48,000	42,000	36,000
20	12.6	54,000	47,300	40,500
12	9.75	60,000	52,500	45,000
5.45	6.53	66,000	57,800	49,500
5	6.24	66,500	58,200	49,800
4	5.57	67,500	59,100	50,600
3	4.80	68,600	60,000	51,400
2	3.87	69,700	61,000	52,300
1	2.65	70,800	62,000	53,100

The compressive strength of steel, F, for structural purposes may be taken at 32 to 28 tons or 72000 to 63000 lbs. per sq. inch and that of wrought iron at about 24 tons or 54000 lbs per sq. inch. The accompanying Table and Diagram give the values of f calculated by Prof. Lilly's equation (L_1) corresponding to these assumed values of F. Granting the appropriateness of using these f's for actual struts and columns, it certainly demands urgent attention that a designer should caution himself against adopting too large a value of the ratio k/t, or against using too high a value of f for a large value of k/t. This remark applies to the use of the Gordon-Rankine formulae or other formulae for columns of uniform section, as well as to the use of the foregoing formulae for columns of uniform strength.

DIE TRANSVERSALE FESTIGKEIT DER DRAHTKANONEN.

VON

M. Ôkôchi, *Kōgakuhakushi.*

THE JOURNAL OF THE COLLEGE OF ENGINEERING.
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN.
Vol. V. No. 9.

Published Sept., 1914.

Die transversale Festigkeit der Drahtkanonen.

Von

M. Ôkôchi, *Kōgakuhakushi.*

Über die transversale Festigkeit der Drahtkanonen ist seit einigen Jahrzehnten schon oftmals gesprochen worden.[1] Die bisherigen Abhandlungen beschränkten sich meines Wissens nur auf eine bestimmte Art des Geschützrohres, und allgemeine Aufstellungen über die Spannungsverteilung im Drahtrohre von moderner komplizierter Konstruktion sind noch nicht veröffentlicht worden.

1. Grundgleichungen.

Wir wählen als Grundgleichungen die Ausdrücke der Spannungsverteilung in einem dickwandigen Hohlzylinder, welcher einem inneren und einem äusseren radialen Drucke P_1 und P_2 auf den Mantelflächen ausgesetzt ist.

Es seien r_1 und r_2 der innere und der äussere Halbmesser des Rohres, E der Elastizitätsmodul, m die Poissonsche Konstante; dann hat man

$$P = \frac{\left(\frac{r_2}{r}\right)^2 - 1}{k_1^2 - 1} P_1 - \frac{\left(\frac{r_2}{r}\right)^2 - k_1^2}{k_1^2 - 1} P_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

1) J. Longridge: ....A treatise on the application of wire to the construction of ordnance.
G. Moch: ....Des canons à fils d'acier.
G. Kaiser: ....Konstruktion der gezogenen Geschützrohre.
P. Alger: ....Elastic strength of guns.
S. Takeda: ....Theorie der Drahtkanonenkonstruktion (japanisch).
G. Whistler: ....Journal of the United States artillery. 1893 p. 177.
W. Crozier: ....Report of the secretary of war; 1883, p. 411.

in radialer Richtung,

$$T=\frac{\left(\frac{r_2}{r}\right)^2+1}{k_1^2-1}P_1-\frac{\left(\frac{r_2}{r}\right)^2+k_1^2}{k_1^2-1}P_2 \quad \text{......................(2)}$$

in tangentialer Richtung, und

$$-T=\frac{d(Pr)}{dr}, \quad \text{...(3)}$$

wo unter r der Abstand irgend eines Punktes von der Symmetrieachse, der Z Achse, unter $k_1=\frac{r_2}{r_1}$ das Verhältnis der Radien zu verstehen ist.

Die Dehnungen ε_r, ε_t, ε_z in den drei Richtungen lassen sich schreiben:

$$\varepsilon_r=-\frac{(m+1)\left(\frac{r_2}{r}\right)^2-m+1}{mE(k_1^2-1)}P_1+\frac{(m+1)\left(\frac{r_2}{r}\right)^2-(m-1)k_1^2}{mE(k_1^2-1)}P_2, \quad \text{....(4)}$$

$$\varepsilon_t=\frac{(m+1)\left(\frac{r_2}{r}\right)^2+m-1}{mE(k_1^2-1)}P_1-\frac{(m+1)\left(\frac{r_2}{r}\right)^2+(m-1)k_1^2}{mE(k_1^2-1)}P_2, \quad \text{......(5)}$$

$$\varepsilon_z=-\frac{2(P_1-k_1^2P_2)}{mE(k_1^2-1)}. \quad \text{...(6)}$$

Die Innenseite des Geschützrohres wird nach aufeinanderfolgenden Schüssen immer erheblich erhitzt.[1] Wir nehmen für unsere Zwecke an, trotzdem Untersuchungen über die Temperaturverteilung innerhalb der Geschützrohrwand mangeln, dass sie der Fourierschen Theorie der Wärmeleitung in homogenem Materiale folgt, und stationär ist, wegen der Kleinheit der Wirkungsdauer des Innendruckes; dann wird

1) Nach T. Yosida beträgt die Temperatur 273° C an der Mündung eines 15 cm marinen Drahtrohres nach 80 Schüssen. Kaheigakkaisi Vol. 1, p. 128, und für andere Literatur siehe C. Cranz, Zs. für das gesamte Schiess- und Sprengw. 1908, p. 301.

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} = 0,$$

wo θ die Temperatur bedeutet, und wir bekommen die Gleichungen der Zugspannungskomponenten σ_r, σ_t und σ_z in der radialen, tangentialen und bzw. axialen Richtung, wenn θ_1 und θ_2 die Temperatur an der inneren bzw. äusseren Seite und α den Temperaturkoeffizient bedeutet,

$$\sigma_r = \frac{mE\alpha}{2(m-1)}\left[\frac{k_1^2\theta_2 - \theta_1}{k_1^2 - 1} + \left(\frac{r_2}{r}\right)^2 \frac{\theta_1 - \theta_2}{k_1^2 - 1} - \theta\right], \dots\dots\dots\dots\dots\dots (7)$$

$$\sigma_t = \frac{mE\alpha}{2(m-1)}\left[\frac{k_1^2\theta_2 - \theta_1}{k_1^2 - 1} - \left(\frac{r_2}{r}\right)^2 \frac{\theta_1 - \theta_2}{k_1^2 - 1} + \frac{\theta_1 - \theta_2}{lg k_1} - \theta\right], \dots\dots (8)$$

$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9)$

With the Compliments of the Director of the Engineering College.

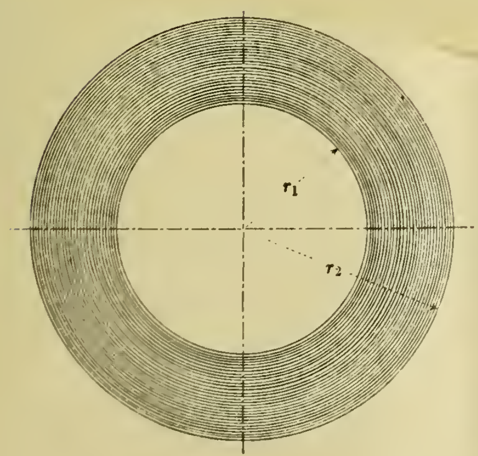


Fig. 1.

Einfachheit ...
vor, welches nur aus Drähten besteht. Das Rohr sei aus sehr dünnen Drähten gewunden, deren Dicke man natürlich im Vergleiche mit dem Halbmesser des Rohres als unendlich klein vernachlässigen kann. Dabei sei die Windungsspannung der Drähte beliebig regulierbar.

in radialer Richtung,

$$T=\frac{\left(\frac{r_2}{r}\right)^2+1}{k_1^2-1}P_1-\frac{\left(\frac{r_2}{r}\right)^2+k_1^2}{k_1^2-1}P_2 \quad\ldots\ldots\ldots\ldots(2)$$

in tangentialer Richtung, und

$$-T=\frac{d\,(Pr)}{dr}, \quad\ldots\ldots\ldots\ldots(3)$$

wo unter r der Abstand irgend eines Punktes von der Symmetrieachse, der Z Achse, unter $k_1=\frac{r_2}{r_1}$ das Verhältnis der Radien zu verstehen ist.

Die Dehnungen ε_r, ε_t, ε_z in den drei Richtungen lassen sich schreiben:

Die Innenseite des Geschützrohres wird nach aufeinanderfolgenden Schüssen immer erheblich erhitzt.[1] Wir nehmen für unsere Zwecke an, trotzdem Untersuchungen über die Temperaturverteilung innerhalb der Geschützrohrwand mangeln, dass sie der Fourierschen Theorie der Wärmeleitung in homogenem Materiale folgt, und stationär ist, wegen der Kleinheit der Wirkungsdauer des Innendruckes; dann wird

1) Nach T. Yosida beträgt die Temperatur 273° C an der Mündung eines 15 cm marinen Drahtrohres nach 80 Schüssen. Kaheigakkaisi Vol. 1, p. 128, und für andere Literatur siehe C. Cranz, Zs. für das gesamte Schiess- und Sprengw. 1908, p. 301.

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}=0,$$

wo θ die Temperatur bedeutet, und wir bekommen die Gleichungen der Zugspannungskomponenten σ_r, σ_t und σ_z in der radialen, tangentialen und bzw. axialen Richtung, wenn θ_1 und θ_2 die Temperatur an der inneren bzw. äusseren Seite und α den Temperaturkoefflzient bedeutet,

$$\sigma_r=\frac{mE\alpha}{2(m-1)}\left[\frac{k_1^2\theta_2-\theta_1}{k_1^2-1}+\left(\frac{r_2}{r}\right)^2\frac{\theta_1-\theta_2}{k_1^2-1}-\theta\right], \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$\sigma_t=\frac{mE\alpha}{2(m-1)}\left[\frac{k_1^2\theta_2-\theta_1}{k_1^2-1}-\left(\frac{r_2}{r}\right)^2\frac{\theta_1-\theta_2}{k_1^2-1}+\frac{\theta_1-\theta_2}{lgk_1}-\theta\right], \ldots\ldots(8)$$

$$\sigma_z=\frac{mE\alpha}{m-1}(\theta_m-\theta), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

wo

$$\theta=\frac{\theta_1 lg\frac{r_2}{r}+\theta_2 lg\frac{r}{r_1}}{lgk_1}, \qquad \theta_m=\frac{k_1^2\theta_2-\theta_1}{k_1^2-1}+\frac{\theta_1-\theta_2}{2lgk_1}$$

ist, und E als konstant betrachtet ist.

2. Drahtkanonen ohne Kern.

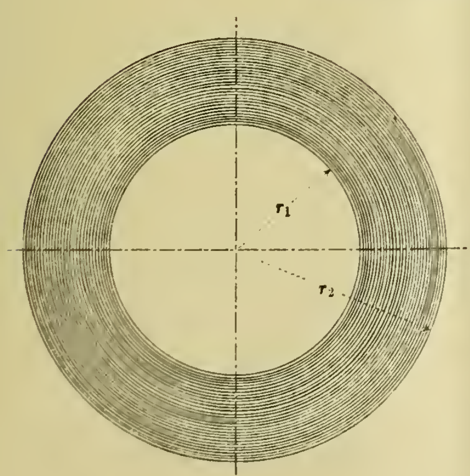


Fig. 1.

Zunächst stellen wir uns der Einfachheit halber ein Drahtrohr vor, welches nur aus Drähten besteht. Das Rohr sei aus sehr dünnen Drähten gewunden, deren Dicke man natürlich im Vergleiche mit dem Halbmesser des Rohres als unendlich klein vernachlässigen kann. Dabei sei die Windungsspannung der Drähte beliebig regulierbar.

Es sei nun das Rohr so erzeugt, dass die Umfangsdehnung unter dem Innendruck P_1 überall gleich sein soll. Wir setzen also

$$\frac{T}{E_d}+\frac{P}{m_d E_d}=\text{konst.},$$

worin E_d und m_d den Elastizitätsmodul und die Poissonsche Konstante des Drahtmaterials bedeutet. Setzen wir für T den Wert aus Gleichung (3) ein, so bekommen wir nach Integration mit der Berücksichtigung, dass die radiale Spannung an der Aussenfläche Null wird,

$$P=\frac{m_d}{m_d-1}S_d\left[\left(\frac{r_2}{r}\right)^{\frac{m_d-1}{m_d}}-1\right], \quad\text{.........................(10)}$$

wo man unter S_d die reduzierte Umfangsspannung zu verstehen hat, oder nach einiger Umformung,

$$T=\frac{m_d}{m_d-1}S_d\left[1-\frac{1}{m_d}\left(\frac{r_2}{r}\right)^{\frac{m_d-1}{m_d}}\right]. \quad\text{.....................(11)}$$

Für den Innendruck P_1 hat man nur in (10) $r=r_1$ zu setzen,

$$P_1=\frac{m_d}{m_d-1}S_d\left(k_1^{\frac{m_d-1}{m_d}}-1\right). \quad\text{..............................(12)}$$

Die Spannungen im Ruhezustande, wo $P_1=0$ ist, zu ermitteln, können wir die Superpositionsgesetze anwenden. Wir brauchen das Rohr nur als aus einem Stück bestehend und Innendruck ausgesetzt anzusehen. Seien P' und T' die Spannungen im oben erwähnten Rohr, dass wir den natürlichen Zylinder nennen wollen; p und t die Spannungen im Ruhezustande, so hat man

$$p = P - P' = \frac{m_d}{m_d - 1} S_d \left[\left(\frac{r_2}{r}\right)^{\frac{m_d - 1}{m_d}} - 1\right] - \frac{\left(\frac{r_2}{r}\right)^2 - 1}{k_1^2 - 1} P_1, \quad \ldots\ldots\ldots(13)$$

$$t = T - T' = \frac{m_d}{m_d - 1} S_d \left[1 - \frac{1}{m_d}\left(\frac{r_2}{r}\right)^{\frac{m_d - 1}{m_d}}\right] - \frac{\left(\frac{r_2}{r}\right)^2 + 1}{k_1^2 - 1} P_1, \quad \ldots\ldots(14)$$

wo man P_1 aus (12) einzuführen hat.

Die Windungsspannungen der Drähte zu ermitteln, stellen wir uns ein halb erzeugtes Drahtrohr vor, dessen innerer und äusserer Radius r_1 bzw. $r < r_2$ ist. An der äussersten Fläche ist jetzt die Umfangsspannung nicht anders als die Windungsspannung und die radiale Spannung Null, während im ganz fertigen Drahtrohr in diesem Punkte sich gerade die in den Gleichungen (13) und (14) ausgedrückten Spannungen p und t befinden sollten.

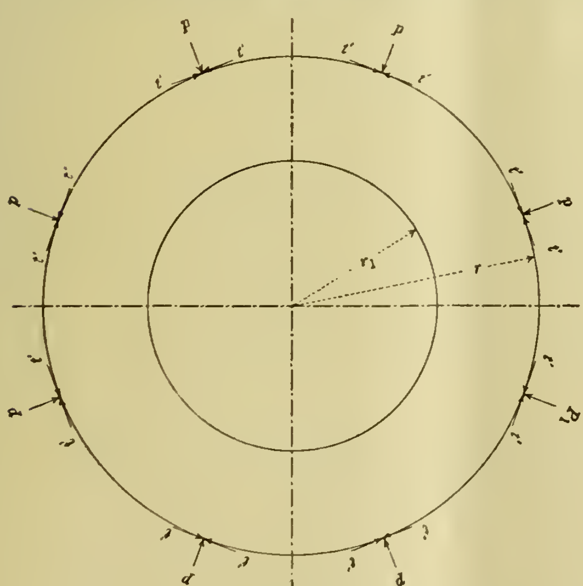


Fig. 2.

Hieraus sehen wir, dass, wenn man p auf der Aussenfläche des halb erzeugten Drahtrohres ansetzt, dann sich die Windungsspannung W zu t ändert. Also, wenn t' die Umfangsspannung an der Aussenfläche eines natürlichen unter dem Aussendruck p befindlichen Zylinders mit den Radien r_1 und r bedeutet, so muss

$$W + t' = t$$

sein.

Nach Einsetzung der Werte von t' und t aus den Gl. (2) und

(14) erhält man

$$W=\frac{m_d}{m_d-1}S_d\left[1-\frac{1}{m_d}\left(\frac{r_2}{r}\right)^{\frac{m_d-1}{m_d}}\right]-\frac{\left(\frac{r_2}{r}\right)^2+1}{k_1^2-1}P_1+\frac{\left(\frac{r}{r_1}\right)^2+1}{\left(\frac{r}{r_1}\right)^2-1}p \quad \ldots(15)$$

$$\lim_{r=r_1} W=0.$$

Ein ganz ähnliche Betrachtung liefern uns auch die Ausdrücke der Spannungsverteilung in verschiedenen Geschützrohren, z. B. in einem Drahtrohre, welches unter innerem Druck gleichmässige Umfangsspannung erfährt, oder im Ruhezustande gleichmässige Umfangsdehnung erfährt, u. s. w.

Übrigens hat man noch eine andere Art des Drahtrohres, wo die Drähte mit konstanter Windungsspannung aufgewickelt worden sind, zu behandeln. In diesem Falle gehen wir zunächst vom Ausdrucke der Windungsspannung aus, also

$$t-t'=W=\text{konst.}$$

Setzt man t' und t aus Gl. (2) und (3) ein, so hat man

$$\frac{dp}{dr}-\frac{2r_1^2}{r(r^2-r_1^2)}p+\frac{W}{r}=0. \quad \ldots\ldots\ldots\ldots(16)$$

Integriert man diese Gleichung mit Rücksicht, dass für $r=r_2, p=o$ ist, so wird

$$p=\frac{W}{2}\left[1-\left(\frac{r_1}{r}\right)^2\right]lg\,\frac{r_2^2-r_1^2}{r^2-r_1^2}, \quad \ldots\ldots\ldots\ldots(17)$$

woraus folgt

$$t=\frac{W}{2}\left[2-\left(1+\frac{r_1^2}{r^2}\right)lg\,\frac{r_2^2-r_1^2}{r^2-r_1^2}\right].$$

Für die Spannungen unter dem Innendrucke P_1 brauchen wir nach dem Superpositionsgesetze nur die Spannungen im natürlichen

Hohlzylinder unter gleichem Innendruck zu den Spannungen im Ruhezustande zu addieren. Es ergiebt sich:

$$P = p + \frac{\left(\frac{r_2}{r}\right)^2 - 1}{k_1^2 - 1} P_1,$$

$$T = t + \frac{\left(\frac{r_2}{r}\right)^2 + 1}{k_1^2 - 1} P_1.$$

Wenn das Drahtrohr aus mehreren Lagen mit variierender Windungsspannung besteht, so stellen wir die Lösung in folgender Weise auf:

Es seien r_1, r_2; r_2, r_3; ....r_{n-1}, r_n; r_n, r_{n+1}; die inneren und äusseren Halbmesser: W_1, W_2, W_3..........W_{n-1}, W_n die Windungsspannungen der beziehungsweise 1^{ten}, 2^{ten}.........n^{ten} Lage, so haben wir für die äusserste Lage nach Gl. (17)

$$p = \frac{W_n}{2}\left[1 - \left(\frac{r_1}{r}\right)^2\right] lg \frac{r_{n+1}^2 - r_1^2}{r^2 - r_1^2},$$

und die radiale Spannung zwischen der n^{ten} und $n-1^{ten}$ Lage ergibt sich, wenn man statt r, r_n schreibt,

$$p_n = \frac{W_n}{2}\left[1 - \left(\frac{r_1}{r_n}\right)^2\right] lg \frac{r_{n+1}^2 - r_1^2}{r_n^2 - r_1^2},$$

Ähnlicherweise für die $n-1^{te}$ Lage,

$$\frac{dp}{dr} - \frac{2r_1^2}{r(r^2 - r_1^2)} p + \frac{W_{n-1}}{r} = 0.$$

Integriert man diese Gleichung mit Rücksicht, dass für $r = r_n$ $p = p_n$ ist, so wird

$$p = \frac{W_n}{2}\left[1 - \left(\frac{r_1}{r}\right)^2\right] lg \frac{r_{n+1}^2 - r_1^2}{r_n^2 - r_1^2} + \frac{W_{n-1}}{2}\left[1 - \left(\frac{r_1}{r}\right)^2\right] lg \frac{r_n^2 - r_1^2}{r^2 - r_1^2}.$$

So gehen wir zu den inneren Lagen über und finden für die m^{te} Lage mit den Radien r_m und r_{m+1}, wo $m<n$ ist,

$$p=\frac{1}{2}\left[1-\left(\frac{r_1}{r}\right)^2\right]\left(\sum_{x=n}^{n+1} W_x\, lg\, \frac{r_{x+1}^2-r_1^2}{r_x^2-r_1^2}+W_m\, lg\, \frac{r_{m+1}^2-r_1^2}{r^2-r_1^2}\right). \ldots(18)$$

Die Umfangsspannung lässt sich schreiben aus Gl. (15)

$$t=W_m-\frac{r^2+r_1^2}{2r^2}\left(\sum_{x=n}^{m+1} W_x\, lg\, \frac{r_{x+1}^2-r_1^2}{r_x^2-r_1^2}+W_m\, lg\, \frac{r_{m+1}^2-r_1^2}{r^2-r_1^2}\right). \ldots\ldots\ldots(19)$$

Für die Spannungen unter Innendruck hat man

$$P=p+\frac{(r_{n+1}^2-r^2)r_1^2}{(r_{n+1}^2-r_1^2)r^2}\,P_1,$$

$$T=t+\frac{(r_{n+1}^2+r^2)r_1^2}{(r_{n+1}^2-r_1^2)r^2}\,P_1.$$

3. Drahtkanonen mit Kernrohr.

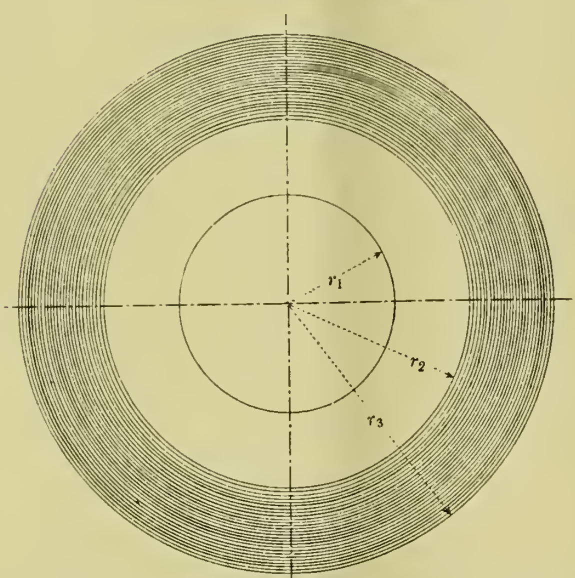


Fig. 3.

Es seien

r_1, r_2 der innere und der äussere Radius des Kernrohres;

r_2, r_3 dieselben des Drahtteils;

P_2 der radiale Druck zwischen Kernrohr und Drahtteil unter Innendruck;

p_2 derselbe im Ruhezustande;

E_1, m_1, a_1 der Elastizitätsmodul, die Poissonsche Konstante und der Temperaturkoëffizient des Kernrohres:

E_d, m_d, a_d dieselben der Drähte.

Zunächst betrachten wir einen Fall, wo die Drähte auf dem Kernrohre so aufgewickelt worden sind, dass im Momente des Schusses am Drahtteil eine überall gleichmässig reduzierte Umfangsspannung S_d auftritt, dann folgt unmittelbar aus Gl. (10) und (11):

$$P = \frac{m_d}{m_d - 1} S_d \left[\left(\frac{r_3}{r} \right)^{\frac{m_d - 1}{m_d}} - 1 \right],$$

$$T = \frac{m_d}{m_d - 1} S_d \left[1 - \frac{1}{m_d} \left(\frac{r_3}{r} \right)^{\frac{m_d - 1}{m_d}} \right]$$

und

$$P_2 = \frac{m_d}{m_d - 1} S_d \left(k_2^{\frac{m_d - 1}{m_d}} - 1 \right),$$

wobei

$$k_2 = \frac{r_3}{r_2}$$

ist.

Die reduzierten Sapnnungen im Kernrohre sind leicht ermittelbar, z. B. für die Spannung nach tangentialer Richtung hat man

$$S = \frac{P_1}{k_1^2 - 1} \left(\frac{m_1 - 1}{m_1} + \frac{m_1 + 1}{m_1} \frac{r_2^2}{r_1^2} \right) - \frac{m_d S_d k_1^2}{(m_d - 1)(k_1^2 - 1)} \left(\frac{m_1 - 1}{m_1} + \frac{m_1 + 1}{m_1} \frac{r_1^2}{r^2} \right) \left(k_2^{\frac{m_d - 1}{m_d}} - 1 \right).$$

Daraus folgt, wenn S_1 die zulässige Inanspruchnahme auf Zug bedeutet, dass der Innendruck, dem das Drahtrohr Widerstand leisten soll,

$$P_1=\frac{S_1(k_1^2-1)}{\frac{m_1-1}{m_1}+\frac{m_1+1}{m_1}k_1^2}+\frac{2m_dS_d\,k_1^2\left(k_2^{\frac{m_d-1}{m_d}}-1\right)}{(m_d-1)\left(\frac{m_1-1}{m_1}+\frac{m_1+1}{m_1}k_1^2\right)}$$

wird.

Wir wollen jetzt die Temperaturspannungen berücksichtigen. Es seien θ_1, θ_2 und θ_3 die Temperaturen in Punkten, deren Abstände r_1, r_2 bzw. r_3 von der Längsachse betragen, α_1 und α_2 die Ausdehnungskoëffizienten des Materials des Kernrohrs und Drahtes. Dann lassen sich die Spannungen in den drei Richtungen innerhalb des Kernrohrs aus Gl. (7), (8) und (9) schreiben:

$$\sigma_{r,1}=\frac{m_1E_1\alpha_1}{2(m_1-1)}\left[\frac{k_1^2\theta_2-\theta_1}{k_1^2-1}+\left(\frac{r_2}{r}\right)^2\frac{\theta_1-\theta_2}{k_1^2-1}-\theta\right],$$

$$\sigma_{t,1}=\frac{m_1E_1\alpha_1}{2(m_1-1)}\left[\frac{k_1^2\theta_2-\theta_1}{k_1^2-1}-\left(\frac{r_2}{r}\right)^2\frac{\theta_1-\theta_2}{k_1^2-1}+\frac{\theta_1-\theta_2}{lgk_1}-\theta\right],$$

$$\sigma_{z,1}=\frac{m_1E_1\alpha_1}{m_1-1}(\theta_m-\theta).$$

An der äusseren und der inneren Fläche gilt

$$\sigma_z=\sigma_t=\sigma_{2,1}=\frac{m_1E_1\alpha_1}{m_1-1}(\theta_2-\theta_1)\left(\frac{1}{k_1^2-1}-\frac{1}{2lgk_1}\right)\qquad \text{für } r=r_2,$$

$$\sigma_z=\sigma_t=\sigma_{1,1}=\frac{m_1E_1\alpha_1}{m_1-1}(\theta_2-\theta_1)\left(\frac{k_1^2}{k_1^2-1}-\frac{1}{2lgk_1}\right)\qquad \text{für } r=r_1,$$

woraus als Umfangsdehnung der Aussenseite der Wert

$$2\pi r_2\frac{\sigma_{2,1}}{E_1}\;\frac{m_1-1}{m_1}$$

folgt.

Diese Dehnung wird aber durch den Drahtteil einigermassen verhindert werden, demnach tritt auf der Grenzfläche ein radialer Druck H_2 ein, den zu ermitteln wir wieder den Drahtteil als aus einem Stück bestehend betrachten. Man hat dann an der äusseren und der inneren Fläche

$$\sigma_{3,d}=\frac{m_d E_d \alpha_d}{m_d-1}(\theta_3-\theta_2)\left(\frac{1}{k_2^2-1}-\frac{1}{2lgk_2}\right),$$

$$\sigma_{2,d}=\frac{m_d E_d \alpha_d}{m_d-1}(\theta_3-\theta_2)\left(\frac{k_2^2}{k_2^2-1}-\frac{1}{2lgk_2}\right),$$

und die Umfangsdehnung der Innenseite wird

$$2\pi r_2\frac{\sigma_{2,d}}{E_d}\,\frac{m_d-1}{m_d}\;.$$

Die beiden Flächen unter dem normalen Druck H_2 müssen immer kontakt bleiben, also nach Vergleichung der Umfangsdehnungen beider Flächen erhält man

$$H_2=\frac{\sigma_{2,1}\dfrac{m_1-1}{m_1E_1}-\sigma_{2,d}\dfrac{m_d-1}{m_dE_d}}{\dfrac{m_1+1+(m_1-1)k_1^2}{m_1E_1(k_1^2-1)}+\dfrac{(m_d+1)k_2^2+m_d-1}{m_dE_d(k_2^2-1)}}. \qquad \ldots\ldots\ldots\ldots(20)$$

Endlich haben wir für die sämtlichen Spannungen im Momente des Schusses in der Drahtspule die Ausdrücke:

$$P=\frac{m_d}{m_d-1}S_d\left[\left(\frac{r_3}{r}\right)^{\frac{m_d-1}{m_d}}-1\right]-\sigma_{r,d}+\frac{\left(\dfrac{r_2}{r}\right)^2-1}{k_2^2-1}H_2, \qquad \ldots\ldots\ldots\ldots(21)$$

$$T=\frac{m_d}{m_d-1}S_d\left[1-\frac{1}{m_d}\left(\frac{r_3}{r}\right)^{\frac{m_d-1}{m_d}}\right]+\sigma_{t,d}+\frac{\left(\dfrac{r_2}{r}\right)^2+1}{k_2^2-1}H_2, \qquad \ldots\ldots(22)$$

$$\sigma_{z,d}=\frac{m_dE_d\alpha_d}{m_d-1}(\theta_m-\theta),$$

und im Kernrohre:

$$P=\frac{\left(\frac{r_2}{r}\right)^2-1}{k_1^2-1}P_1-\frac{\left(\frac{r_2}{r}\right)^2-k_1^2}{k_1^2-1}(P_2+H_2)-\sigma_{r,1},$$

$$T=\frac{\left(\frac{r_2}{r}\right)^2+1}{k_1^2-1}P_1-\frac{\left(\frac{r_2}{r}\right)^2+k_1^2}{k_1^2-1}(P_2+H_2)+\sigma_{t,1},$$

$$\sigma_{z,1}=\frac{m_1E_1a_1}{m_1-1}(\Theta_m-\Theta).$$

In diesen Gleichugen bedeuten $\sigma_{r,d}$, $\sigma_{t,d}$ und $\sigma_{z,d}$ die radiale, tangentiale und axiale Spannung in der Drahtlage: $\sigma_{r,1}$, $\sigma_{t,1}$, und $\sigma_{z,1}$ die gleichen im Kernrohre.

Wenn wir nur die reduzierte Umfangsspannung S in der Drahtlage beachten, so bekommen wir

$$S=S_d+\sigma_{t,d}+\frac{\left(\frac{r_3}{r}\right)^2+1}{k_2^2-1}H_2-\frac{1}{m_d}\left(\sigma_{z,d}+\sigma_{r,d}-\frac{\frac{r_3^2}{r^2}-1}{k_2^2-1}H_2\right),$$

und es wird an der Aussenseite

$$S_3=S_d+E_da_d(\Theta_3-\Theta_2)\left(\frac{1}{k_2^2-1}-\frac{1}{2lgk_2}\right)+\frac{2H_2}{k_2^2-1}.$$

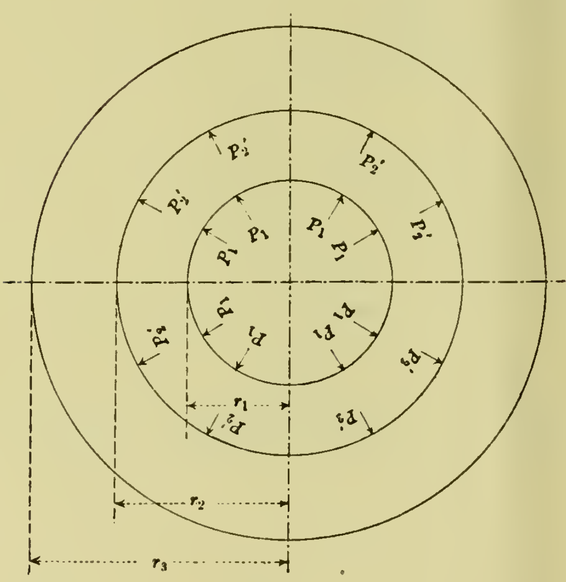


Fig. 4.

Man sieht aus diesem Ausdrucke, dass, wenn $\Theta_3<\Theta_2$ ist, Bruchgefahr an der äussersten Schichte der Drahtlage im Momente des Schusses eintreten wird.

Die Spannungsverteilung im Ruhezustande ist ohne weiteres ermittelbar, wenn man die Spannungen im natürlichen Zylin-

der unter Innendruck[1] bestimmt. Dazu brauchen wir nur den normalen Druck P_2' auf der Grenzfläche zu finden. Nach der Vergleichung der Umfangsdehnungen beider Kontaktflächen hat man:

$$P_2' = B_1 P_1,$$

$$B_1 = \frac{\dfrac{2}{E_1(k_1^2-1)}}{\dfrac{m_1+1+(m_1-1)k_1^2}{m_1 E_1(k_1^2-1)} + \dfrac{m_d-1+(m_d+1)k_2^2}{m_d E_d(k_2^2-1)}}.$$

Dann wird wieder nach dem Superpositionsgesetze

$$p_2 = P_2 - B_1 P_1,$$

und die Spannungen im Drahtteile

$$p = \frac{m_d}{m_d-1} S_d \left[\left(\frac{r_3}{r}\right)^{\frac{m_d-1}{m_d}} - 1\right] - \frac{B_1 P_1}{k_2^2-1}\left(\frac{r_3^2}{r^2} - 1\right),$$

$$t = \frac{m_d}{m_d-1} S_d \left[1 - \frac{1}{m_d}\left(\frac{r_3}{r}\right)^{\frac{m_d-1}{m_d}}\right] - \frac{B_1 P_1}{k_2^2-1}\left(\frac{r_3^2}{r^2} + 1\right).$$

Berücksichtigen wir wieder die Temperaturspannungen, so tritt die grösste Druckspannung an der Innenseite des Kernrohres mit der Grösse

$$t = -\frac{2k_1^2}{k_1^2-1}(p_2 + H_2) - \frac{m_1 E_1 \alpha_1}{m_1-1}(\theta_1 - \theta_2)\left(\frac{k_1^2}{k_1^2-1} - \frac{1}{2 lg k_1}\right) \ldots\ldots (23)$$

auf.

Die Windungsspannung wird nach Gl. (15) ermittelt:

$$W = \frac{m_d}{m_d-1} S_d \left[1 - \frac{1}{m_d}\left(\frac{r_3}{r}\right)^{\frac{m_d-1}{m_d}}\right] - \frac{B_1 P_1}{k_2^2-1}\left(\frac{r_3^2}{r^2} + 1\right) + \frac{ar^2 + br_2^2}{ar^2 - br_2^2} p,$$

---

1) L. Boltzmann, Sitzgsbr. d. k. Akad. d. Wissensch. in Wien 59 (1859) p. 679.

falls

$$a=\frac{m_a E_a[(m_1-1)k_1^2+m_1+1]}{m_1 E_1(k_1^2-1)}+m_a+1, \qquad b=a-2m_a$$

ist.

Ähnlicherweise können wir die Spannungen in den Drahtgeschützen ermitteln, die die gegebenen anderen Bedingungen erfüllen. Von diesen wollen wir nur zwei Arten behandeln.

Sind die Drähte auf dem Kernrohre mit gleicher Windungsspannung aufgewickelt worden, so bekommen wir

$$W=t+\frac{ar^2+br_2^2}{ar^2-br_2^2}\,p=\text{konst.}$$

Folglich wird der Druck in der Drahtlage

$$p=\frac{W}{2ar^2}(ar^2-br_2^2)\,lg\frac{ar_3^2-br_2^2}{ar^2-br_2^2},$$

und der an der Kontaktfläche

$$p_2=\frac{m_a W}{a}\,lg\frac{ak_2^2-b}{2m_a},$$

woraus als Spannung im Momente des Schusses folgt

$$P=\frac{W}{2ar^2}(ar^2-br_2^2)\,lg\frac{ar_3^2-br_2^2}{ar^2-br_2^2}+\frac{B_1P_1}{k_2^2-1}\left(\frac{r_3^2}{r^2}-1\right),$$

$$T=\frac{W}{2ar^2}\left[2ar^2-(ar^2+br_2^2)\,lg\frac{ar_3^2-br_2^2}{ar^2-br_2^2}\right]+\frac{B_1P_1}{k_2^2-1}\left(\frac{r_3^2}{r^2}+1\right)$$

und

$$P_2=\frac{m_a W}{a}\,lg\frac{ak_2^2-b}{2m_a}+B_1P_1.$$

Man kann sehr leicht die Temperaturspannungen berücksichtigen, weil wir schon H_2 in der Gl. (20) gefunden haben, so z.B. im Kernrohr unter Innendruck:

$$P=\frac{\left(\frac{r_2}{r}\right)^2-1}{k_1^2-1}P_1-\frac{\left(\frac{r_2}{r}\right)^2-k_1^2}{k_1^2-1}\left(\frac{m_a W}{a}lg\frac{ak_2^2-b}{2m_a}+B_1P_1+H_2\right)-\sigma_{r,1},$$

$$T=\frac{\left(\frac{r_2}{r}\right)^2+1}{k_1^2-1}P_1-\frac{\left(\frac{r_2}{r}\right)^2+k_1^2}{k_1^2-1}\left(\frac{m_a W}{a}lg\frac{ak_2^2-b}{2m_a}+B_1P_1+H_2\right)+\sigma_{t,1}.$$

Aus Formeln für die Spannungen in der Drahtlage im Momente des Schusses sowie im Ruhezustande sehen wir, dass wir verschiedene Spannungszustände mit der konstant bleibenden Windungsspannung erhalten können.

Vergleichen wir die reduzierten Umfangsspannungen der äusseren und inneren Seite der Drahtlage im Momente des Schusses, so bekommen wir eine Windungsspannung

$$W=\frac{\left(1+\frac{1}{m_a}\right)aB_1P_1}{\left(a-\frac{1}{m_a}-1\right)lg\frac{ak_2^2-b}{2m_a}},$$

die die gleiche tangentiale Dehnung erzeugt. Es bietet auch keine Schwierigkeiten, eine Windungsspannung, die die gegebene Druckspannung an der Innenseite des Kernrohrs im Ruhezustande erfährt, zu ermitteln und andere Fälle zu behandeln.

Der Spannungszustand im Drahtgeschütze, wo die Drahtspule aus mehreren Lagen mit verschiedenen Windungsspannungen besteht, kann leicht nach Formeln (18) und (19) ausgedrückt werden:

$$p=\frac{ar^2-br_2^2}{2ar^2}\left(\sum_{x=n}^{m+1}W_x lg\frac{ar_{x+1}^2-br_2^2}{ar_x^2-br_2^2}+W_m lg\frac{ar_{m+1}^2-br_2^2}{ar^2-br_2^2}\right),$$

$$t=W_m-\frac{ar^2+br_2^2}{2ar^2}\left(\sum_{x=n}^{m+1}W_x lg\frac{ar_{x+1}^2-br_2^2}{ar_x^2-br_2^2}+W_m lg\frac{ar_{m+1}^2-br_2^2}{ar^2-br_2^2}\right),$$

wo r_{n+1} und r_2 den äusseren und den inneren Radius der Drahtspule bedeuten.

Es sei also W_2 die Windungsspannung der innersten Drahtlage mit den Radien r_2 und r_3; W_3 diejenige der Drahtlage mit r_3 und r_4 u.s.w., so wird der Druck auf der äusseren Fläche des Kernrohres

$$p_2 = \frac{m_d}{a}\left(\sum_{x=n}^{3} W_x lg \frac{ar_{x+1}^2 - br_2^2}{ar_x^2 - br_2^2} + W_2 lg \frac{ak_2^2 - b}{2m_d}\right),$$

während die Spannungen unter innerem Druck sich schreiben lassen;

$$P = p + \frac{B_1 P_1 r_2^2}{r_{n+1}^2 - r_2^2}\left(\frac{r_{n+1}^2}{r^2} - 1\right),$$

$$T = t + \frac{B_1 P_1 r_2^2}{r_{n+1}^2 - r_2^2}\left(\frac{r_{n+1}^2}{r^2} + 1\right).$$

Der Spannungszustand mit Rücksicht auf die Temperaturdifferenz zwischen den Mantelflächen ist leicht ermittelbar, da ohne weiteres Gl. (20) sich anwenden lässt, und man kann die ähnlichen Gleichungen wie (21), (22), (23), u.s.w. finden.

4. Die Festigkeit der grosskalibrigen Drahtkanonen mit komplizierter Konstruktion.

Wir wollen jetzt die oben erwähnten Formeln für komplizierte Konstruktionen praktisch anwenden, in denen die Drähte auf dem Doppelkern aufgewickelt, und ein Mantelzylinder auf dem Drahtteile aufgeringt worden ist. Zunächst wollen wir die Bedingung des grössten Widerstandes auf Zug gegen Innendruck aufstellen.

Es seien P_2, P_3, P_{n+1}, die auf der Berührungsfläche einwirkenden Normalspannungen unter dem Innendruck P_1, wobei der Index die Lage anzeigt, in Übereinstimmung mit dem der Radien, also ist jede dieser Spannungen bezüglich der nach innen gelegenen Lage als Aussendruck aufzufassen,

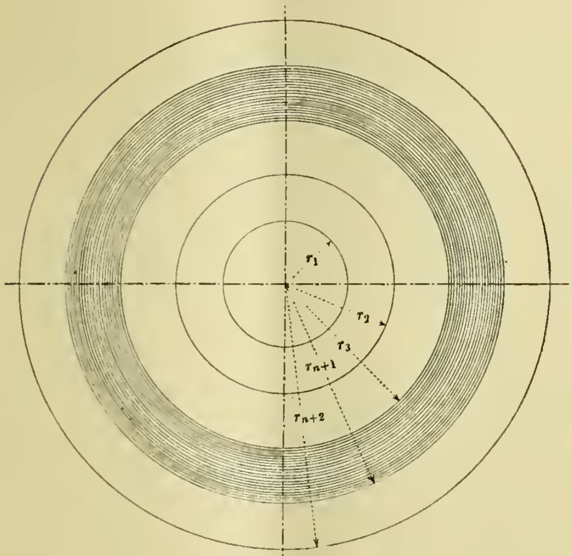


Fig. 5.

p_2, p_3, p_{n+1},	dieselben Spannungen im Ruhezustande,
E_1, E_2, E_d, E_4,	die Elastizitätsmoduln,
m_1, m_2, m_d, m_4,	die Poissonschen Konstanten,
S_1, S_2, S_d, S_4,	die zulässigen Inanspruchnahmen auf Zug des inneren und des äusseren Kerns, der Drähte und bzw. des Mantelzylinders. Ferner setzen wir

$$k_1 = \frac{r_2}{r_1}, \quad k_2 = \frac{r_3}{r_2}, \quad k_3 = \frac{r_{n+1}}{r_3}, \quad k_{n+1} = \frac{r_{n+2}}{r_{n+1}}.$$

Der Mantelzylinder erfährt einen Innendruck P_{n+1}, also besteht die folgende Beziehung nach (5)

$$P_{n+1} = \frac{m_4(k_{n+1}^2 - 1)S_4}{(m_4+1)k_{n+1}^2 + m_4 - 1},$$

welche die grösste Normalspannung für den Mantelzylinder ist.

Führen wir die Bedingung der gleichmässigen Umfangsdeh-

nung der Drahtlage ein, so ergibt sich die Normalspannung

$$P_3=\frac{m_d}{m_d-1}S_o\left(k_3^{\frac{m_d-1}{m_d}}-1\right)+k_3^{\frac{m_d-1}{m_d}}P_{n+1},$$

und für die Normalspannung zwischen dem beiden Zylindern des Doppelkerns hat man aus der Grundgleichung;

$$P_2=\frac{m_2(k_2^2-1)S_2}{(m_2+1)k_2^2+m_2-1}+\frac{2m_2k_2^2}{(m_2+1)k_2^2+m_2-1}P_3,$$

$$P_1=\frac{m_1(k_1^2-1)S_1}{(m_1+1)k_1^2+m_1-1}+\frac{2m_1k_1^2}{(m_1+1)k_1^2+m_1-1}P_2.$$

Die letzte Gleichung zeigt den Innendruck, dem das Geschütz Widerstand leisten soll.

Die Spannungen im Ruhezustande zu ermitteln, brauchen wir nur die Spannungen im natürlichen Zylinder unter Innendruck P_1, von den oben gefundenen Spannungen im Geschütze im Momente des Schusses zu subtrahieren; dazu ist erforderlich, die Normalspannungen auf den Berührungsflächen im natürlichen Zylinder auszudrücken.

Seien P_2', P_3', und P'_{n+1} der Normaldruck, wo $r=r_2$, $r=r_3$ und $r=r_{n+1}$ ist, so bekommen wir nach Vergleichung der Umfangsdehnungen

$$P'_{n+1}=B_3P_3',$$

$$P_3'=B_2P_2',$$

$$P_2'=B_1P_1, \quad \text{.....................................}(24)$$

wo

$$B_3=\frac{\dfrac{2}{E_d(k_3^2-1)}}{\dfrac{m_4-1+(m_4+1)k_{n+1}^2}{m_4E_4(k_{n+1}^2-1)}+\dfrac{m_d+1+(m_d-1)k_3^2}{m_dE_d(k_3^2-1)}},$$

$$B_2 = \frac{\dfrac{2}{E_2(k_2^2-1)}}{\dfrac{m_d-1+(m_d+1)k_3^2}{m_dE_d(k_3^2-1)} - \dfrac{2B_3k_3^2}{E_d(k_3^2-1)} + \dfrac{m_2+1+(m_2-1)k_2^2}{m_2E_2(k_2^2-1)}},$$

$$B_1 = \frac{\dfrac{2}{E_1(k_1^2-1)}}{\dfrac{m_2-1+(m_2+1)k_2^2}{m_2E_2(k_2^2-1)} - \dfrac{2B_2k_2^2}{E_2(k_2^2-1)} + \dfrac{m_1+1+(m_1-1)k_1^2}{m_1E_1(k_1^2-1)}}$$

ist.

Daraus folgt

$$P_3' = B_1B_2P_1, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

$$P'_{n+1} = B_1B_2B_3P_1, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

und

$$p_2 = P_2 - B_1P_1\,, \quad p_3 = P_3 - B_1B_2P_1\,, \quad p_{n+1} = P_{n+1} - B_1B_2B_3P_1\,.$$

Die Spannungen in der tangentialen Richtung im Ruhezustande sind sehr leicht zu ermitteln, so wird z. B. die Druckspannung an der Innenseite des Innenkerns;

$$t_1 = -\frac{2k_1^2}{k_1^2-1}p_2\,.$$

Es bleibt uns noch übrig die Temperaturspannungen zu finden. Bedeuten α_1, α_2, α_d und α_4 die Ausdehnungskoëffizienten des Innen- und des Aussenkerns, der Drähte, und des Mantelzylinders, θ_1, θ_2, θ_3, θ_{n+1} und θ_{n+2} die Temperaturen an den Stellen $r=r_1$, $r=r_2$, $r=r_3$, $r=r_{n+1}$ und $r=r_{n+2}$; Π_2, Π_3, und Π_{n+1} die von der Temperaturspannung erzeugten Drucke auf den Berührungsflächen, dann haben wir wieder nach Vergleichung der Dehnungen,

$$\Pi_{n+1} = \mathfrak{A}_3 + \mathfrak{B}_3\Pi_3,$$

$$\Pi_3 = \mathfrak{A}_2 + \mathfrak{B}_2\Pi_2,$$

$$\Pi_2 = \mathfrak{A}_1, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

wo

$$\mathfrak{A}_3 = \frac{\frac{m_d - 1}{m_d E_d}\sigma_{n+1,d} - \frac{m_4 - 1}{m_4 E_4}\sigma_{n+1,4}}{\frac{m_4 - 1 + (m_4 + 1)k_{n+1}^2}{m_4 E_4 (k_{n+1}^2 - 1)} + \frac{m_d + 1 + (m_d - 1)k_3^2}{m_d E_d (k_3^2 - 1)}},$$

$$\mathfrak{A}_2 = \frac{\frac{m_2 - 1}{m_2 E_2}\sigma_{3,2} - \frac{m_d - 1}{m_d E_d}\sigma_{3,d} + \frac{2k_3^2}{E_d(k_3^2 - 1)}\mathfrak{A}_3}{\frac{m_d - 1 + (m_d + 1)k_3^2}{m_d E_d (k_3^2 - 1)} - \frac{2k_3^2}{E_d(k_3^2 - 1)}\mathfrak{B}_3 + \frac{m_2 + 1 + (m_2 - 1)k_2^2}{m_2 E_2 (k_2^2 - 1)}},$$

$$\mathfrak{A}_1 = \frac{\frac{m_1 - 1}{m_1 E_1}\sigma_{2,1} - \frac{m_2 - 1}{m_2 E_2}\sigma_{2,2} + \frac{2k_2^2}{E_2(k_2^2 - 1)}\mathfrak{A}_2}{\frac{m_2 - 1 + (m_2 + 1)k_2^2}{m_2 E_2 (k_2^2 - 1)} - \frac{2k_2^2}{E_2(k_2^2 - 1)}\mathfrak{B}_2 + \frac{m_1 + 1 + (m_1 - 1)k_1^2}{m_1 E_1 (k_1^2 - 1)}},$$

$$\mathfrak{B}_3 = \frac{\frac{2}{E_d(k_3^2 - 1)}}{\frac{m_4 - 1 + (m_4 + 1)k_{n+1}^2}{m_4 E_4 (k_{n+1}^2 - 1)} + \frac{m_d + 1 + (m_d - 1)k_3^2}{m_d E_d (k_3^2 - 1)}},$$

$$\mathfrak{B}_2 = \frac{\frac{2}{E_2(k_2^2 - 1)}}{\frac{m_d - 1 + (m_d + 1)k_3^2}{m_d E_d (k_3^2 - 1)} - \frac{2k_3^2}{E_d(k_3^2 - 1)}\mathfrak{B}_3 + \frac{m_2 + 1 + (m_2 - 1)k_2^2}{m_2 E_2 (k_2^2 - 1)}}$$

ist, wonach

$$H_3 = \mathfrak{A}_2 + \mathfrak{B}_2\mathfrak{A}_1, \quad \text{...} (28)$$

$$H_{n+1} = \mathfrak{A}_3 + \mathfrak{B}_3(\mathfrak{A}_2 + \mathfrak{B}_2\mathfrak{A}_1). \quad \text{...........................} (29)$$

In diesen Gleichungen ist σ die Temperaturspannung, wo der vordere Index die Lage der Fläche anzeigt, während der hintere den Innen- und Aussenkern, den Drahtteil und bzw. den Mantelzylinder bedeutet. Natürlich sind diese Ausdrücke auch für die aus fünf Hohlzylindern bestehende beringte Geschützkonstruktion verwendbar, in welcher bis jetzt, so viel ich weiss, die Temperaturspannungen noch nicht berücksichtigt worden sind. Nun lassen sich die Spannungen unter Innendruck mit Rücksicht

auf die Temperaturdifferenz ohne weiteres schreiben; es ergiebt sich nämlich die reduzierte Spannung in tangentialer Richtung an der äusseren Drahtschichte

$$S=S_d+\frac{m_d-1}{m_d E_d}\sigma_{n+1,d}+\frac{2}{E_d(k_3^2-1)}H_3-\frac{m_d+1+(m_d-1)k_3^2}{m_d E_d(k_3^2-1)}H_{n+1},$$

indem statt H_3 und H_{n+1}, die Werte aus Gleichungen (25) und (26) einzuführen sind.

Man sieht aus diesem Resultate, dass wenn die Innentemperatur höher als die äussere ist, die reduzierte Spannung nach tangentialer Richtung die zulässige Inanspruchnahme auf Zug überschreitet, und zwar, dass sich die Bruchgefahr an der äussersten Schichte der Drahtlage findet.

Für die Windungsspannung der Drähte erhält man aus Gl. (15),

$$W=t+\frac{a_1 r^2+b_1 r_3^2}{a_1 r^2-b_1 r_3^2}p, \quad \text{..................................}(30)$$

wo

$$t=\frac{m_d}{m_d-1}S_d\left[1-\frac{1}{m_d}\left(\frac{r_{n+1}}{r}\right)^{\frac{m_d-1}{m_d}}\right]-\frac{k_3^{\frac{m_d-1}{m_d}}}{m_d}P_{n+1}-\frac{\left(\frac{r_{n+1}}{r}\right)^2+1}{k_3^2-1}P_3' \\ +\frac{\left(\frac{r_{n+1}}{r}\right)^2+k_3^2}{k_3^2-1}P'_{n+1},$$

$$p=\frac{m_d}{m_d-1}S_d\left[\left(\frac{r_{n+1}}{r}\right)^{\frac{m_d-1}{m_d}}-1\right]+k_3^{\frac{m_d-1}{m_d}}P_{n+1}-\frac{\left(\frac{r_{n+1}}{r}\right)^2-1}{k_3^2-1}P_3' \\ +\frac{\left(\frac{r_{n+1}}{r}\right)^2-k_3^2}{k_3^2-1}P'_{n+1},$$

$$a_1=\frac{m_d E_d}{m_2 E_2(k_2^2-1)}\left[m_2+1+(m_2-1)k_2^2-2m_2 A_1\right]+m_d+1,$$

$$b_1=a_1-2m_d,$$

und

$$A_1=\frac{\dfrac{2k_2^2}{E_2(k_2^2-1)}}{\dfrac{m_1+1+(m_1-1)k_1^2}{m_1E_1(k_1^2-1)}+\dfrac{m_2-1+(m_2+1)k_2^2}{m_2E_2(k_2^2-1)}}$$

zu setzen ist.

Es sei D_{n+1} die Schrumpfmasse zwischen Drahtteil und Mantelzylinder, die den Druck P_{n+1} im Momente des Schusses erzeugen möge, D_2 dieselbe zwischen Innen- und Aussenkern. Ferner sei

$r_{n+1,4}$ der Innenradius des Mantelzylinders,

$r_{n+1,d}$ der Aussenradius der Drahtspule,

$r_{2,2}$ der Innenradius des Aussenkerns,

$r_{2,1}$ der Aussenradius des Innenkerns vor der Beringung, so bestehen, wenn man näherungsweise

$$\frac{r_{n+2}}{r_{n+1,4}}=k_{n+1}\,,\qquad \frac{r_{n+1,d}}{r_3}=k_3\,,\qquad \frac{r_{2,1}}{r_1}=k_1\,,$$

setzt, die folgenden Beziehungen:

$$D_{n+1}=2(r_{n+1,d}-r_{n+1,4})\,,$$

$$r_{n+1,4}\left(1+\frac{S_4}{E_4}\right)=r_{n+1,d}\left(1+\frac{S_d}{E_d}\right).$$

Daraus folgt, dass

$$D_{n+1}=2r_{n+1,d}\frac{S_4E_d-S_dE_4}{E_d(S_4+E_4)}$$

ist, und ähnlicherweise hat man

$$D_2=2r_{2,1}\left[1-\frac{1+\dfrac{2}{E_1(k_1^2-1)}P_1-\dfrac{m_1+1+(m_1-1)k_1^2}{m_1E_1(k_1^2-1)}P_2}{1+\dfrac{S_2}{E_2}}\right].$$

In den vorhergehenden Fällen haben wir nur die Wider-

standsfähigkeit auf Zug betrachtet, aber eine ganz ähnliche Betrachtung ermöglicht die Formeln aufzustellen, wo die Inanspruchnahme des Materials auf Druck berücksichtigt ist.

Wenn die Schrumpfmassen D_{n+1} und D_2 als vorher bekannt vorausgesetzt sind, können wir die Spannungen im Mantelzylinder und Aussenkern nicht mehr beliebig nehmen, während die Spannungen in der Drahtlage und im Innenkern beliebig annehmbar sind. Setzen wir voraus, dass die Drähte so aufgewickelt sind, dass eine gleichmässige Umfangsdehnung unter Innendruck herrscht.

Vergleicht man die Dehnungen der Berührungsfläche, so bekommt man die Drucke als Funktionen der gegebenen Schrumpfmasse

$$P_{n+1}=\frac{m_4E_4(k_4^2-1)\left(2r_{n+1,d}\frac{S_d}{E_d}+D_{n+1}\right)}{[(m_4+1)k_4^2+m_4-1](2r_{n+1,d}-D_{n+1})}, \dots\dots\dots\dots(31)$$

$$P_3=\frac{m_d}{m_d-1}S_d\left(k_3^{\frac{m_d-1}{m_d}}-1\right)+k_3^{\frac{m_d-1}{m_d}}P_{n+1}. \dots\dots\dots\dots(32)$$

Es bleibt uns noch übrig, den Innendruck, der die gegebene Inanspruchnahme im Innenkern erfährt, zu bestimmen. Hierbei haben wir drei Gleichungen mit drei Unbekannten, nämlich

$$S_1=\frac{(m_1+1)k_1^2+m_1-1}{m_1(k_1^2-1)}P_1-\frac{2k_1^2}{k_1^2-1}P_2,$$

$$S_2=\frac{(m_2+1)k_2^2+m_2-1}{m_2(k_2^2-1)}P_2-\frac{2k_2^2}{k_2^2-1}P_3,$$

$$D_2=2r_{2,1}\left[1-\frac{1+\frac{2}{E_1(k_1^2-1)}P_1-\frac{(m_1-1)k_1^2+m_1+1}{m_1E_1(k_1^2-1)}P_2}{1+\frac{S_2}{E_2}}\right],$$

woraus

$$P_1=\frac{\dfrac{D_2}{2r_{2,1}}+\Big(1-\dfrac{D_2}{2r_{2,1}}\Big)\dfrac{(m_2-1)k_2^2+m_2+1}{m_2E_2(k_2^2-1)}P_3+\Big[\dfrac{(m_1-1)k_1^2+m_1+1}{m_1E_1(k_1^2-1)}}{\dfrac{\Big[\dfrac{(m_1-1)k_1^2+m_1+1}{m_1E_1(k_1^2-1)}+\Big(1-\dfrac{D_2}{2r_{2,1}}\Big)\dfrac{(m_2-1)k_2^2+m_2+1}{m_2E_2(k_2^2-1)}\Big]+\dfrac{(m_2+1)k_2^2+m_2-1}{m_2E_2(k_2^2-1)}\Big(1-\dfrac{D_2}{2r_{2,1}}\Big)\Big]\dfrac{S_1(k_1^2-1)}{2k_1^2}}{\dfrac{(m_1+1)k_1^2+m_1-1}{2m_1k_1^2}-\dfrac{2}{E_1(k_1^2-1)}}} \quad \text{.........(33)}$$

wird.

Für die Windungsspannungen der Drähte ist Gl. (30) auch gültig, wo nur statt P_{n+1} und P_1 die Werte aus Gl. (31) und (33) einzusetzen sind.

Wenn die Windungsspannungen und die Schrumpfmasse nicht nach den gegebenen Bedingungen, sondern schon vorher festgestellt worden sind, dann stellen wir uns die Aufgabe wie folgt vor:

Es seien D_{n+1}, D_2 die Schrumpfmassen, W_3, $W_4, \ldots\ldots W_{n-1}$, W_n die Windungsspannungen der Drähte an der Lage mit den Radien r_3 und r_4, r_4 und $r_5, \ldots\ldots r_n$ und r_{n+1}, dann beträgt der Druck an der Aussenseite des Drahtteils nach der Beringung des Mantelzylinders mit der Schrumpfmasse D_{n+1},

$$p_{n+1}=\frac{\dfrac{D_{n+1}}{2r_{n+1,\,d}}}{\dfrac{(m_4+1)k_{n+1}^2+m_4-1}{E_4(k_{n+1}^2-1)}\Big(1-\dfrac{D_{n+1}}{2r_{n+1,\,d}}\Big)-\dfrac{1}{m_dE_d}\Big[1-\dfrac{m_d(a_1k_3^2+b_1)}{a_1k_3^2-b_1}\Big]}\,.$$

Die radiale Spannung an r, wo $r_n \lesseqgtr r < r_{n+1}$ ist, wird

$$p=\frac{W_n}{2a_1r^2}(a_1r^2-b_1r_3^2)\,lg\,\frac{a_1r_{n+1}^2-b_1r_3^2}{a_1r^2-b_1r_3^2}+\Big(\frac{r_{n+1}}{r}\Big)^2\frac{a_1r^2-b_1r_3^2}{a_1r_{n+1}^2-b_1r_3^2}p_{n+1},$$

und an der Lage mit den Radien r_m und r_{m+1} geht sie über in

$$p=\frac{a_1r^2-b_1r_3^2}{2a_1r^2}\Big(\sum_{x=n}^{m+1}W_x\,lg\,\frac{a_1r_{x+1}^2-b_1r_3^2}{a_1r_x^2-b_1r_3^2}+W_m\,lg\,\frac{a_1r_{m+1}^2-b_1r_3^2}{a_1r^2-b_1r_3^2}+\frac{2a_1k_3^2}{a_1k_3^2-b_1}p_{n+1}\Big),$$

$$\text{................................(34)}$$

woraus folgt

$$t = W_m - \frac{a_1 r^2 + b_1 r_3^2}{a_1 r^2 - b_1 r_3^2} p_n \text{.} \qquad (35)$$

An der Mantelfläche des Aussenkerns ergibt sich die radiale Spannung

$$p_3 = \frac{m_d}{a_1}\left(\sum_{x=n}^{4} W_x lg \frac{a_1 r_{x+1}^2 - b_1 r_3^2}{a_1 r_x^2 - b_1 r_3^2} + W_3 lg \frac{a_1 r_4^2 - b_1 r_3^2}{2 m_d r_3^2} + \frac{2 a_1 k_3^2}{a_1 k_3^2 - b_1} p_{n+1}\right),$$

dann bekommt man nach Vergleichung der Umfangsdehnungen an der Berührungsfläche des Aussen- und Innenkerns,

$$p_2 = \frac{\frac{D_2}{2r_{2,1}} + \left(1 - \frac{D_2}{2r_{2,1}}\right)\frac{2k_1^2}{E_2(k_2^2-1)} p_3}{\left(1 - \frac{D_2}{2r_{2,1}}\right)\frac{(m_2+1)k_2^2 + m_2 - 1}{m_2 E_2 (k_2^2-1)} + \frac{(m_1-1)k_1^2 + m_1 + 1}{m_1 E_1 (k_1^2 - 1)}}.$$

Die Spannungen im Momente des Schusses sind ohne weiteres ermittelbar, weil die Drucke P_2', P_3' und P'_{n+1} schon in den Gl. (24), (25) und (26) aufgestellt worden sind. Hier wollen wir z. B. nur die Spannungen in der Drahtlage ausdrücken:

$$P = p + \left[\left(\frac{r_{n+1}}{r}\right)^2 - \left(\frac{r_{n+1}^2}{r^2} - k_3^2\right) B_3 - 1\right] \frac{B_1 B_2 P_1}{k_3^2 - 1},$$

$$T = t + \left[\left(\frac{r_{n+1}}{r}\right)^2 - \left(\frac{r_{n+1}^2}{r^2} + k_3^2\right) B_3 + 1\right] \frac{B_1 B_2 P_1}{k_3^2 - 1},$$

wo man p und t aus den Gl. (34) und (35) einzuführen hat.

Mit Rücksicht auf die Temperaturunterschiede findet man die Spannungen im Momente des Schusses sowie im Ruhezustande in derselben Weise, wie wir im vorhergehenden Falle getan haben; so lassen sie sich z. B. im Mantelzylinder unter Innendruck schreiben

$$P=\frac{\left(\frac{r_{n+2}}{r}\right)^2-1}{k_{n+1}^2-1}\left(p_{n+1}+B_1B_2B_3P_1+H_{n+1}\right)-\frac{m_4E_4a_4}{2(m_4-1)}\left[\frac{k_{n+1}^2\theta_{n+2}-\theta_{n+1}}{k_{n+1}^2-1}\right.$$
$$\left.+\left(\frac{r_{n+2}}{r}\right)^2\frac{\theta_{n+1}-\theta_{n+2}}{k_{n+1}^2-1}-\theta\right],$$

$$T=\frac{\left(\frac{r_{n+2}}{r}\right)^2+1}{k_{n+1}^2-1}\left(p_{n+1}+B_1B_2B_3P_1+H_{n+1}\right)+\frac{m_4E_4a_4}{2(m_4-1)}\left[\frac{k_{n+1}^2\theta_{n+2}-\theta_{n+1}}{k_{n+1}^2-1}\right.$$
$$\left.-\left(\frac{r_{n+2}}{r}\right)^2\frac{\theta_{n+1}-\theta_{n+2}}{k_{n+1}^2-1}+\frac{\theta_{n+1}-\theta_{n+2}}{lg\,k_{n+1}}-\theta\right],$$

$$\sigma_z=\frac{m_4E_4a_4}{m_4-1}(\theta_m-\theta).$$

In den oben erwähnten Formeln zur Festigkeit der Drahtkanonen treten immer die Verhältnisse der Radien hinein. Die Berechnung der Formeln zu erleichtern ist die Tabelle von k und seine Funktionen, nach der Annahme, dass $m=3$ ist, zusammengestellt.[1]

Waffentechnisches Institut d. Kaiserl. Universität zu Tokyo. März 1914.

1) Diese Tabellen für beringte Geschütze siehe:
L. Tsoucalas; Revue d'Artillerie T. 48 p. 567.
A. Mattei; Rivista di artig. e genio 1895.
O. Dirmoser; Mitteilung. ü. Gegenstände d. Artillerie u. Geniew. 1907 p. 311.

k	k^2	Diff.	$k^{\frac{2}{3}}$	Diff.	$\frac{3k^2}{2k^2+1}$	Diff.	$\frac{2k^2+1}{k^2-1}$	Diff.	$\frac{k^2+2}{k^2-1}$
1,10	1,2100	221	1,0655	65	1,0614	56	16,2857	1,3602	15,2857
1,11	1,2321	223	1,0720	65	1,0670	55	14,9255	1,1330	13,9255
1,12	1,2544	225	1,0785	64	1,0725	54	13,7925	9583	12,7925
1,13	1,2769	227	1,0849	64	1,0779	53	12,8342	8208	11,8342
1,14	1,2996	229	1,0913	64	1,0832	52	12,0134	7111	11,0134
1,15	1,3225	231	1,0977	63	1,0884	52	11,3023	6217	10,3023
1,16	1,3456	233	1,1040	64	1,0936	51	10,6806	5483	9,6806
1,17	1,3689	235	1,1104	63	1,0987	50	10,1323	4870	9,1323
1,18	1,3924	237	1,1167	63	1,1037	49	9,6453	4355	8,6453
1,19	1,4161	239	1,1230	63	1,1086	48	9,2098	3946	8,2098
1,20	1,4400	241	1,1293	62	1,1134	47	8,8152	3511	7,8152
1,21	1,4641	243	1,1355	62	1,1181	47	8.4641	3216	7,4641
1,22	1,4884	245	1,1417	63	1,1228	46	8,1425	2934	7,1425
1,23	1,5129	247	1,1480	62	1,1274	45	7,8491	2687	6,8491
1,24	1,5376	249	1,1542	62	1,1319	44	7,5804	2471	6,5804
1,25	1,5625	251	1,1604	62	1,1363	44	7,3333	2278	6,3333
1,26	1,5876	253	1,1666	61	1,1407	43	7,1055	2107	6,1055
1,27	1,6129	255	1,1727	61	1,1450	43	6,8948	1956	5,8948
1,28	1'6384	257	1,1788	62	1,1493	42	6,6992	1818	5,6992
1,29	1,6641	259	1,1850	61	1,1535	41	6,5174	1696	5,5174
1,30	1,6900	261	1,1911	61	1,1576	40	6,3478	1584	5,3478
1,31	1,7161	263	1,1972	61	1,1616	39	6,1894	1485	5,1894
1,32	1,7424	265	1,2033	61	1,1655	39	6,0409	1392	5,0409
1,33	1,7689	267	1,2094	60	1,1694	39	5,9017	1310	4,9017
1,34	1,7956	269	1,2154	61	1,1733	38	5,7707	1233	4,7707
1,35	1,8225	271	1,2215	60	1,1771	37	5,6474	1163	4,6474
1,36	1,8496	273	1,2275	60	1,1808	37	5,5311	1100	4,5311
1,37	1,8769	275	1,2335	60	1,1845	36	5,4211	1040	4,4211
1,38	1,9044	277	1,2395	60	1,1881	35	5,3171	986	4,3171
1,39	1,9321	279	1,2455	60	1,1916	35	5,2185	935	4,2185
1,40	1,9600	281	1,2515	59	1,1951	35	5,1250	889	4,1250
1,41	1,9881	283	1,2574	60	1,1986	34	5,0361	846	4,0361
1,42	2,0164	285	1,2634	59	1,2020	33	4,9515	804	3,9515
1,43	2,0449	287	1,2693	59	1,2053	33	4,8711	768	3,8711
1,44	2,0736	289	1,2752	59	1,2086	32	4,7943	732	3,7943
1,45	2,1025	291	1,2811	58	1,2118	32	4,7211	700	3,7211

k	k^2	*Diff.*	$k^{\frac{2}{3}}$	*Diff.*	$\frac{3k^2}{2k^2+1}$	*Diff.*	$\frac{3k^2}{2k^2+1}$	*Diff.*	$\frac{k^2+2}{k^2-1}$
1,46	2,1316	293	1,2869	59	1,2150	31	4,6511	669	3,6511
1,47	2.1609	295	1.2928	59	1,2181	31	4,5842	640	3,5842
1.48	2.1904	297	1,2987	58	1,2212	31	4,5202	614	3,5202
1,49	2,2201	299	1,3045	59	1,2243	30	4,4588	588	3,4588
1.50	2,2500	301	1,3104	58	1,2273	29	4,4000	564	3,4000
1,51	2.2801	303	1,3162	58	1,2302	29	4,3436	542	3,3436
1.52	2.3104	305	1,3220	58	1,2331	29	4,2894	521	3,2894
1,53	2.3409	307	1,3278	58	1,2360	28	4,2373	500	3,2373
1,54	2.3716	309	1,3336	58	1,2388	28	4,1873	483	3,1873
1,55	2,4025	311	1,3394	58	1,2416	27	4,1390	464	3,1390
1,56	2.4336	313	1,3452	57	1,2443	27	4,0926	447	3,0926
1.57	2,4649	315	1,3509	57	1,2470	27	4,0479	431	3,0479
1,58	2.4964	317	1,3566	57	1,2497	26	4,0048	416	3,0048
1,59	2,5281	319	1,3623	57	1,2523	26	3.9632	401	2,9632
1,60	2,5600	321	1,3680	57	1,2549	25	3,9231	388	2,9231
1,61	2,5921	323	1,3737	57	1,2574	25	3,8843	375	2,8843
1,62	2,6244	325	1,3794	56	1,2599	25	3,8468	362	2,8468
1,63	2,6569	327	1,3850	57	1,2624	25	3,8106	350	2,8106
1,64	2,6896	329	1,3907	56	1,2659	24	3,7756	339	2,7756
1.65	2,7225	331	1,3963	57	1,2673	24	3,7417	329	2,7417
1,66	2,7556	333	1,4020	56	1,2697	23	3,7088	318	2,7088
1,67	2,7889	335	1,4076	56	1,2720	23	3,6770	308	2,6770
1,68	2,8224	337	1,4132	56	1,2743	23	3,6462	299	2,6462
1,69	2,8561	339	1,4188	56	1,2766	22	3,6163	290	2,6163
1,70	2,8900	341	1,4244	56	1,2788	22	3,5873	281	2,5873
1,71	2,9241	343	1,4300	56	1,2810	22	3,5592	273	2,5592
1,72	2,9584	345	1,4356	55	1,2832	21	3,5319	265	2,5319
1,73	2,9929	347	1,4411	56	1,2853	21	3,5054	258	2,5054
1,74	3,0276	349	1,4467	55	1,2874	21	3,4796	251	2,4796
1,75	3,0625	351	1,4522	55	1,2895	21	3,4545	243	2,4545
1,76	3,0976	353	1,4577	55	1,2916	21	3,4302	237	2,4302
1,77	3,1329	355	1,4632	55	1,2936	20	3,4065	230	2,4065
1,78	3,1684	357	1,4687	55	1,2956	20	3,3835	224	2,3835
1,79	3,2041	359	1,4742	55	1,2976	20	3,3611	218	2,3611
1,80	3,2400	361	1,4797	55	1,2995	19	3,3393	213	2,3393
1,81	3,2761	363	1,4852	55	1,3014	19	3,3180	207	2,3180

k	k^2	Diff.	$k^{\frac{2}{3}}$	Diff.	$\frac{3k^2}{2k^2+1}$	Diff.	$\frac{2k^2+1}{k^2-1}$	Diff.	$\frac{k^2+2}{k^2-1}$
1,82	3,3124	365	1,4907	55	1,3033	19	3,2973	202	2,2973
1,83	3,3489	367	1,4962	54	1,3051	18	3,2771	196	2,2771
1,84	3,3856	369	1,5016	54	1,3070	19	3,2575	191	2,2575
1,85	3,4225	371	1,5070	54	1,3088	18	3,2384	187	2,2384
1,86	3,4596	373	1,5124	54	1,3106	18	3,2197	182	2,2197
1,87	3,4969	375	1,5178	54	1,3124	18	3,2015	178	2,2015
1,88	3,5344	377	1,5232	54	1,3141	17	3,1837	173	2,1837
1,89	3,5721	379	1,5286	54	1,3158	17	3,1664	170	2,1664
1,90	3,6100	381	1,5340	54	1,3175	17	3,1494	165	2,1494
1,91	3,6481	383	1,5394	54	1,3192	17	3,1329	161	2,1329
1,92	3,6864	385	1,5448	54	1,3209	17	3,1168	158	2,1168
1,93	3,7249	387	1,5502	54	1,3225	16	3,1010	154	2,1010
1,94	3,7636	389	1,5556	53	1,3241	16	3,0856	151	2,0858
1,95	3,8025	391	1,5609	53	1,3257	16	3,0705	148	2,0706
1,96	3,8416	393	1,5662	53	1,3273	16	3,0557	144	2,0557
1,97	3,8809	395	1,5715	53	1,3288	15	3,0413	141	2,0413
1,98	3,9204	397	1,5768	53	1,3303	15	3,0272	137	2,0272
1,99	3,9601	399	1,5821	53	1,3318	15	3,0135	135	2,0135
2,00	4,0000	401	1,5874	53	1,3333	15	3,0000	132	2,0000
2,01	4,0401	403	1,5927	53	1,3348	15	2,9868	129	1,9868
2,02	4,0804	405	1,5980	53	1,3363	14	2,9739	126	1,9739
2,03	4,1209	407	1,6033	53	1,3377	14	2,9613	123	1,9613
2,04	4,1616	409	1,6086	52	1,3391	14	2,9490	122	1,9490
2,05	4,2025	411	1,6138	52	1,3405	14	2,9368	119	1,9368
2,06	4,2436	413	1,6191	52	1,3419	14	2,9249	116	1,9249
2,07	4,2849	415	1,6243	52	1,3433	13	2,9133	114	1,9133
2,08	4,3264	417	1,6295	52	1,3446	13	2,9019	112	1,9019
2,09	4,3681	419	1,6347	52	1,3459	13	2,8907	110	1,8907
2,10	4,4100	421	1,6399	52	1,3472	13	2,8797	107	1,8797
2.11	4,4521	423	1,6351	52	1,3485	13	2,8690	105	1,8690
2,12	4,4944	425	1,6403	52	1.3498	13	2,8585	103	1,8585
2,13	4,5369	427	1,6555	52	1,3511	12	2,8482	101	1,8482
2,14	4,5796	429	1,6607	52	1,3523	12	2,8381	99	1,8381
2,15	4,6225	431	1,6659	52	1,3535	13	2,8282	97	1,8282
2,16	4,6656	433	1,6711	51	1,3548	12	2,8185	96	1,8185
2,17	4,7089	435	1,6762	52	1,3560	12	2,8089	94	1,8089

k	k^2	Diff.	$k^{\frac{2}{3}}$	Diff.	$\frac{3k^2}{2k^2+1}$	Diff.	$\frac{2k^2+1}{k^2-1}$	Diff.	$\frac{k^2+2}{k^2-1}$
2,18	4,7524	437	1,6814	51	1,3572	12	2,7995	93	1,7995
2,19	4,7961	439	1,6865	51	1,3584	12	2,7902	90	1,7902
2,20	4,8400	441	1,6916	51	1,3596	11	2,7812	88	1,7812
2,21	4,8841	443	1,6967	51	1,3607	11	2,7724	87	1,7724
2,22	4,9284	445	1,7018	51	1,3618	11	2,7637	86	1,7637
2,23	4,9729	447	1,7069	51	1,3629	11	2,7551	84	1,7551
2,24	5,0176	449	1,7120	51	1,3640	11	2,7467	82	1,7467
2,25	5,0625	451	1,7171	51	1,3651	11	2,7385	81	1,7385
2,26	5,1076	453	1,7222	50	1,3662	11	2,7304	80	1,7304
2,27	5,1529	455	1,7273	51	1,3673	11	2,7224	78	1,7224
2,28	5,1984	457	1,7324	51	1,3684	11	2,7146	77	1,7146
2,29	5,2441	459	1,7374	51	1,3695	10	2,7069	76	1,7069
2,30	5,2900	461	1,7425	50	1,3705	10	2,6993	74	1,6993
2,31	5,3361	463	1,7475	50	1,3715	10	2,6910	73	1,6919
2,32	5,3824	465	1,7526	50	1,3725	10	2,6846	72	1,6846
2,33	5,4289	467	1,7576	50	1,3735	10	2,6774	71	1,6774
2,34	5,4756	469	1,7626	50	1,3745	10	2,6703	70	1,6703
2,35	5,5225	471	1,7676	50	1,3755	09	2,6633	68	1,6633
2,36	5,5696	473	1,7726	50	1,3764	10	2,6565	67	1,6565
2,37	5,6169	475	1,7776	50	1,3774	09	2,6498	66	1,6498
2,38	5,6644	477	1,7826	50	1,3783	10	2,6432	65	1,6432
2,39	5,7121	479	1,7876	50	1,3793	09	2,6367	64	1,6367
2,40	5,7600	481	1,7926	50	1,3802	09	2,6303	63	1,6303
2,41	5,8081	483	1,7976	50	1,3811	09	2,6240	62	1,6240
2,42	5,8564	485	1,8026	49	1,3820	09	2,6178	61	1,6178
2,43	5,9049	487	1,8075	50	1,3829	09	2,6117	60	1,6117
2,44	5,9536	489	1,8125	49	1,3838	09	2,6057	59	1,6057
2,45	6,0025	491	1,8174	50	1,3847	09	2,5996	58	1,5996
2,46	6,0516	493	1,8224	49	1,3856	08	2,5938	57	1,5938
2,47	6,1009	495	1,8373	49	1,3864	09	2,5881	56	1,5881
2,48	6,1504	497	1,8322	49	1,3873	08	2,5825	56	1,5825
2,49	6,2001	499	1,8371	49	1,3881	08	2,5769	55	1,5769
2,50	6,2500		1,8420		1,3889		2,5714		1,5714

GRAPHICAL STUDY

OF

THE CENTRIFUGAL PUMP

ERRATA

Page 291, line 11 from bottom, *for* ne *read* line.

,, 302, ,, 2 ,, ,, ,, $\frac{w_0^2}{2}$,, $\frac{\overline{w_0}^2}{2}$.

k	k^2	Diff.	$k^{\frac{2}{3}}$	Diff.	$\frac{3k^2}{2k^2+1}$	Diff.	$\frac{2k^2+1}{k^2-1}$	Diff.	$\frac{k^2+2}{k^2-1}$
2,18	4,7524	437	1,6814	51	1,3572	12	2,7995	93	1,7995
2,19	4,7961	439	1,6865	51	1,3584	12	2,7902	90	1,7902
2,20	4,8400	441	1,6916	51	1,3596	11	2,7812	88	1,7812
2,21	4,8841	443	1,6967	51	1,3607	11	2,7724	87	1,7724
2,22	4,9284	445	1,7018	51	1,3618	11	2,7637	86	1,7637
2,23	4,9729	447	1,7069	51	1,3629	11	2,7551	84	1,7551
2,24	5,0176	449	1,7120	51	1,3640	11	2,7467	82	1,7467
2,25	5,0625	451	1,7171	51	1,3651	11	2,7385	81	1,7385
2,26	5,1076	453	1,7222	50	1,3662	11	2,7304	80	1,7304
2,27	5,1529	455	1,7273	51	1,3673	11	2,7224	78	1,7224
2,2									
2,2									
2,3									
2,3									
2,3									
2,3									
2,3									
2,3									
2,3									
2,37									
2,3									
2,39	5,7121	479	1,7876	50	1,3793	09	2,6367	64	1,6367
2,40	5,7600	481	1,7926	50	1,3802	09	2,6303	63	1,6303
2,41	5,8081	483	1,7976	50	1,3811	09	2,6240	62	1,6240
2,42	5,8564	485	1,8026	49	1,3820	09	2,6178	61	1,6178
2,43	5,9049	487	1,8075	50	1,3829	09	2,6117	60	1,6117
2,44	5,9536	489	1,8125	49	1,3838	09	2,6057	59	1,6057
2,45	6,0025	491	1,8174	50	1,3847	09	2,5996	58	1,5996
2,46	6,0516	493	1,8224	49	1,3856	08	2,5938	57	1,5938
2,47	6,1009	495	1,8373	49	1,3864	09	2,5881	56	1,5881
2,48	6,1504	497	1,8322	49	1,3873	08	2,5825	56	1,5825
2,49	6,2001	499	1,8371	49	1,3881	08	2,5769	55	1,5769
2,50	6,2500		1,8420		1,3889		2,5714		1,5714

GRAPHICAL STUDY

OF

THE CENTRIFUGAL PUMP

BY

O. Miyagi, *Kōgakushi*

THE JOURNAL OF THE COLLEGE OF ENGINEERING,
TOKYO IMPERIAL UNIVERSITY, TOKYO, JAPAN
Vol. V. No. 10

Published December, 1914

Graphical Study of the Centrifugal Pump

By

O. Miyagi, *Kōgakushi*

Many practical engineers and designers, who have attempted to determine the shape or to discuss the form of the blades of a water-turbine rotor or of a centrifugal pump impeller, have long confined their attention to mathematical formulæ based upon hydraulic investigations. Although some of the graphical solutions have in certain cases been applied, it has been only to a small portion of the whole process.

The method I propose in this paper, is a pure graphical solution throughout the whole process of discussion, and greatly simplifies studies on the rotor or impeller blades of these machines.

Almost everyone knows the so-called entropy-diagrams and how to make the best use of them in solving many problems relating to steam-turbine vanes, but I wonder why no one has ever proposed a diagram relating to water turbines and centrifugal pumps, as convenient as an entropy diagram. This is why I have thought of constructing a new diagram relating to water turbines and centrifugal pumps, which is analogous in many respects to an entropy-diagram in the case of a steam turbine. The diagram, I here suggest, may not be the best one, but I believe it to be one

of the best. I shall be only too pleased if some one will improve upon it.

The centrifugal pump is, in all respects, the reverse of the water turbine, and the discussion of the centrifugal pump will equally be applicable at once to the water turbine. I shall, therefore, proceed to show the applications only to the centrifugal pump, for the sake of brevity in statement.

Fundamental Hydraulic Equations

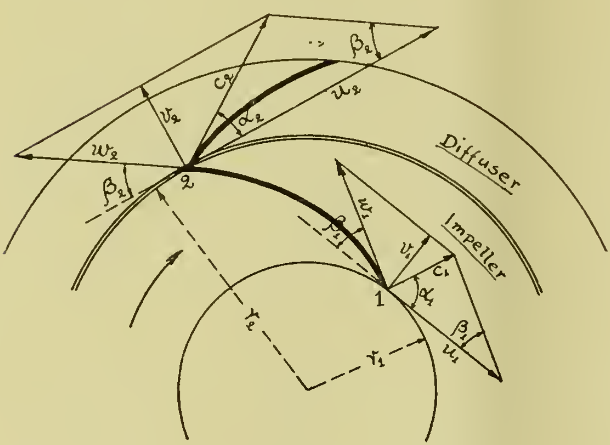


Fig. 1

Referring to Fig. 1, the notations with the suffix 1 signify those at the point of entrance into the impeller channel, and those with the suffix 2 signify those at the point of exit from the impeller channel and entrance into the diffuser. Let

c_1 and c_2 = absolute velocities of water,

w_1 and w_2 = velocities of water relative to the impeller,

u_1 and u_2 = peripheral or circumferential velocities of the impeller,

v_1 and v_2 = radial velocities of water,

α_1 = angle between u_1 and c_1, denoting the direction of the absolute entrance velocity,

α_2 = angle between u_2 and c_2, denoting the direction of the absolute exit velocity, or the entrance angle of diffuser,

β_1 and β_2 = entrance and exit angles, respectively, of the impeller,

r_1 and r_2 = inside and outside radii of the impeller,

H = actual total head of water, or the actual height between the higher and lower water levels,

g = acceleration due to gravity,

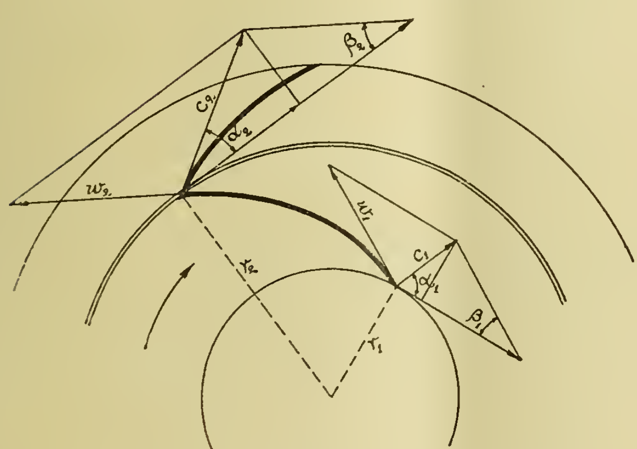


Fig. 2

all the units being conveniently expressed in the foot-pound-second system of measurement.

From Fig. 2 the tangential component of the change of momentum per second through the impeller, if m be the mass of water flowing per second, is given by

$$m\, c_2 \cos \alpha_2 - m\, c_1 \cos \alpha_1.$$

This is no other than the force of rotation of the impeller. The moment imparted to the water by the impeller is, therefore,

$$r_2\, m\, c_2 \cos \alpha_2 - r_1\, m\, c_1 \cos \alpha_1.$$

Hence, if ω be the angular velocity of the impeller per second, the energy E given to the water by the impeller per second, will be

$$E = (r_2\, m\, c_2 \cos \alpha_2 - r_1\, m\, c_1 \cos \alpha_1)\, \omega,$$

or since $\qquad \omega\, r_2 = u_2 \qquad$ and $\qquad \omega\, r_1 = u_1,$

$$E = m\,(u_2\,c_2 \cos \alpha_2 - u_1\,c_1 \cos \alpha_1). \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

If there were no resistance to the flow of the water, the energy given to the water per second would be equal to mgH. But actually the flow is always exposed to hydraulic resistances due to friction, bends, sudden changes of section, &c., which require additional energy. Moreover, the water must be carried through the delivery pipe with the velocity c_d, called the residual velocity, and the energy must be increased by the amount $\frac{c_d^2}{2g}$. The sum of heads due to hydraulic resistances is usually denoted by the expression $\Sigma\zeta\frac{c_2^2}{2g}$, ζ being the coefficient of resistance. The head, therefore, through which the pump strives to lift the water is always greater than H, and is

$$H + \Sigma\zeta\frac{c_2^2}{2g} + \frac{c_d^2}{2g},$$

which is usually called the theoretical total head. We may generally consider the lost head $\left(\Sigma\zeta\frac{c_2^2}{2g} + \frac{c_d^2}{2g}\right)$ to be some percentage of the actual total head H, and to be represented by the equation

$$\Sigma\zeta\frac{c_2^2}{2g} + \frac{c_d^2}{2g} = \lambda H,$$

λ being some coefficient. Hence the theoretical total head is expressed by

$$H + \lambda H \qquad \text{or} \qquad (1 + \lambda) H,$$

or calling $1 + \lambda = \phi$, the theoretical total head is now

$$\phi H.$$

It will, therefore, be seen that the value of ϕ is always greater than unity for a centrifugal pump, and $\frac{1}{\phi}$ is called the hydraulic efficiency of the pump, which is generally represented by the symbol η.

For a water turbine the theoretical total head is obviously equal to

$$H - \left(\Sigma \zeta \frac{c_2^2}{2g} + \frac{c_d^2}{2g} \right) \quad \text{or} \quad (1-\lambda) H.$$

Calling $(1-\lambda)=\eta$, η is always less than unity and is called at once the hydraulic efficiency of the turbine. The distinction of η for a centrifugal pump and for a water turbine should be clearly distinguished.

From the above reasoning the energy actually given to the water per second by the impeller will be equal to $mg\phi H$, which is no other than the value of E. thus

$$E = mg\phi H. \qquad \text{.................................(2)}$$

Equating the values of E given in the equations (1) and (2), we obtain

$$u_2 c_2 \cos\alpha_2 - u_1 c_1 \cos\alpha_1 = g\phi H. \qquad \text{................................(A)}$$

This is a well-known hydraulic equation relating to the theory of the centrifugal pump and the water turbine, so popular that no further explanation of it is needed.

The direction of the absolute entrance velocity c_1 is indeterminable unless guide vanes are used at the entrance, as in usual practice. It is obvious that the angle α_1 is somewhat less than 90°, for the water in the vicinity of the entrance tip is in some kind of rotational motion due to the viscosity of water; but as this rotational motion is practically an indeterminate one, we can not give a true angle of α_1. The angle α_1 is evidently dependent on the peripheral velocity u_1, and increases as u_1 decreases, approaching gradually to 90°. In all practical cases, however, it will not differ so very much from 90°, and it will be better to assume $\alpha_1=90°$, or the direction of the absolute entrance velocity to be radial, rather than to give a probable indeterminate angle. Assuming this, all

the solutions become greatly simplified, for in this case the equation (A) becomes simply

$$u_2 c_2 \cos\alpha_2 = g\phi H. \quad \text{...(B)}$$

When the pump is in the best condition the direction of the relative entrance velocity w_1 is the direction of the impeller blade at the entrance, and the direction of the absolute exit velocity c_2 is the direction of the diffuser blade at its entrance. When the quantity of flow is changed the directions of w_1 and c_2 are also changed, and there arise additional losses due to shock at the entrance into the impeller and into the diffuser. If the directions of w_1 and c_2 do not coincide with the directions of the blades, resolve them into directions tangential and normal to the vanes, as shown in Fig. 3. The tangential components w'_1 and c'_2 are the velocities with which the flow takes place through the channels, and the normal components w_0 and c_0 are the velocities entirely lost by shock. The additional loss of head due to shock, therefore, at the entrance into the impeller is $\frac{w_0^2}{2g}$, and at the entrance into the diffuser is $\frac{c_0^2}{2g}$. Hence, in this case, the equation (B) will take the form

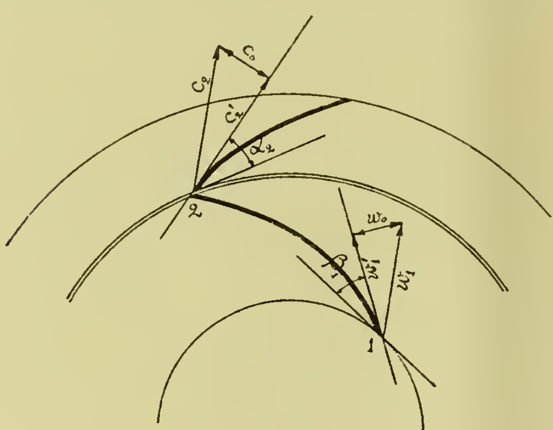


Fig. 3

$$u_2 c_2 \cos\alpha_2 = g\left(\phi H + \frac{w_0^2}{2g} + \frac{c_0^2}{2g}\right),$$

or in another form

$$u_2 c_2 \cos\alpha_2 = g\phi H + \frac{w_0^2}{2} + \frac{c_0^2}{2} \quad \text{.........................(C)}$$

The hydraulic efficiency η in this case is evidently

$$\eta=\frac{gH}{g\phi H+\frac{w_0^2}{2}+\frac{c_0^2}{2}}. \qquad \text{.........................(D)}$$

The pressure head h_1 existing at the entrance into the impeller is given by

$$h_1=h_a-h_s-\frac{c_s^2}{2g}-\zeta_s\frac{c_s^2}{2g},$$

and the pressure head h_2 existing at the exit from the impeller is given by

$$h_2=h_a+h_d+\frac{c_d^2}{2g}-\frac{c_2^2}{2g}+\zeta_d\frac{c_d^2}{2g}.$$

In these equations,

h_a =atmospheric pressure expressed in terms of the head of water,

h_s and h_d=suction and delivery heads respectively, whose sum gives the total head H,

c_s and c_d=velocities of water in the suction and the delivery pipes respectively,

ζ_s =coefficient of lost heads in the suction pipe from its inlet opening to the point of entrance into the impeller,

and ζ_d =coefficient of lost heads in the delivery pipe from the point of exit from the impeller to the discharge orifice of the pipe.

The difference of pressures existing in the clearance space between the side of the impeller and the casing, in terms of the head of water, is evidently the difference of heads h_2 and h_1. Hence calling this pressure difference existing in the clearance space, h_c, we have

$$h_c = h_2 - h_1 = h_s + h_d + \frac{c_s^2}{2g} + \frac{c_d^2}{2g} - \frac{c_2^2}{2g} + \zeta_s \frac{c_s^2}{2g} + \zeta_d \frac{c_d^2}{2g},$$

or $$h_c = H - \frac{c_2^2}{2g} + \zeta_s \frac{c_s^2}{2g} + \zeta_d \frac{c_d^2}{2g} + \frac{c_s^2}{2g} + \frac{c_d^2}{2g}.$$

Practically, we may assume with approximate correctness that the head $\frac{c_s^2}{2g}$ in the suction pipe is entirely lost by friction, bends, and various shocks in the impeller channel. Under this assumption, the quantity $\left(\zeta_s \frac{c_s^2}{2g} + \zeta_d \frac{c_d^2}{2g} + \frac{c_s^2}{2g}\right)$ is the total hydraulic loss through the whole system which is simply represented by the expression $\Sigma\zeta \frac{c_2^2}{2g}$ as given above. Consequently the above equation becomes

$$h_c = H - \frac{c_2^2}{2g} + \Sigma\zeta \frac{c_2^2}{2g} + \frac{c_d^2}{2g}.$$

The value $\left(\Sigma\zeta \frac{c_2^2}{2g} + \frac{c_d^2}{2g}\right)$ is equal to λH shown above.

Hence we have

$$h_c = H - \frac{c_2^2}{2g} + \lambda H = (1+\lambda)H - \frac{c_2^2}{2g},$$

and since $1+\lambda = \phi$, we get finally

$$h_c = \phi H - \frac{c_2^2}{2g}. \qquad \text{(E)}$$

which is an expression for the pressure difference existing in the clearance space. The leakage of water from the delivery side to the suction occurs through this clearance, and its amount is expressed by the equation

$$q = Ca\sqrt{2gh_c}, \qquad \text{(F)}$$

in which

q=quantity of water leaking in a unit of time,

a=area of the clearance between the impeller and casing at the smallest section,

and C=coefficient of discharge through that section.

From this it will be seen that the amount of leakage is proportional to the square-root of h_c for a given pump.

For a pump or turbine of the impulse type, the pressure in the channel is uniform throughout or rather less at the exit than at the inlet, and this means h_c is zero or negative. Hence from the equation (E), the necessary condition of the impulse type is

$$\phi H \gtreqless \frac{c_2^2}{2g} \quad \text{.................................(G)}$$

and, in this case, the leakage will not occur or rather outward.

Graphical Representation of the Equation $uc\cos\alpha = g\phi H$, Together with Various Velocities and Angles

The equation $uc\cos\alpha = g\phi H$, which is the generalized form of the equation (B), can be conveniently represented graphically by means of rectangular coordinate axes as shown in Fig. 4.

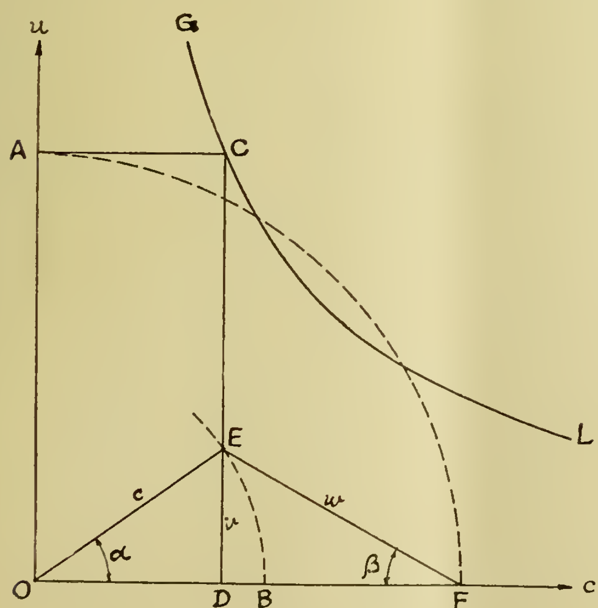


Fig. 4

Let the ordinate be the axis of u, and the abscissa that of c. Take $OA = u$, $OB = c$, and draw a circular arc BE with its centre at O. Draw a line OE from O, making an angle α with the axis of c, and meeting at the point E with

the circular arc BE. Through E draw a vertical line CD, and from A a horizontal line AC so as to form a rectangle OACD. Then, since $OE=OB=c$, OD is equal to $OE\cos\alpha$, which is equal to $c\cos\alpha$. Consequently, the area of the rectangle OACD thus formed, is numerically equal to the value of $uc \cos\alpha$, which is equal to $g\phi H$.

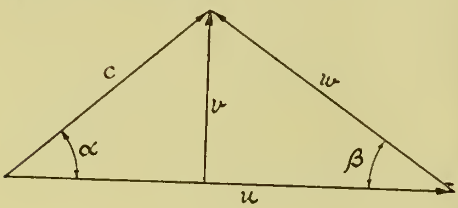


Fig. 5

Now, referring to Fig. 1, the entrance or exit velocity diagram is like the one shown in Fig. 5, and referring to Figs. 4 and 5, we may at once infer that in Fig. 4, DE represents the radial velocity v. Also, if we take OF equal to OA and join F and E, the triangle OFE will be the same velocity diagram as the one given in Fig 5.

The lengths OA and OD, that is u and $c\cos\alpha$, are conveniently adjusted in case of need, giving the same value of $g\phi H$, by drawing a rectangular hyperbola GCL passing through C and containing an area OACD, which is equal to $g\phi H$. Then, as the area of the rectangle formed by the abscissa and its corresponding ordinate of any point on this curve gives the same area of $g\phi H$, the point C will be properly established on this curve having suitable values of $OA=u$ and $OD=c\cos\alpha$.

In Diagrams I and II annexed to this paper, a series of rectangular hyperbolas are drawn on a squared paper giving different values of $g\phi H$. Diagram I will be used for a pump of great head, and Diagram II for one of lesser head. In these diagrams, if we add some radial lines from O giving various angles of α with the horizontal axis, and also add some concentric circular arcs corresponding to arcs like AF and BE (Fig. 4), the diagram in

each case will become more convenient; but to avoid complication I have omitted them at this time.

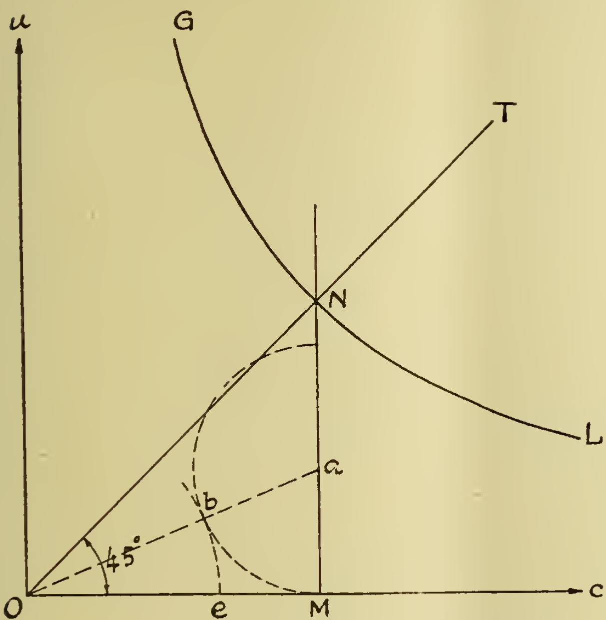


Fig. 6

In Fig. 6, it will be seen that, if OT be a radial line making an angle of 45° with the axes and intersecting at N with the rectangular hyperbola GL, which has a given value of $g\phi H$, and if we let fall a vertical line NM from N meeting the horizontal axis at M, OM will be equal to MN. Hence

$$OM^2 = OM \times MN = g\phi H.$$

Take a point a on the line MN so that Ma equals $\frac{DF}{2}$ (Fig. 4), which is equal to $\frac{w\cos\beta}{2}$ or $\frac{v\cot\beta}{2}$. With the centre at a and the radius aM, draw a circular arc Mb. Again, draw another circular arc bc with its centre at O, touching the circular arc Mb and meeting the horizontal axis at e. Then Oe will be equal to OD (Fig. 4.) which gives the value of $c\cos\alpha$.

To prove this, let Fig. 7 be a diagram composed of those shown in Figs. 4 and 6, with a rectangle OD′C′F touching the hyperbola at a point C′. As OF equals OA, OD′ must be equal to OD. Consequently the rectangle OD′SD will be a square,

S being the point of intersection of DC with D′C′ and also with OT. Now,

$$\text{area}\,OD'C'F = \text{area}\ OD'SD + \text{area}\ DSC'F,$$

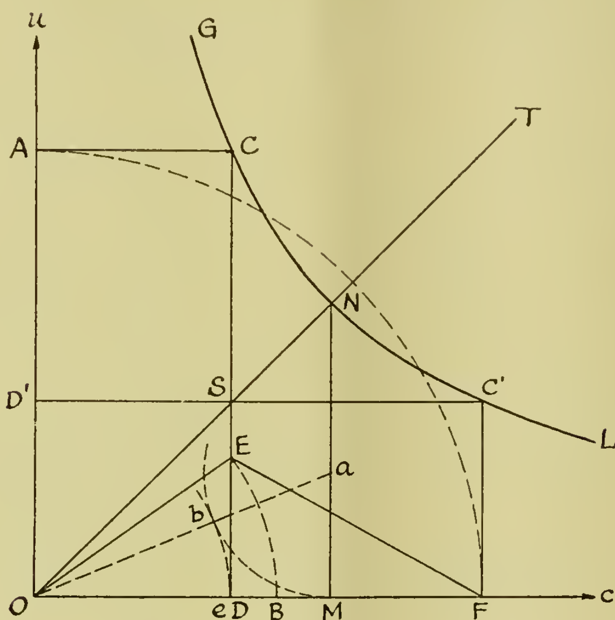


Fig. 7

or since

$$\text{area}\ OD'SD = OD^2,$$

$$\text{area}\ DSC'F = DF \times DS = DF \times OD,$$

and

$$\text{area}\ OD'C'F = \text{area}\,OACD = OM^2,$$

therefore

$$OD^2 + DF \times OD - OM^2 = 0.$$

The two roots of OD out of this quadratic equation will be

$$OD = -\frac{DF}{2} \pm \sqrt{\left(\frac{DF}{2}\right)^2 + OM^2}.$$

But as OD which represents $c\cos\alpha$ is always considered positive in the present diagram, or practically generally so, the quantity with the sign of square-root in the above expression must be positive. It will be,

therefore

$$OD=\sqrt{\left(\frac{DF}{2}\right)^2+OM^2}-\frac{DF}{2},$$

or since $\dfrac{DF}{2}=Ma$

and also $\dfrac{DF}{2}=ab=Oa-Ob=Oa-Oe,$

therefore

$$\begin{aligned}OD&=\sqrt{Ma^2+OM^2}-Oa+Oe,\\&=Oa-Oa+Oe,\end{aligned}$$

and finally we have

$$OD=Oe.$$

Hence the proposition is proved.

Fig. 8 shows a diagram composed of the diagrams shown in Figs. 4 and 6, in order to give a clear idea in the case of the practical use of this diagram. It will easily be seen from Fig. 8, that if we take Mf equal to DE, the radial velocity v, and construct a triangle Mfi, whose one side Mi is the diameter of the circular arc Mb, this triangle will be equal to that of DEF, showing $fi=EF=w$, and the angle $fiM=\beta$.

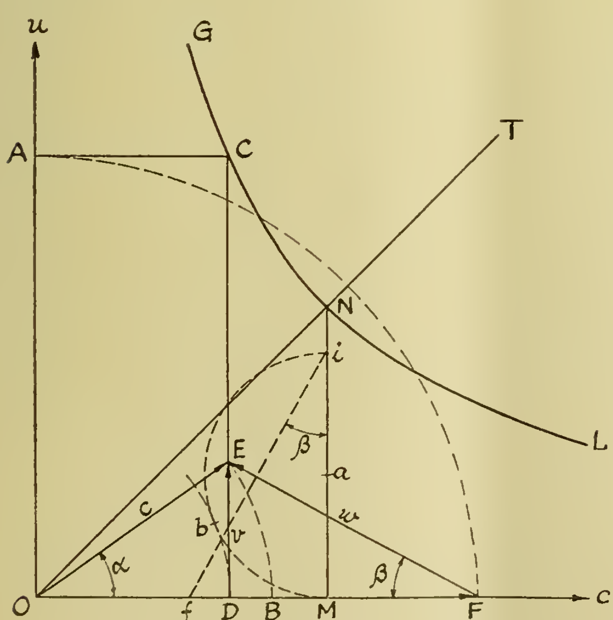


Fig. 8

By the graphical methods shown above, we can always determine all the values or quantities, that is ϕH, u, c, w, v, a, and β, if any three of them be

given. The arrow-heads shown in Fig. 8 give the positive directions of velocities.

Diagrams I and II annexed to this paper serve to the practical use of this diagram.

Determination of the Entrance and Exit Velocity Diagrams under the Best Working Conditions

When the direction of the relative entrance velocity w_1 is the direction of the impeller blade at that point, it will be considered that no shock occurs. When the direction of the absolute exit velocity c_2 from the impeller is the direction of the diffuser blade at its entrance, it will also be considered that no shock occurs. The triangles OPQ and OEF shown in Fig. 9 are the velocity diagrams at the entrance and exit respectively, constructed according to the method explained in the preceding section. Here the absolute entrance velocity is assumed to be radial, that is, $\alpha_1 = 90°$. The directions of QP and OE must be the directions of the

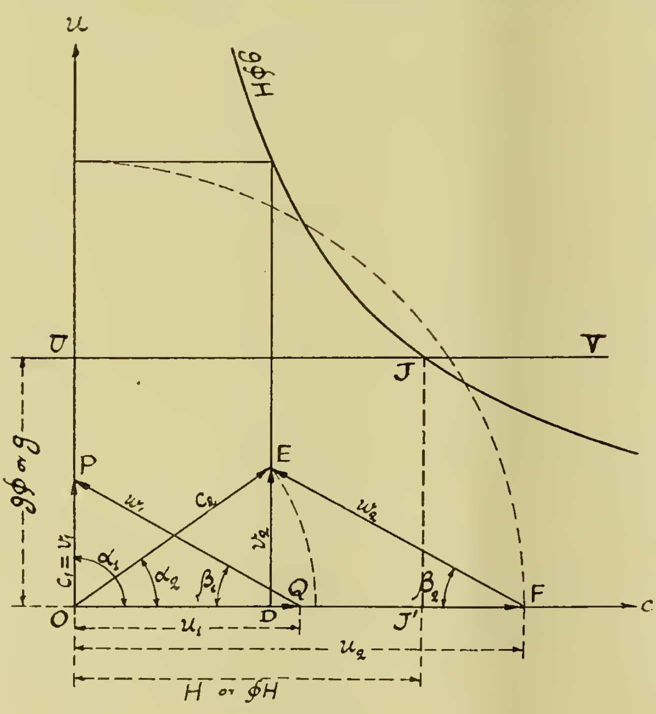


Fig. 9

entrance tips of the impeller and diffuser blades respectively, in

order to fulfil the condition of no shocks there, and to obtain the greatest hydraulic efficiency possible.

In order to find the head H from the value $g\phi H$ directly without calculation, it will be convenient to draw in the diagram a horizontal line UV at such a distance from the horizontal axis that OU equals $g\phi$, meeting in J with the hyperbola giving the required value of $g\phi H$. Then dropping the vertical line JJ′, the horizontal amount UJ or OJ′ will obviously be that value of H. In the same way, if we take OU such that OU equals g, the horizontal amount UJ or OJ′ will be the value of ϕH. In Diagrams I and II, a horizontal line corresponding to the line UV is drawn so that OU equals g or 32·15, so that the horizontal amount will give the value of ϕH.

Velocity Diagrams when there are Additional Shock Losses at the Entrance and Exit

When the quantity of water, head, velocities, or angles are varied for a given pump, the directions of the velocities w_1 and c_2 become non-coincident with the directions of the entrance tips of the impeller and the diffuser respectively, and there arise additional shock losses at those points. The equation (C) is the fundamental hydraulic equation for this case.

For a given pump the angles α_1, α_2, β_1, and β_2 are obviously unchanged in all cases (although it is assumed that α_1 is always constant and equal to 90°). It will be well to assume, now, that the value of ϕ is maintained constant throughout all the changes of practical importance, although strictly it varies with the velocity of the water.

From these standpoints, the discussions relating to a given centrifugal pump can be easily made by means of the diagram.

To illustrate the method, the following two cases are here shown as examples.

Case 1.—Variable Quantity of Flow with Constant Peripheral Velocity.

The quantity of flow directly affects the radial velocities v_1 and v_2. Consequently it will be the same thing, whether we say the change in flow or the change in radial velocities.

When the quantity of flow undergoes a change, the original radial velocities v_1 and v_2 are also changed as shown in Fig. 10

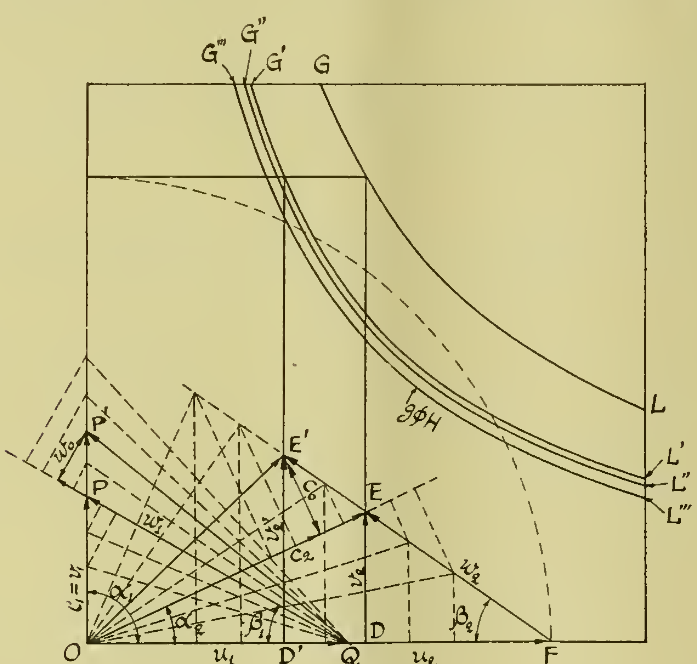


Fig. 10

by $OP'=v'_1=c'_1$ and $D'E'=v'_2$ respectively in the same proportion as the quantity, and the value $g\phi H$ originally represented by a hyperbola GL is now changed to one represented by another hyperbola G'L', the peripheral velocities u_1 and u_2 being unchanged. The direction of $w_1=QP$ at the entrance now changes to another

as shown by QP′, and there occurs unavoidably a loss of head $\frac{w_0^2}{2g}$ (see Fig. 3). At the exit, the direction of c_2=OE also changes to another indicated by OE′, and if the diffuser blades are made movable to fit this direction there will occur no appreciable loss at this section. If the diffuser blades, however, are fixed as is usual in centrifugal pumps, an additional loss of head $\frac{c_0^2}{2g}$ (see Fig. 3) will result.

An additional but unavoidable loss of head for a pump having a fixed diffuser, therefore, will be $\left(\frac{w_0^2}{2g}+\frac{c_0^2}{2g}\right)$ when the quantity of flow undergoes a change, and consequently the value represented by the hyperbola G′L′ will be $\left(g\phi H+\frac{w_0^2}{2}+\frac{c_0^2}{2}\right)$ as can be seen from the equation (C). Hence, finding the values of $\frac{w_0^2}{2}$ and $\frac{c_0^2}{2}$ separately, then adding and then subtracting from the value read by the hyperbola G′L′, the resulting value will be the value of $g\phi H$ for this case, from which the head H may soon be determined by a calculation or by means of the horizontal ne UV as shown in Fig. 9.

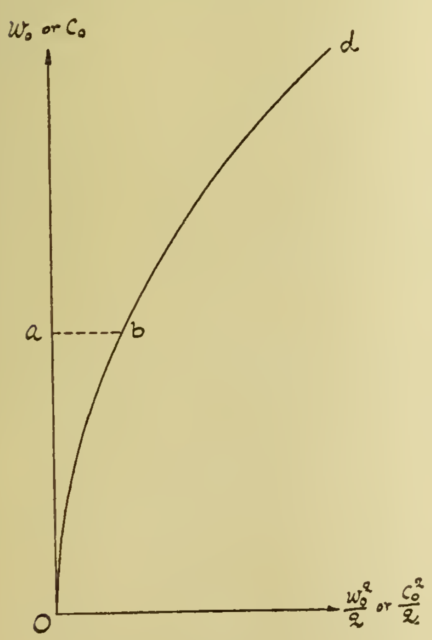


Fig. 11

To find directly the values of $\frac{w_0^2}{2}$ and $\frac{c_0^2}{2}$ for different values of w_0 and c_0, a diagram as shown in Fig. 11 may be conveniently used. In this diagram the vertical axis is the axis of w_0 or c_0 and the horizontal axis is that of $\frac{w_0^2}{2}$ or $\frac{c_0^2}{2}$. By taking the axes thus, the curve Obd will be drawn, by which, for instance, the value of

$\frac{w_0^2}{2}$ is directly read from the abscissa *ab* corresponding to the ordinate Oa, which gives the value of w_0. In Diagrams I and II annexed to this paper, the additional diagram on the left-hand of the main diagram is one drawn according to this principle on the same scale as the main diagram.

By the use of this auxiliary diagram shown in Fig 11, find the value of $\frac{w_0^2}{2}$ and set this value on the left-hand side of the point G′ (Fig. 10) horizontally on the line of the margin of the diagram. This having been done, the new point, say G″ such as $G'G'' = \frac{w_0^2}{2}$, will be settled, and the value represented by the hyperbola G″L″ passing through G″ will be $\left(g\phi H + \frac{c_0^2}{2}\right)$. Then again finding the value of $\frac{c_0^2}{2}$ by the use of the auxiliary diagram and setting this value on the left-hand side of the point G″ horizontally as above explained, getting the new point G‴, the value represented by the hyperbola G‴L‴ passing through G‴ will be the required value of $g\phi H$, corresponding to the flow in question. Now dividing this value by $g\phi$ which is assumed to be a constant value for a given pump, the new head H will be immediately obtained. It is hardly necessary to add that this dividing is done directly by using the horizontal line UV as shown in Fig. 9.

Velocities like w_0 and c_0, which are entirely lost in forming eddies, being all perpendicular to the directions QP and OE respectively, are respectively parallel as shown by the dotted lines in Fig. 10.

Case 2.—Variable Peripheral Velocities, with Constant Quantity of Flow.

When the peripheral velocity u_2 is changed, u_1 is also changed proportionally. When these velocities are changed while the quantity of flow or the radial velocities remain constant, the

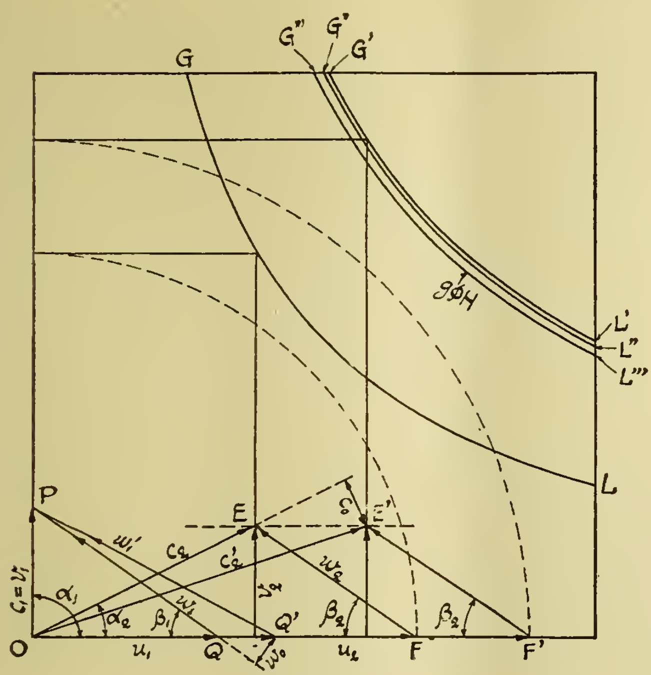


Fig. 12

original entrance velocity w_1 and the exit velocity c_2 are changed as shown in Fig. 12 by $Q'P = w_1'$ and $OE' = c_2'$, and the value $g\phi H$, originally represented by the hyperbola GL, is now changed to that represented by the hyperbola G'L'. At the entrance the direction of Q'P now becomes non-coincident with that of the entrance tip of the impeller blade, and the loss of head $\frac{w_0^2}{2g}$ results. Similarly at the exit another loss of head $\frac{c_0^2}{2g}$ occurs, if the diffuser blades are not made movable so they can be set in the direction OE'. Consequently the resulting value of $g\phi H$ in this case will be that represented by the hyperbola G'L' less by the amount $\left(\frac{w_0^2}{2}+\frac{c_0^2}{2}\right)$, and by applying the same processes explained in the preceding case, it will be seen that the value represented by the hyperbola G'''L''' is that required.

Conditions for Driving a Given Pump with Variable Speeds and with Greatest Efficiency

Let OEF and OPQ in Fig. 13, be the original velocity diagrams with which the given pump is designed. Take a point E'

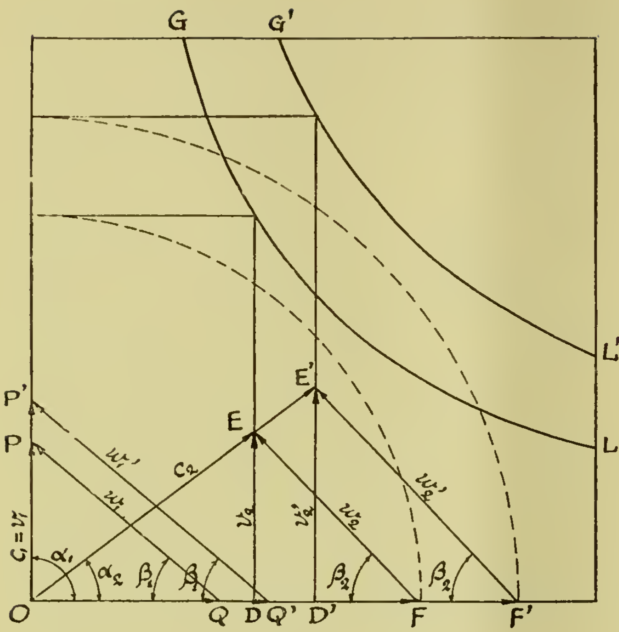


Fig. 13

on the line OE or on its prolongation and draw a line E'F' parallel to EF, the perpendicular from E' to OF' being E'D'. Then if the peripheral and radial velocities are both changed as represented by $OF'=u_2'$ and $D'E'=v_2'$ respectively, so as to form a triangle OE'F' as its transformed exit velocity diagram, there will occur no additional shock loss by this transformation at the exit from the impeller and at the entrance into the diffuser.

When the peripheral velocity u_2 at the exit is changed to another like u_2', the periperal velocity $OQ=u_1$ at the entrance edge will also be changed to another like $OQ'=u_1'$ in the same proportion as u_2' is to u_2, and at the same time the radial entrance velocity $OP=v_1=c_1$, will now pass into another like $OP'=v_1'=c_1'$ in the same proportion as v_2' is to v_2.

Now from Fig. 13 it will easily be seen that the following proportion holds, viz.,

$$\frac{OF'}{OF}=\frac{OE'}{OE}=\frac{D'E'}{DE},$$

that is, $$\frac{u_2'}{u_2}=\frac{v_2'}{v_2}.$$

But, as shown above, as there always exist the relations

$$\frac{u_2'}{u_2}=\frac{u_1'}{u_1} \quad \text{and} \quad \frac{v_2'}{v_2}=\frac{v_1'}{v}$$

we shall have the following relation

$$\frac{u_1'}{u_1}=\frac{v_1'}{v_1}$$

that is, $$\frac{\mathrm{OQ}'}{\mathrm{OQ}}=\frac{\mathrm{OP}'}{\mathrm{OP}}.$$

This shows us that the lines Q′P′ and QP are parallel to each other, or in other words the relative entrance velocities are always parallel. It will, therefore, also be seen that there occurs no additional loss at the entrance edge.

The above reasoning shows that, if the peripheral and radial velocities at the exit are changed so as to form a triangle OE′F′, there do not occur any additional losses at all and the value $g\phi H$ in this transformed state is represented directly by the hyperbola G′L′. This indicates the proper method for driving a pump with the greatest efficiency in case of variable head and variable flow.

Graphical Solution of the Pressure Difference existing in the Clearance Space

The fundamental equation (E) is

$$h_c=\phi H-\frac{c_2^2}{2g}.$$

This may be written in another form, thus

$$h_c=\frac{1}{g}\left(g\phi H-\frac{c_2^2}{2}\right).$$

From this equation the pressure difference h_c in terms of the head of water in feet, may be easily determined by means of a graphical solution by using the same diagrams as those used in the foregoing sections.

In Fig. 14, let OEF and OPQ be the exit and entrance velocity diagrams respectively, and the corresponding value of $g\phi H$ be represented by the hyperbola GL. Find the value of $\frac{c_2^2}{2}$ by using the auxiliary diagram as shown in Fig. 11, whose practical diagram is given on the left of Diagrams I and II annexed to this paper, and set this value on the main diagram so that GG′ equals $\frac{c_2^2}{2}$ getting G′ on the left-hand side of the point G. Then the value represented by the hyperbola G′L′ passing through G′ will be $\left(g\phi H-\frac{c_2^2}{2}\right)$. Consequently, if the point J be the intersection of this hyperbola with the horizontal line UV which is drawn parallel to the horizontal axis and distant from it by the amount g or $32{\cdot}15$, the horizontal amount UJ or OJ′ will evidently be $\frac{1}{g}\left(g\phi H-\frac{c_2^2}{2}\right)$ which gives the value of the pressure difference h_c.

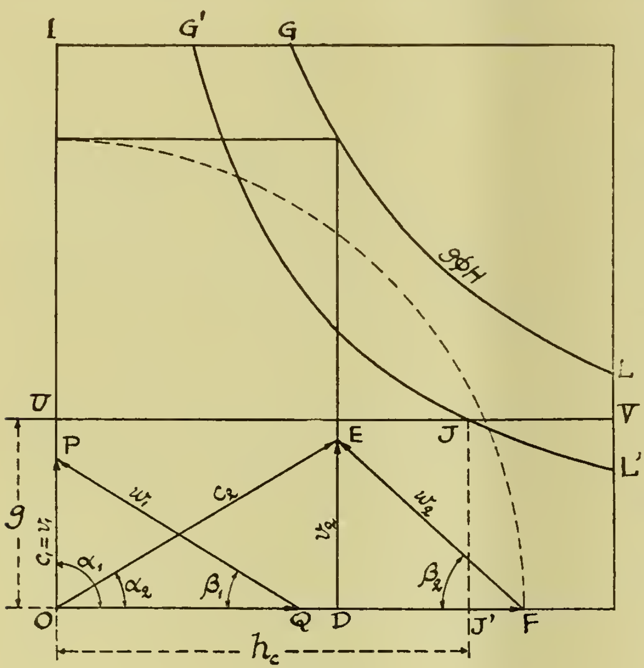


Fig. 14

If we know the value of h_c, the loss due to leakage may be calculated by the formula (F), and the amount of the axial thrust caused by the impeller may be determined, if necessary, by multiplying h_c by the area of the impeller wall exposed to the clearance and a certain coefficient depending on the whirling due to the viscosity of the water.

If the value of $\frac{c_2^2}{2}$ or GG′ be equal to or even greater than GI (Fig. 14), the point J will coincide with U or fall on the left of U; this means respectively that the pressure difference between the entrance and exit is zero or even negative, which corresponds to a turbine of the impulse type as given by the equation (G). Pumps of the impulse type are rarely manufactured in practice, since in this type the velocities of flow through the impeller channel being necessarily great, the hydraulic losses become serious, though the loss of flow due to leakage may be absolutely avoided.

For a given pump of a given revolution, the loss due to leakage increases as the absolute exit velocity c_2 or the exit angle of impeller β_2 decreases, as will be seen from the diagram.

Entrance and Exit Velocity Diagrams when the Absolute Entrance Velocity is not Radial

Let the hyperbolas GL and G_1L_1 in Fig. 15, be those corresponding to the exit and entrance velocity diagrams OEF and OPQ respectively, in a case where the absolute entrance velocity c_1 is not radial. Then the difference of values represented by these hyperbolas will be the value of $g\phi H$ in this case, as is obvious from the fundamental equation (A).

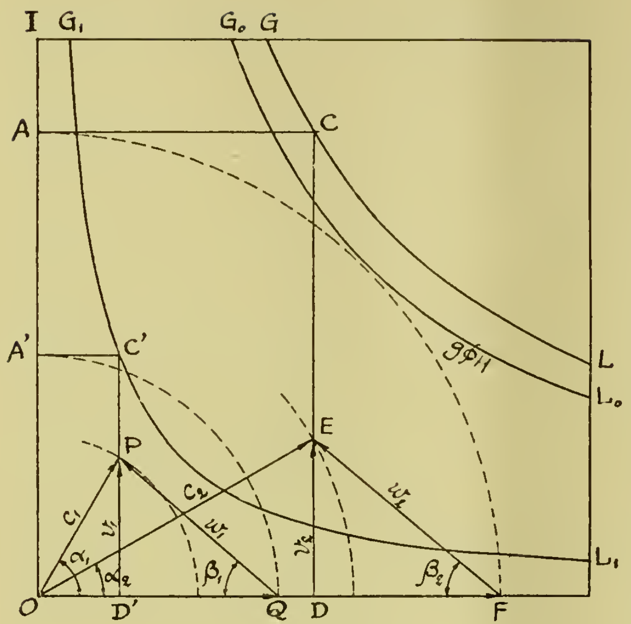


Fig. 15

Various discussions as explained previously at length under the assumption of $\alpha_1=90°$, that is, that the absolute entrance velocity is radial, are equally applicable if we substitute the hyperbola G_oL_o instead of GL. The hyperbola G_oL_o is so taken that its value is equal to $g\phi H$ or the difference of the values of the hyperbolas GL and G_1L_1. This hyperbola G_oL_o may be directly established without calculation by taking GG_o equal G_1I, in which I is the apex of the right-angled corner of the diagram.

Numerical Example

Let us consider an example in which a three-stage centrifugal pump has to lift 160 cub. ft. of water per min. through 350 ft. of pipe against a head of 300 ft., using an electric motor running at 1,000 revolutions per min.

Here the total required head H is 300 ft. Assuming now the total hydraulic loss of head through the 350 ft. pipe together with the pump proper to be 30 ft., the theoretical total head through which the pump strives to lift water will be 330 ft. The pump being some three-stage type, if we consider the theoretical head of

330 ft. to be equally distributed and borne among these three impellers, which assumption is practically approximately true, each impeller has to be worked under the theoretical head of 110 ft. This head of 110 ft. is the value of ϕH used in the previous discussions. Consequently, by taking the value of g at 32·15 ft. per sec. per sec., we have

$$g\phi H = 32{\cdot}15 \times 110 = 3{,}540.$$

Since $\phi H = 110$ and $H = 100$ for each impeller, the value of ϕ for this example will be

$$\phi = \frac{\phi H}{H} = \frac{110}{100} = 1{\cdot}1,$$

and the maximum efficiency obtainable by this pump will be

$$\eta = \frac{1}{\phi} = \frac{1}{1{\cdot}1} = 0{\cdot}91 \quad \text{or} \quad 91\%.$$

The quantity of water 160 cub. ft. per min. is evidently the required quantity to be lifted up to the required place ; but the quantity through the impeller must be more than this, because some water undoubtedly goes back through the clearance space between the impeller and the casing in addition to the loss of water through stuffing boxes and various joints. Assuming the total amount of the lost water to be 5% of the required quantity, the actual quantity passing through the impeller channel will be

$$1{\cdot}05 \times 160 = 168^{\text{cub. ft.}}/_{\text{min.}} \quad \text{or} \quad 2{\cdot}8^{\text{cub. ft.}}/_{\text{sec.}}$$

From these conditions and referring the impeller of 1,000 revolutions per min. or 16·7 revolutions per sec., we shall generally be able to determine suitable values of the velocities v_1, v_2, u_1 and u_2 as in the usual design of a centrifugal pump. Let us assume that suitable values of them are as follows.—

Radial velocities, $v_1 = 9^{\text{ft.}}/_{\text{sec.}}$ and $v_2 = 7^{\text{ft.}}/_{\text{sec.}}$

Peripheral velocities, $u_1 = 36^{\text{ft.}}/_{\text{sec.}}$ and $u_2 = 64^{\text{ft.}}/_{\text{sec.}}$

Now the value of $g\phi H$ being 3,540 and assuming $\alpha_1 = 90°$, we can directly determine, by using Diagram I, the exit velocity

diagram knowing the three values $g\phi H$, v_2 and u_2, and the entrance velocity diagram by giving v_1 and u_1. Fig. 16 shows these diagrams directly obtained from Diagram I, from which we read the following results :—

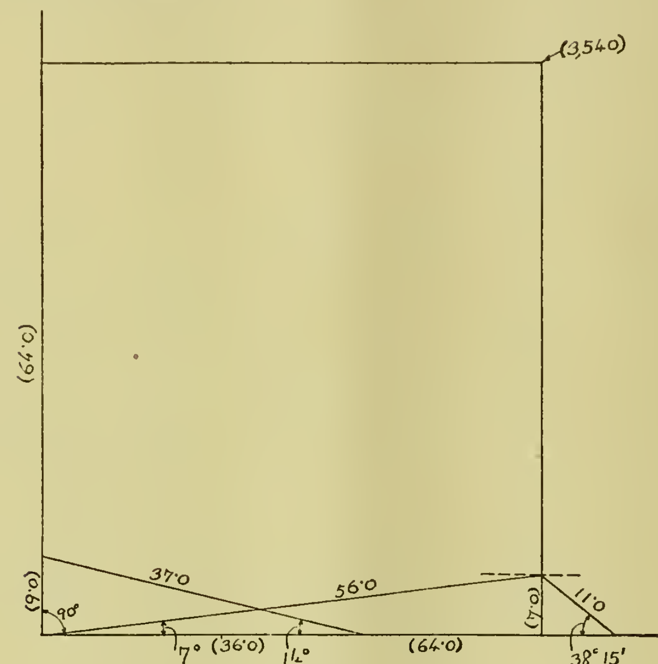


Fig. 16

Absolute entrance velocity, $c_1 = v_1 = 9^{\text{ft.}}/_{\text{sec.}}$

,, exit ,, $c_2 = 56^{\text{ft.}}/_{\text{sec.}}$

Relative entrance velocity, $w_1 = 37^{\text{ft.}}/_{\text{sec.}}$

,, exit ,, $w_2 = 11^{\text{ft.}}/_{\text{sec.}}$

Entrance angle of impeller, $\beta_1 = 14^\circ$

Exit ,, ,, $\beta_2 = 38^\circ\ 15'$

Entrance angle of diffuser, $\alpha_2 = 7^\circ$

Pressure difference in the clearance space, $h_c = 62^{\text{ft.}}$

To use Diagrams I and II, set a tracing paper on it and draw your required diagrams on that paper.

The variations in H, h_c, the hydraulic efficiency η, and

several other quantities or values with respect to the variations in the quantity of flow or the radial outlet velocity v_2 are shown by curves in Fig. 17. These curves have been plotted by reading

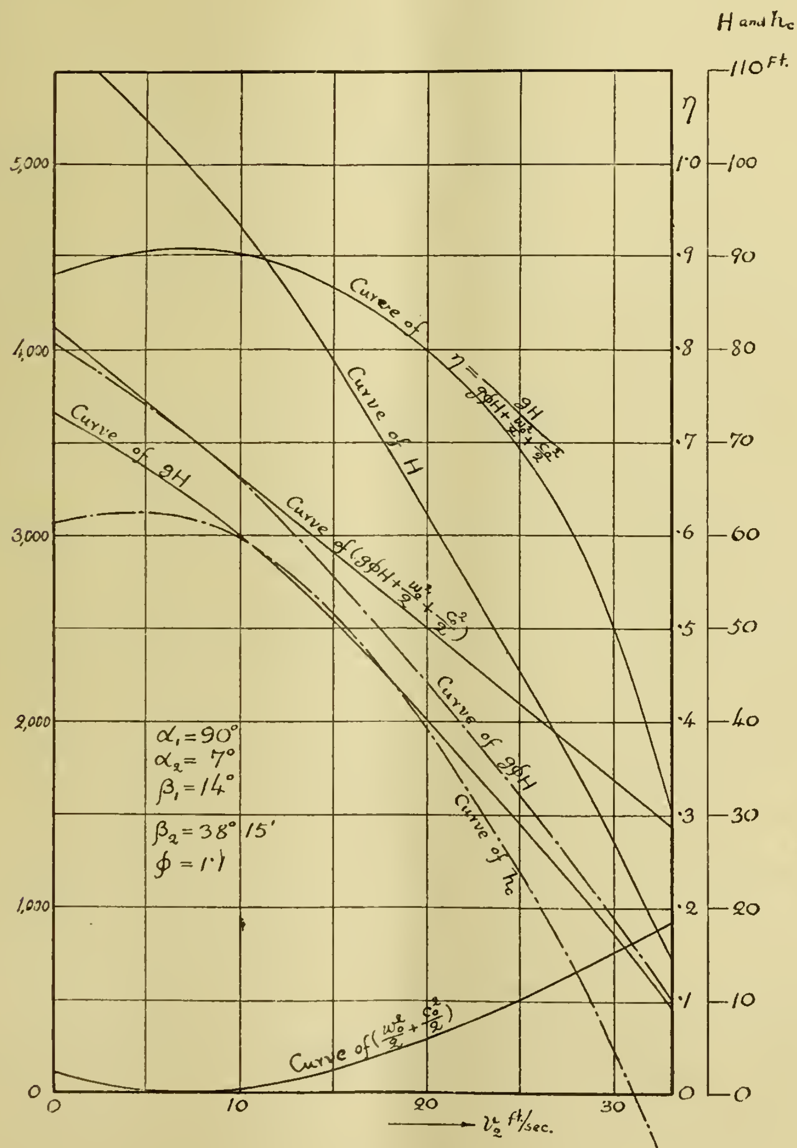


Fig. 17

the corresponding values direct on Diagram I which I have used to solve the above example, and the process applied here is that explained with Fig. 10. The relations in the variations in various quantities or values may easily be obtained in a similar manner by the convenient use of Diagram I or II.

Specific Speed

Velocities in hydraulic machines are sometimes conveniently expressed in terms of specific speeds. The ratio of a velocity to that due to the total head is called the specific speed of that velocity. Consequently the specific speed is a mere number always less than unity.

If the total head H is entirely converted into velocity, it will acquire a velocity equal to $\sqrt{2gH}$. So, the specific speeds of velocities u, c, and v, for instance, will be respectively $\frac{u}{\sqrt{2gH}}$, $\frac{c}{\sqrt{2gH}}$, and $\frac{v}{\sqrt{2gH}}$. Let the specific speed be represented by a notation with a bar on it, as for instance $\bar{u}$, $\bar{c}$ and $\bar{v}$ be the specific speeds of velocities u, c, and v respectively, and be substituted instead of velocity. Then the fundamental equations (A), (B), (C), (D), (E), and (G) will respectively transform into

$$u_2 \bar{c}_2 \cos \alpha_2 - u_1 \bar{c}_1 \cos \alpha_1 = \frac{\phi}{2} \quad \text{..........................} (A')$$

$$u_2 \bar{c}_2 \cos \alpha_2 = \frac{\phi}{2} \quad \text{..............................} (B')$$

$$\bar{u}_2 \bar{c}_2 \cos \alpha_2 = \frac{\phi}{2} + \frac{w_0^2}{2} + \frac{\bar{c}_0^2}{2} \quad \text{.........................} (C')$$

$$\eta = \frac{1}{\phi + \bar{w}_0^2 + \bar{c}_0^2} \quad \text{..........................} (D')$$

$$\left.\begin{aligned} h_c &= (\phi - \bar{c}_2^2)\,H \\ \text{or} \qquad h_c &= \left(\frac{\phi}{2} - \frac{\bar{c}_2^2}{2}\right) 2H \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (E')$$

$$\phi \gtreqless \bar{c}_2^2 \quad \cdots\cdots\cdots\cdots (G')$$

From the equation (B′), it will at once be seen that the application of the specific speed to the present diagram can soon be done, which is sometimes convenient. Here the horizontal and the vertical axes are the axes of $\bar{c}$ and $\bar{u}$, and the rectangular hyperbolas give the values of $\frac{\phi}{2}$ simply. The methods of applications and the uses of such a diagram are self-evident from the above equations and referring the foregoing explanations, and consequently I will repeat no further.

The chief reason that the diagram in terms of specific speeds is sometimes convenient, is that the specific speed is a mere number ranging from 0 to 1, and therefore the values written in the diagram are comparatively simplified and also one diagram serves always for all speeds.

To Draw a Series of Rectangular Hyperbolas

There is a series of rectangular hyperbolas in the diagram here introduced, and consequently it may not be unnecessary to add a few notes on the drawing of these hyperbolas.

The most common and convenient way to draw a rectangular hyperbola having a given value of $g\phi H$ is as shown in Fig. 18. Let OX, OY be the horizontal and vertical axes respectively, and construct a rectangle OXMY. These axes are the axes of velocities and they must be drawn with the same scale. Draw a

vertical line AB so that the product of OY and OA equals $g\phi H$ and thus fix a point B on the line YM. Draw any number of radial lines from O such as OB′, OB″, meeting AB in a, b respectively. Then the points a', b' which are the intersections of the horizontal and vertical lines drawn from a, b and B′, B″ respectively will be the points determining the hyperbola $Ba'b'C$ having the given value of $g\phi H$.

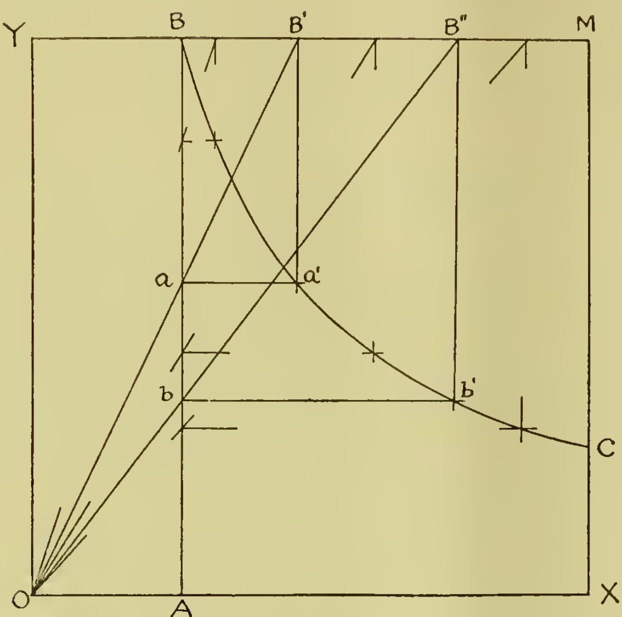


Fig. 18

Now, in Fig. 19, let BC be a rectangular hyperbola obtained by such a method as above explained, then the series of such hyperbolas will be easily drawn by the method following.—

Draw a number of horizontal (or vertical) lines such as ll', mm', nn' cutting the hyperbola BC at 4, 4, 4. Divide the left-hand portions $l4$, $m4$, $n4$ into an equal number of equal divisions respectively, and proceed to divide the right-hand portions equally with the same pitches as the left-hand divisions. Then the curves connecting these corresponding divisions will determine the series of hyperbolas required. The curves shown in broken lines are such hyperbolas.

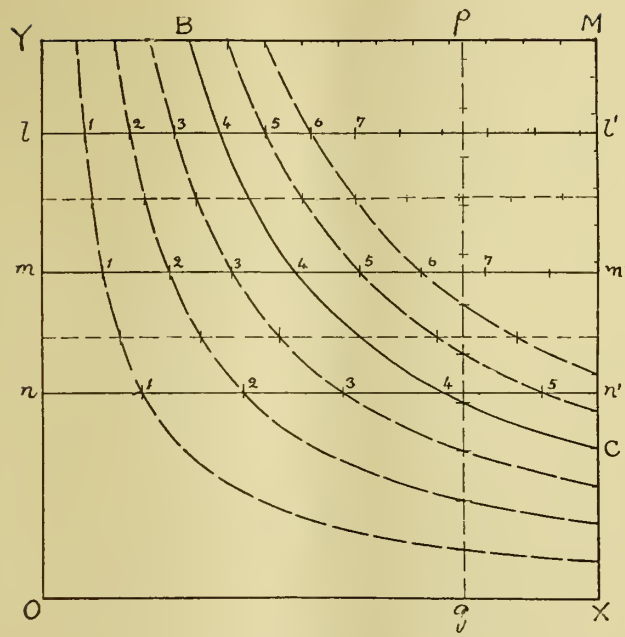


Fig. 19

If the value of the hyperbola BC is $g\phi H$, or in general K, and the number of divisions on the left of BC is N, the values of these hyperbolas counting from the left will clearly be $\frac{K}{N}$, $\frac{2K}{N}$, $\frac{3K}{N}$, $\frac{4K}{N}$, &c. For instance, if the number of divisions on the left of BC is four as shown in Fig. 19, the values of the hyperbolas 111, 222, 333, 444, &c. will be respectively $\frac{K}{4}$, $\frac{K}{2}$, $\frac{3K}{4}$, K, $\frac{5K}{4}$, &c.

The above method is equally applicable if we use vertical lines such as pq, instead of horizontal lines.

Sendai, June, 1914.

Dec. 24th, 1914 Vol. V., No. 10

東京帝國大學

工科大學紀要

第五冊第十號

大正三年十二月

JOURNAL

OF THE

COLLEGE OF ENGINEERING,

IMPERIAL UNIVERSITY OF TOKYO

TOKYO
Published by the University
3RD. YEAR OF TAISYO
(1914)

EDITING COMMITTEE

All communications relating to this Journal should be addressed to the Chairman of the Committee.

AGENT FOR THE SALE OF THIS JOURNAL.

Z. P. MARUYA & Co., Ltd.

Tôri Sanchome, Nihonbashi,

TOKYO

大正三年十二月二十一日印刷
大正三年十二月二十四日發行

編纂兼發行者　東京帝國大學

東京市日本橋區兜町二番地
印刷者　神谷岩次郎

東京市日本橋區兜町二番地
印刷所　東京印刷株式會社

東京市日本橋區通三丁目十四番地
賣捌所　丸善株式會社書店

Contents of Latest Publications.

Vol. V.

Page.

1. On Vibration of steamers. By SEINEN YOKOTA, *Kōgakuhakushi* 1
2. The Action of Acids in the Enzymic Decomposition of Oil by Castor Seeds. By YOSHIO TANAKA, *Kōgakushi* 25
3. The Characteristics of Synchronous Motors. By M. SIBUSAWA, *Kōgakushi.* 43
   The Protective Action of the Ground Wire. By H. HŌ, *Kōgakuhakushi* ... 93
4. The Preparation of "Lipase Powder" acting in Neutral Medium and Its Technical Application. By YOSHIO TANAKA, *Kōgakuhakushi*... 125
   Influence of the Products of Change on the Action of Lipase. By YOSHIO TANAKA, *Kōgakuhakushi* 137
   Influence of Some Neutral Salts, Nitrogenous Matters, and Castor Seed Extract on Lipase. By YOSHIO TANAKA, *Kōgakuhakushi* 142
   The Action of Lipase on Oxidised and Polymerised Oils. By YOSHIO TANAKA, *Kōgakuhakushi* 152
5. On the Determination of Actual Stresses in a Metallic Bridge. By I. HIROI, *Kōgakuhakushi* 163
6. On Shearing Stress in a Ship's Structure. By K. SUYEHIRO, *Kōgakuhakushi.* 181
7. On the Fineness of the 800 Standard Silver Bar. By O. YAMAGATA, *Rigakushi* 191
8. The Relation between the Horse Power and the Weight of an Engine. By Prof. A. INOKUTY. 213
   Column of Uniform Strength. By Prof. A. INOKUTY. 223
9. Die transversale Festigkeit der Drahtkanonen. By M. ÔKÔCHI, *Kōgakuhakushi*.... 245

These publications are issued at irregular intervals. When about 300 pages are reached, they are formed into one volume.

PRINTED BY THE TOKYO PRINTING CO. L'D.

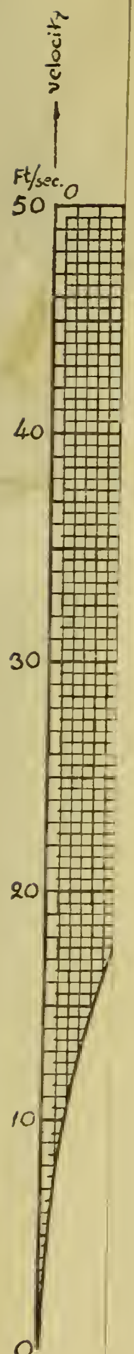

velocity
Ft/sec.
0
50
40
30
20
10
0

DIAGRAM II

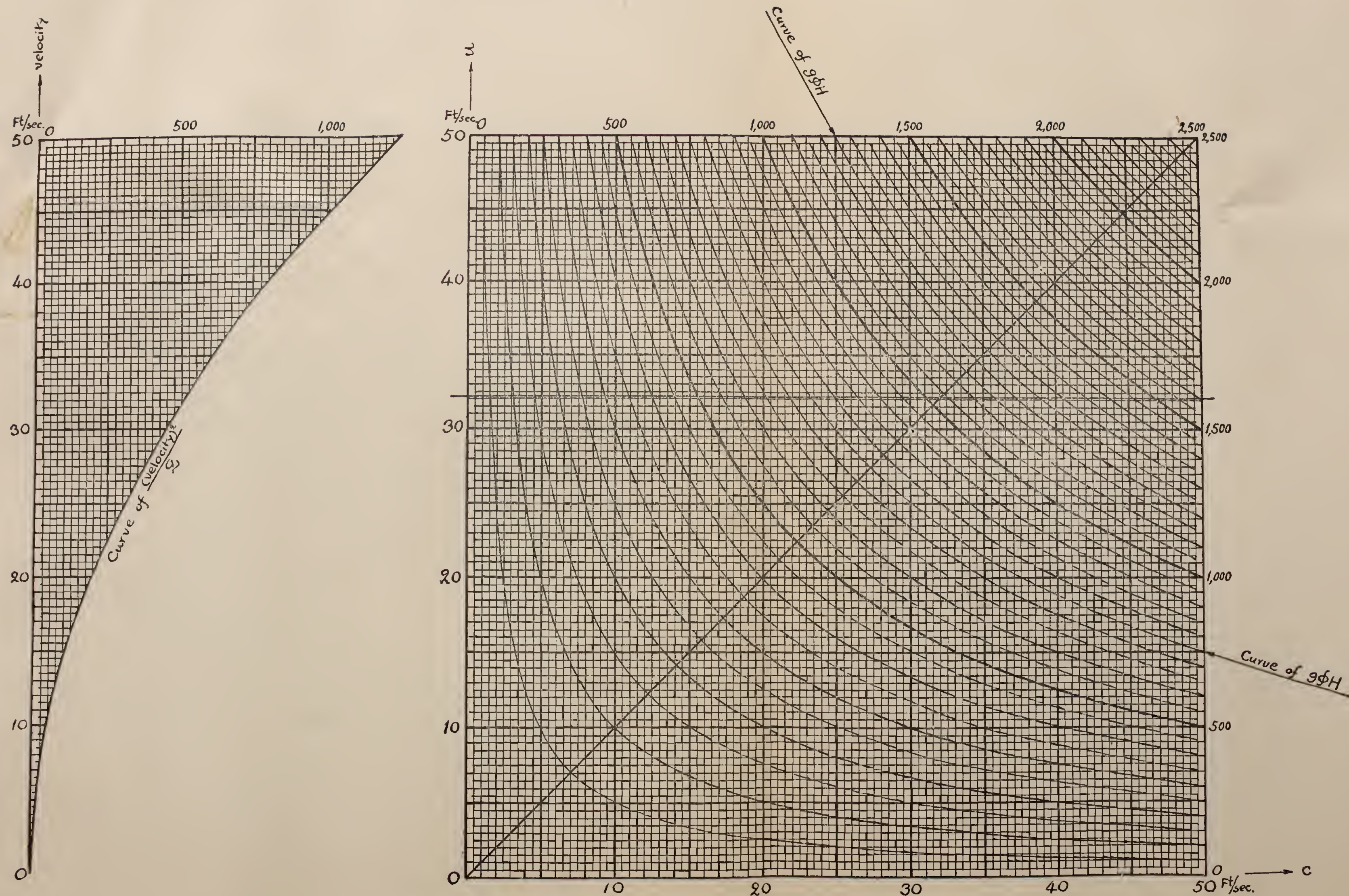
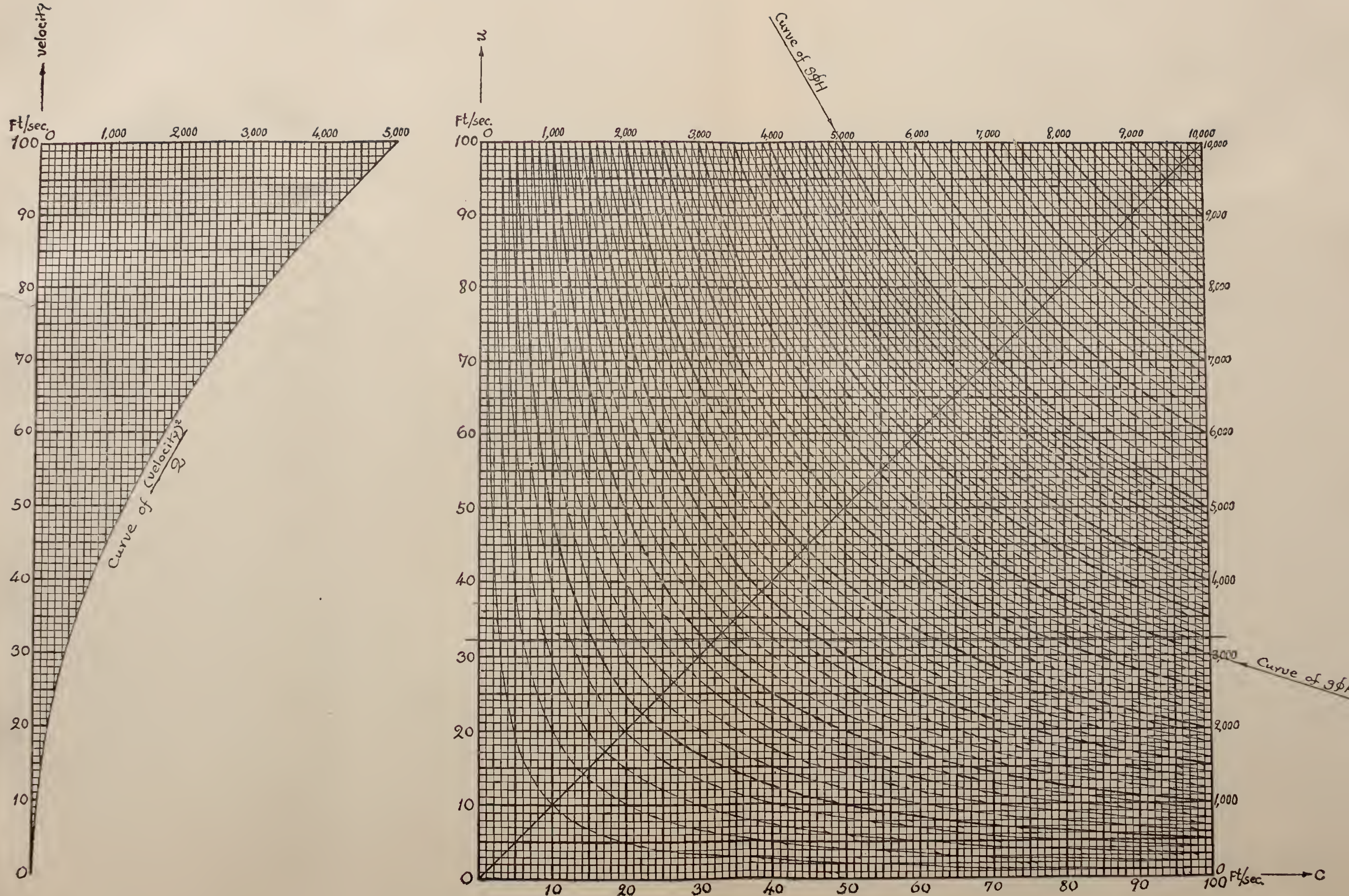

velocity
Ft/sec.
500
1,000
50
40
30
20
10
0
Curve of (velocity)²/2g
u
Ft/sec.
500
1,000
1,500
2,000
2,500
50
40
30
20
10
0
2,500
2,000
1,500
1,000
500
0
Curve of gφH
Curve of gφH
10
20
30
40
50
Ft/sec.
c

DIAGRAM I



velocity
Ft/sec.
0
1,000
2,000
3,000
4,000
5,000
100
90
80
70
60
50
40
30
20
10
0
Curve of (velocity)²/2
u
Ft/sec.
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
10,000
Curve of gφH
100
90
80
70
60
50
40
30
20
10
0
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
Curve of gφH
10
20
30
40
50
60
70
80
90
100
Ft/sec.
C